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# A Tale of Fire-Sales and Liquidity Hoarding

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# A Tale of Fire-Sales and Liquidity Hoarding*

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**Abstract**

We extend the analysis of the interbank market model of Gale and Yorulmazer (2013) by introducing randomized trading (lotteries). In contrast to Gale and Yorulmazer, we find that fire-sale asset prices are efficient and that no liquidity hoarding occurs in equilibrium. While Gale and Yorulmazer find that the market provides insufficient liquidity, we find that it provides too much liquidity. We also show how to decentralize the efficient lottery mechanism.

JEL Classification: G12, G21, G33.

Keywords: Fire-sales, lotteries, liquidity hoarding, interbank markets, indivisibility.

# 1 Introduction

Liquidity hoarding and fire-sales are seemingly closely related phenomena. Consider an economy populated by many banks, which face random liquidity needs. Suppose further that banks anticipate that tomorrow many banks will face liquidity shortages and in order to avoid default are forced to sell assets at fire-sale prices. If assets are cheaper tomorrow than today, liquid banks prefer to hoard liquidity today. This is inefficient, since banks that are illiquid today have no access to liquidity and default. This is essentially the tale of fire-sales and liquidity hoarding developed in the paper by Gale and Yorulmazer (2013, henceforth GY).

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Swiss National Bank.

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The crucial question with this tale is why today's defaulting banks are not selling their assets at fire-sale prices in order to prevent default. In GY, the reason is that the price of assets, $q$, cannot always adjust to market conditions. This is due to two assumptions: First, GY assume that in period 1 the portfolio of a liquid bank consist of one unit of the indivisible cash and one unit of the divisible asset, and the portfolio of an illiquid bank consist of one unit of the divisible asset only. Second, they consider a trading mechanism, where an indivisible unit of cash is exchanged for $x \leq 1$ units of the assets. Hence, the asset price cannot fall below one in period 1, since it is constrained to satisfy $q = 1/x \geq 1$. In contrast, the expected asset price in period 2 can fall below one, in which case it is optimal for liquid banks to hoard cash in period 1.[1]

In this paper, we extend the interbank market model of GY by introducing randomized trading (lotteries) and we derive a lottery mechanism that restores efficiency. The intuition for our result is straightforward. With lotteries, the asset price is $q = \widetilde{\tau}/x$, where $\widetilde{\tau}$ is the probability that the indivisible unit of cash is exchanged for $x \leq 1$ units of assets. Apparently, with lotteries there is no lower bound on the asset price, since for $\widetilde{\tau} < x$, we have $q < 1$. We show that under the efficient lottery mechanism, if banks anticipate a fire-sale tomorrow, there is a fire-sale today, which eliminates the incentives to inefficiently hoard liquidity. Thus, we show that fire-sales are efficient and restore efficiency in the GY framework.

After characterizing the efficient lottery mechanism, we show how the efficient allocation can be decentralized. In particular, we show that profit maximizing financial intermediaries endogenously emerge to supply the two lotteries that are needed to restore efficiency. The first lottery is a lottery $\ell$, which pays out one unit of cash with probability $\tau$. It is sold at price $x$, where $x$ is the quantity of assets to be paid for one unit of lottery $\ell$. In equilibrium, this lottery is acquired by banks that need cash to avoid default. The second lottery is a lottery $\widetilde{\ell}$. It pays out $\tilde{x}$ units of the asset against one unit of cash which has to be paid with probability $\tilde{\tau}$. This second lottery is acquired by banks that have no current need for their cash.[2]

It is important to note that the hoarding inefficiency in GY does not arise from the assumption of indivisible cash. Rather it arises from the fact that there is a discontinuity in payoffs when a bank defaults. This discontinuity introduces a nonconvexity into the GY environment. As already discussed in GY, it is not sufficient to make cash divisible to restore efficiency. Rather, as shown by us, randomized trading is needed to restore efficiency.

[1] A price below one in period 2 is interpreted by GY as a fire-sale price.

[2] A manifestation of our lottery mechanism in reality is 'gambling for resurrection' (see, for instance, Dewatripont and Tirole, 1994). A distressed bank (the cash demander in our case), has an incentive to gamble if there is a chance to survive. In the context of our model, 'gambling for resurrection' means to use the remaining assets to make risky investments; i.e., to buy 'lotteries'. Note, though, that in contrast to this literature, gambling for resurrection is efficient in our paper.

It is well known from theory that in economic environments with nonconvexities, agents can often do better using randomized rather than deterministic trading mechanisms. Analyses of nonconvexities and lotteries include Prescott and Townsend (1984a, 1984b), Rogerson (1988), Diamond (1990), Shell and Wright (1993), and Chatterjee and Corbae (1995). These authors typically justify the use of lotteries because of welfare considerations as expressed by Rogerson (1988, p. 11): "One of the reasons for adding lotteries to the consumption set was the potential gain in welfare. In essence, making labor indivisible creates a barrier to trade and the introduction of lotteries is one way to overcome part of this barrier." The literature on fire-sale prices includes Allen and Gale (1998) who show that fire-sale prices can also serve a useful role by encouraging agents to hold liquid assets in the first place. Our story, however, is different since it focuses on the role of fire-sales in reallocating assets away from those who are insolvent to those who are solvent.

Models of monetary exchange that allow for randomized trading include Berentsen, Molico and Wright (2002) and Berentsen and Rocheteau (2002). In a model where agents meet pairwise at random, Berentsen and Rocheteau demonstrate that indivisible money generates a *no-trade* inefficiency, where no trade takes place in bilateral meetings even though it would be socially efficient to trade. They show that agents prefer not to trade if they expect to receive more goods for the indivisible unit of cash tomorrow. This *no-trade* inefficiency is closely related to the *hoarding* inefficiency in GY, since liquid banks prefer not to trade if they expect to receive more assets for the indivisible unit of cash tomorrow. Berentsen and Rocheteau also show that randomized trading eliminates the *no-trade* inefficiency. Along the same line, we show that lotteries eliminate the *hoarding* inefficiency in GY, too.

Our paper does not refute the phenomenon of liquidity hoarding, since empirical evidence suggests that liquidity hoarding was indeed a problem during the latest financial crisis.[3] However, we do not think that the pricing frictions, emphasized by GY, were at the origin of this phenomenon. Rather, we speculate that private information problems generated severe counterparty risks (see, for instance, Afonso, Kovner and Schoar; 2011 or Alvarez and Barlevy; 2014).

The remainder of this paper is structured as follows. Section 2 summarizes the GY environment. Section 3 studies the banks' decisions. Section 4 derives an efficient incentive-feasible mechanism for the GY environment. Section 5 describes the portfolio choice of banks in the initial period. Section 6 shows how the efficient allocation can be decentralized. Section 7 relates our results to GY. Finally, Section 8 concludes.

[3] See, for instance, Heider, Hoerova and Holthausen (2009), Acharya and Merrouche (2013) or Ashcraft, McAndrews and Skeie (2011).

## 2 Environment

Our environment is identical to GY. Nevertheless, for ease of reference we summarize it below.

The economy is populated by a continuum of identical and risk neutral utility-maximizing banks indexed by $i \in [0,1]$. There are four periods ($t = 0,1,2,3$) and two assets: An indivisible asset (henceforth called *cash*) which has a return of one unit of consumption at date 3 and an divisible asset (henceforth called *asset*) which has a return of $R > 1$ units of consumption at date 3. Both assets are storable.

In period 0, banks have an initial portfolio consisting of one unit of the asset and one unit of cash $\{1,1\}$. A bank's utility function is defined as follows:

$$U(c_0, c_3) = \rho c_0 + c_3. \tag{1}$$

The interpretation of the utility function is as follows: banks can either consume the indivisible unit of cash in period 0 or in period 3. The asset can only be turned into consumption in period 3.[4] Banks prefer to consume cash in period 0 because of the opportunity cost $\rho > 1$ of holding cash after period 0. Banks which consume their cash in period 0 are called *illiquid* banks and those which keep it, *liquid* banks.

Every bank is required to pay one unit of cash either in period $t = 1,2,3$. Formally, in period 1 and 2 banks experience a liquidity shock which is modelled as a random cost for a bank to maintain its portfolio. Banks which experience a liquidity shock have to deliver one unit of cash. Otherwise, the bank's assets are liquidated and the liquidation cost is exactly equal to the remaining value of the portfolio. In order to avoid default a bank can either use its initial cash endowment or sell assets to acquire one unit of cash in a competitive interbank market which opens in period $t = 1,2$. If a bank is required to pay in period 3, it can make the repayment out of the asset return $R$.

Let $\theta_t$ denote the probability that a liquidity shock arrives at date $t$. The random variable $\theta_t$ has a density function $f(\theta_t)$ and a cumulative distribution function $F(\theta_t)$, where $t = 1,2$. The liquidity shocks $\theta_1$ and $\theta_2$ are assumed to be independent. A bank can only receive one liquidity shock. With probability $\theta_1$ a bank receives a liquidity shock in period 1. With probability $(1-\theta_1)\theta_2$ the liquidity shock arrives in period 2. With probability $(1-\theta_1)(1-\theta_2)$ a bank receives no shock in period 1 and 2 and repays in period 3.

[4] GY call cash the liquid asset because cash can be turned into consumption utility in period 0 already, while the asset is called illiquid because it yields utility in period three only.

## 2.1 Planner allocation

As GY, we assume that the planner's objective is to maximize the total expected surplus. From (1), utility can be generated by consuming cash in period 0 and by carrying forward assets to period 3, where the return can be turned into consumption.

In contrast to GY, we characterize the first-best allocation which refers to the *unconstrained*-efficient allocation in GY. In particular, we allow the planner to redistribute assets between banks in period $t = 1, 2, 3$. As suggested by GY, if the planner is able to redistribute assets between banks, the planner can assign all assets to those banks with no liquidity shock in a period. In this case, all assets can be carried forward to period 3 and, since no cash is needed to do so, all cash holdings can be consumed in the initial period. Hence, there is no waste of assets and cash. Thus, welfare of the *unconstrained*-efficient allocation is

$$W^P = R + \rho. \tag{2}$$

GY restrict the planner's actions to accumulating cash at date 0, distributing cash at dates 1 and 2, and redistributing the consumption good at date 3. In contrast to GY, our planner is endowed with more power since he can redistribute assets between banks. For this reason, we refer to the solution of the planner's problem as an *unconstrained*-efficient allocation, whereas we refer to the solution of the planner's problem in GY as a *constrained*-efficient allocation.

## 2.2 Lottery mechanism

In GY, the trading mechanism in the interbank market is restricted to the exchange of one unit of indivisible cash for $x$ units of divisible assets. Here, we allow for lotteries which is a more general trading mechanism. In particular, we introduce lotteries $\ell$ and $\tilde{\ell}$, where $\ell$ denotes the lottery of receiving one unit of cash with probability $\tau \in [0, 1]$, and $\tilde{\ell}$ denotes the lottery of delivering one unit of cash with probability $\widetilde{\tau} \in [0, 1]$. Without loss in generality, we do not consider lotteries on the asset, since the asset is divisible. Furthermore, we assume that the lotteries $\ell$ and $\tilde{\ell}$ are indivisible.[5]

Let $x_t$ denote the quantity of assets delivered in order to acquire lottery $\ell_t$ in periods $t = 1, 2$. Furthermore, let $\widetilde{x}_t$ denote the quantity of assets received in exchange for lottery $\tilde{\ell}_t$ in periods $t = 1, 2$. As in GY, $x_t$ ($\widetilde{x}_t$) is a price. Here, $x_t$ is the quantity of asset to be paid for one unit of the indivisible lottery $\ell_t$, where $\ell_t$ pays out one unit of indivisible cash with probability $\tau \in [0, 1]$. In GY, $x_t$ is the quantity of assets to be paid for one unit of indivisible cash with certainty; i.e., $\tau_t = 1$.

In what follows, we introduce lotteries into the GY framework and present an incentive-feasible lottery mechanism that restores efficiency. In Section 6, we then

[5] This means that banks cannot acquire fractions of lotteries. This is without loss in generality, since we will show that efficiency can be restored with indivisible lotteries in the GY-model.

show that this mechanism can be decentralized. In particular, we show that the efficient allocation can be attained as the allocation of a competitive equilibrium.

**Definition 1** *A mechanism for period $t$ is denoted by the list $(x_t, \tau_t, \widetilde{x}_t, \widetilde{\tau}_t)$, where $t = 1, 2$. A mechanism for the entire game is denoted by the list $(x_t, \tau_t, \widetilde{x}_t, \widetilde{\tau}_t)_{t=1,2}$.*

A feasible mechanism is a mechanism that satisfies physical constraints. Feasibility in period $t$ requires that the measure of banks which receive one unit of cash is equal to the measure of banks which deliver one unit of cash. Then, feasibility requires that

$$m_t^s \widetilde{\tau}_t = m_t^d \tau_t. \tag{3}$$

The left-hand side of equation (3) represents the cash delivered in period $t$: $m_t^s$ is the measure of banks that supply cash and they have to deliver it with probability $\widetilde{\tau}_t$. The right-hand side of equation (3) represents the cash received in period $t$: $m_t^d$ is the measure of banks that demand cash and they receive it with probability $\tau_t$.

Feasibility in period 1 also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility requires that

$$m_t^s \widetilde{x}_t = m_t^d x_t. \tag{4}$$

The left-hand side of equation (4) represents the assets received, since $m_t^s$ is the measure of banks that receive $\widetilde{x}_t$ units of assets. The right-hand side of equation (4) represents the assets delivered, since $m_t^d$ is the measure of banks that deliver $x_t$ units of assets.

Finally, feasibility also requires that

$$x_t \leq e_t \text{ and } \tau_t, \widetilde{\tau}_t \leq 1, \tag{5}$$

where $e_t$ denotes the asset holdings of a bank which supplies assets in period $t = 1, 2$. Since banks have an initial endowment of one unit of the asset, $e_1 = 1$ in the first period. However, $e_2$ can exceed one, since liquid banks can acquire assets in exchange for cash in period 1.[6]

**Definition 2** *A feasible mechanism is a mechanism $(x_t, \tau_t, \widetilde{x}_t, \widetilde{\tau}_t)_{t=1,2}$ that satisfies (3), (4) and (5).*

[6] Our mechanism shares elements of an all-pay auction. The difference is that with our mechanism the prize (i.e. cash) is not awarded to the highest bidder, but each bidder has a chance to get cash, which is proportional to his bid. In particular, when the number of banks needing cash is larger than the number of banks supplying it, the revenue of the cash sellers can be maximized by having all potential cash buyers offer their asset as a payment for which they receive a probability of getting the cash. Then the limited amount of cash is randomly allocated among the buyers. This allows the selling banks to obtain a higher price for their cash as in GY.

An implication from (3) and (4) is

$$\frac{\widetilde{x}_t}{\widetilde{\tau}_t} = \frac{x_t}{\tau_t}. \tag{6}$$

The quantities $\widetilde{x}_t/\widetilde{\tau}_t$ and $x_t/\tau_t$ can be interpreted as the expected price of cash. A cash supplier receives $\widetilde{x}_t$ assets in exchange for one unit of cash with probability $\widetilde{\tau}_t$. With risk neutral agents, this is similar to a trade, where a bank receives $\widetilde{x}_t/\widetilde{\tau}_t$ assets for one unit of cash. Similarly, a cash demander delivers $x_t$ assets for one unit of cash with probability $\tau_t$. With risk neutral agents this is similar to a trade, where a bank delivers $x_t/\tau_t$ assets for one unit of cash. Thus, feasibility implies that the expected price of cash is the same for cash suppliers and cash demanders.

Note that our trading mechanism encompasses the mechanism applied in GY. In their paper, $\tau_t = \widetilde{\tau}_t = 1$ which reduces the set of feasible mechanisms and leads to liquidity hoarding.

# 3 Decisions

We now characterize the banks' decisions in period $t = 1, 2, 3$.

**Period 0** In period 0, banks choose whether to consume their cash holdings or not. Let $0 \leq \alpha \leq 1$ measure of illiquid banks. The $\alpha$ illiquid banks end period 0 with portfolio $\{1, 0\}$ and the $(1-\alpha)$ liquid banks with portfolio $\{1, 1\}$.

**Period 1** Consider a mechanism $(x_1, \tau_1, \widetilde{x}_1, \widetilde{\tau}_1)$ for period 1. At the beginning of period 1, a fraction $\theta_1$ of banks receives a liquidity shock. Figure 1 displays the game tree.

First, illiquid banks with a shock in period 1 have measure $\alpha\theta_1$ and pre-trade portfolio $\{1, 0\}$. To avoid default, they need to acquire cash in period 1. If they trade, they have portfolio $\{1 - x_1, 0\}$ with probability $\tau_1$ and portfolio $\{0, 0\}$ with probability $(1 - \tau_1)$. Otherwise, they default and the portfolio is $\{0, 0\}$.

Second, illiquid banks without a shock in period 1 have measure $\alpha(1 - \theta_1)$. Since they hold no cash, they remain inactive and end period 1 with portfolio $\{1, 0\}$.

Third, liquid banks with a shock in period 1 have measure $(1-\alpha)\theta_1$ and pre-trade portfolio $\{1, 1\}$. Since they hold cash, they can avoid default by using their own cash holdings and end period 1 with portfolio $\{1, 0\}$.

Finally, liquid banks with no shock in period 1 have measure $(1 - \alpha)(1 - \theta_1)$ and pre-trade portfolio $\{1, 1\}$. They can either supply cash in exchange for assets in period 1 or not trade. We call the former banks cash suppliers. GY call the latter *hoarders* and we will keep their language. With probability $\widetilde{\tau}_1$, cash suppliers have to deliver one unit of cash and they end period 1 with portfolio $\{1+\widetilde{x}_1, 0\}$. With probability $(1-\widetilde{\tau}_1)$, cash suppliers don't need to deliver cash and they end period 1 with portfolio $\{1+\widetilde{x}_1, 1\}$. Hoarders end period 1 with portfolio $\{1, 1\}$.

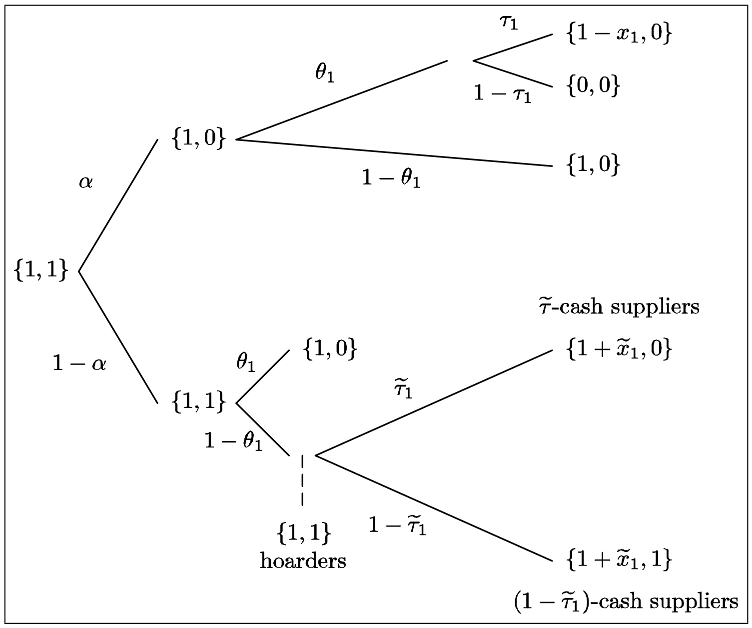

Figure 1: Event tree for $t = 1$

We will focus on equilibria, where all banks are willing to trade. Hence, 'not trading' is an out-of-equilibrium action. The 'not trading' decision which is labelled as 'hoarding' in GY is indicated by the dashed line in Figure 1. All other possible 'not trading' actions are ignored for the moment ('not trading' is a possible action at any node of the tree, where a trading decision has to be made).

**Period 2** Consider a mechanism $(x_2, \tau_2, \widetilde{x}_2, \widetilde{\tau}_2)$ for period 2. At the beginning of period 2, a fraction $\theta_2$ of banks receive a liquidity shock. Banks with a shock in the previous period remain inactive in period 2, since they hold no cash. Figure 2 displays the game tree for period 2.

First, illiquid banks with no shock in period 1, but with a shock in period 2 have measure $\alpha(1-\theta_1)\theta_2$ and pre-trade portfolio $\{1, 0\}$. To avoid default, they need to acquire cash in period 2. If they trade, they have portfolio $\{1-x_2, 0\}$ with probability $\tau_2$ and portfolio $\{0, 0\}$ with probability $(1-\tau_2)$. Otherwise, they default and the portfolio is $\{0, 0\}$.

Second, illiquid banks with no shock in period 1 and 2 have measure $\alpha(1-\theta_1)(1-\theta_2)$. Since they hold no cash, they remain inactive and end period 2 with portfolio $\{1, 0\}$.

Third, $\widetilde{\tau}_1$-cash suppliers with a shock in period 2 have measure $(1-\alpha)(1-\theta_1)\widetilde{\tau}_1\theta_2$ and pre-trade portfolio $\{1+\widetilde{x}_1, 0\}$. They need to acquire one unit of cash in period 2 to avoid default. If they trade, they have portfolio $\{1+\widetilde{x}_1 - x_2, 0\}$ with probability $\tau_2$ and portfolio $\{0, 0\}$ with probability $(1-\tau_2)$. If they don't trade, they default and the portfolio is $\{0, 0\}$.

Fourth, $\widetilde{\tau}_1$-cash suppliers with no shock in period 2 have measure $(1-\alpha)(1-\theta_1)\widetilde{\tau}_1(1-\theta_2)$. Since they hold no cash, they remain inactive and end period 2 with portfolio $\{1+\widetilde{x}_1, 0\}$.

Fifth, $(1-\widetilde{\tau}_1)$-cash suppliers with a shock in period 2 have measure $(1-\alpha)(1-\theta_1)(1-\widetilde{\tau}_1)\theta_2$ and pre-trade portfolio $\{1+\widetilde{x}_1, 1\}$. Since they hold cash, they can avoid default by using their own cash holdings and end period 2 with portfolio $\{1+\widetilde{x}_1, 0\}$.

Sixth, $(1-\widetilde{\tau}_1)$-cash suppliers with no shock in period 2 have measure $(1-\alpha)(1-\theta_1)(1-\widetilde{\tau}_1)(1-\theta_2)$ and pre-trade portfolio $\{1+\widetilde{x}_1, 1\}$. If they trade, they have portfolio $\{1+\widetilde{x}_1+\widetilde{x}_2, 0\}$ with probability $\widetilde{\tau}_2$ and portfolio $\{1+\widetilde{x}_1+\widetilde{x}_2, 1\}$ with probability $(1-\widetilde{\tau}_2)$.

Finally, consider the hoarders (recall, this is out-of-equilibrium) with pre-trade portfolio $\{1, 1\}$. If they receive a shock in period 2, they use their cash holdings to meet their required payment and end the period with portfolio $\{1, 0\}$. If they do not receive a liquidity shock they can supply cash. If they trade, they have portfolio $\{1+\widetilde{x}_2, 0\}$ with probability $\widetilde{\tau}_2$ and portfolio $\{1+\widetilde{x}_2, 1\}$ with probability $(1-\widetilde{\tau}_2)$.

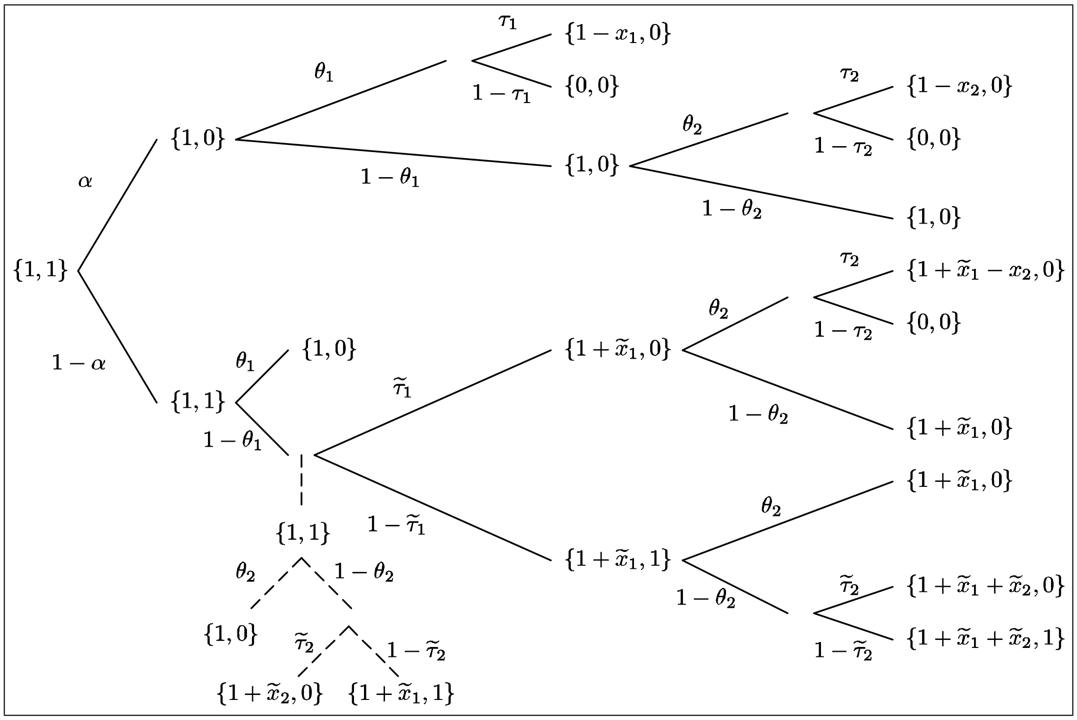


Figure 2: Event tree for $t = 1, 2$

**Period 3** In period 3, the return of the various portfolios carried forward from period 2 realizes. Each bank that has not yet received a liquidity shock meets its required payment either from the realization of the asset return or the cash holdings. Then, the economy ends.

# 4 An efficient incentive-feasible mechanism

An efficient incentive-feasible mechanism is a feasible mechanism that satisfies participation constraints. The participation constraints require that banks are willing to go along with the proposed mechanism; i.e., we allow them to opt out of a mechanism. Efficiency requires that there is no waste of assets or cash.

## 4.1 An efficient incentive-feasible mechanisms for period 1

**Participation constraints in period 1** The supply of cash comes from liquid banks without a shock in period 1. They can either supply cash or hoard. They are willing to supply cash if

$$X_1^S \geq Y_1^S, \tag{7}$$

where $X_1^S$ is a cash supplier's expected payoff and $Y_1^S$ is a hoarder's expected payoff.

The demand for cash comes from the illiquid banks with a shock in period 1. They are willing to demand cash in exchange for assets if

$$X_1^D \geq Y_1^D, \tag{8}$$

where $X_1^D$ is the expected payoff for trading and $Y_1^D$ the expected payoff of default.

In what follows and without loss in generality, we restrict our attention to incentive-feasible mechanisms. Hence, all banks accept the mechanism. Later on we will verify that the participation constraints (7) and (8) are satisfied.

**Feasibility in period 1** Feasibility in period 1 requires that the measure of banks which receive one unit of cash is equivalent to the measure of banks which deliver one unit of cash. If (7) and (8) hold, then from (3), feasibility in period 1 requires that

$$(1-\alpha)(1-\theta_1)\widetilde{\tau}_1 = \alpha\theta_1\tau_1. \tag{9}$$

The left-hand side of equation (9) represents the cash supply in period 1: $(1-\alpha)(1-\theta_1)$ is the measure of cash suppliers that deliver cash with probability $\widetilde{\tau}_1$. The right-hand side of equation (9) represents the cash demand in period 1: $\alpha\theta_1$ is the measure of cash demanders that receive cash with probability $\tau_1$.

From (4), feasibility also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility in period 1 requires also that

$$(1-\alpha)(1-\theta_1)\widetilde{x}_1 = \alpha\theta_1 x_1. \tag{10}$$

The left-hand side of equation (10) represents the assets received: $(1-\alpha)(1-\theta_1)$ is the measure of cash suppliers and each of them receives $\widetilde{x}_1$ units of assets. The right-hand side of equation (10) represents the assets delivered: $\alpha\theta_1$ is the measure of cash demanders and each of them delivers $x_1$ units of assets.

Finally, from (5), feasibility also requires that

$$x_1, \tau_1, \widetilde{\tau}_1 \leq 1. \tag{11}$$

As discussed above, there is no such constraint on $\widetilde{x}_1$, since a cash supplier can receive more than one unit of assets in exchange for one unit of cash.

An implication from (9) and (10) is

$$\frac{\widetilde{x}_1}{\widetilde{\tau}_1} = \frac{x_1}{\tau_1}.$$

Hence, feasibility implies that the expected price of cash in period 1 is the same for cash demanders and cash suppliers.

**Incentive-feasible mechanism for period 1** An incentive-feasible mechanism for period 1 is a mechanism $(x_1, \tau_1, \widetilde{x}_1, \widetilde{\tau}_1)$ that satisfies the participation constraints (7) and (8) and the feasibility conditions (9) through (11).

It is important to note that there are many mechanisms for period 1 that will satisfy these conditions. In what follows, we will choose an incentive-feasible mechanism that will get us as close to an efficient allocation as possible.

**An efficient incentive-feasible mechanism for period 1** Efficiency in period 1 requires that no assets and no cash holdings are wasted.

Assets are wasted if a bank that holds assets defaults. Recall that in GY, by assumption the asset holdings of a defaulting bank are liquidated and the liquidation cost is equal to the remaining portfolio value. There are two possibilities to avoid a waste of assets in period 1. First, if each cash demander receives one unit of cash, there is no default and no assets are wasted.[7] Second, if all assets are transferred from illiquid banks to liquid banks, then default is not costly, since defaulting banks hold portfolio $\{0, 0\}$. Cash is wasted if it is used to save a portfolio with zero value.

We will restore efficiency with a mechanism that transfers all assets from illiquid banks to liquid banks and which ensures that all cash rests with the liquid banks. In particular, consider the mechanism

$$(x_1, \tau_1, \widetilde{x}_1, \widetilde{\tau}_1) = \left(1, 0, \frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}, 0\right). \tag{12}$$

This mechanism is efficient, since all assets are transferred from illiquid to liquid banks; i.e., $x_1 = 1$, which implies from (10) that $\widetilde{x}_1 = \frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}$. Furthermore, no cash is wasted. That is, all cash should rest with the liquid banks, since they might have a need for it in period 2. This is attained by setting $\widetilde{\tau}_1 = 0$, which implies from (9) that $\tau_1 = 0$. Furthermore, it is easy to verify that the mechanism is feasible, since it satisfies conditions (9) through (11).

## 4.2 An efficient incentive-feasible mechanisms for period 2

Now, we derive an incentive-feasible mechanism for period 2 given that the mechanism in period 1 satisfies (12).

**Participation constraints in period 2** The supply of cash comes from the $(1-\widetilde{\tau}_1)$-cash suppliers without a shock in period 2. They are willing to supply cash in

[7] Since in GY, one unit of cash has to be delivered in order to get assets, the only way to obtain efficiency in market 1 is to transfer $\alpha\theta_1$ units of cash to the $\alpha\theta_1$ illiquid banks. If there is not enough cash, efficiency cannot be obtained.

exchange for assets if

$$X_2^S \geq Y_2^S, \tag{13}$$

where $X_2^S$ is the expected payoff of trading and $Y_2^S$ the expected payoff of remaining inactive.

The demand for cash comes from illiquid banks with a shock in period 2. They are willing to demand cash in exchange for assets if

$$X_2^D \geq Y_2^D, \tag{14}$$

where $X_2^D$ is the expected payoff for trading and $Y_2^D$ the expected payoff of default.

In what follows and without loss in generality, we restrict our attention to incentive-feasible mechanisms. Hence, all banks accept the mechanism. Later on we will verify that the participation constraints (13) and (14) are satisfied.

**Feasibility in period 2** Feasibility in period 2 requires that the measure of banks which receive one unit of cash is equivalent to the measure of banks which deliver one unit of cash. If (13) and (14) hold, then, feasibility requires that

$$(1-\alpha)(1-\theta_1)(1-\widetilde{\tau}_1)(1-\theta_2)\widetilde{\tau}_2 = \alpha(1-\theta_1)\theta_2\tau_2.$$

Given the mechanism for period 1, $\widetilde{\tau}_1 = 0$. Then, the above equation simplifies to

$$(1-\alpha)(1-\theta_1)(1-\theta_2)\widetilde{\tau}_2 = \alpha(1-\theta_1)\theta_2\tau_2. \tag{15}$$

The left-hand side of equation (15) represents the cash supply in period 2: $(1-\alpha)(1-\theta_1)(1-\theta_2)$ is the measure of cash suppliers that deliver cash with probability $\widetilde{\tau}_2$. The right-hand side of equation (15) represents the cash demand in period 2: $\alpha(1-\theta_1)\theta_2$ is the measure of cash demanders with a shock in period 2 that receive cash with probability $\tau_2$.

From (4), feasibility also requires that the quantity of assets delivered is equal to the quantity of assets received. Then, feasibility in period 2 also requires that

$$(1-\alpha)(1-\theta_1)(1-\widetilde{\tau}_1)(1-\theta_2)\widetilde{x}_2 = \alpha(1-\theta_1)\theta_2 x_2.$$

Again, taking into account that $\widetilde{\tau}_1 = 0$, the above equation yields

$$(1-\alpha)(1-\theta_1)(1-\theta_2)\widetilde{x}_2 = \alpha(1-\theta_1)\theta_2 x_2. \tag{16}$$

The left-hand side of equation (16) represents the assets received: $(1-\alpha)(1-\theta_1)(1-\theta_2)$ is the measure cash suppliers and each of them receives $\widetilde{x}_2$ units of assets. The right-hand side of equation (16) represents the assets delivered: $\alpha(1-\theta_1)\theta_2$ is the measure of illiquid banks with a shock in period 2 and each of them delivers $x_2$ units of assets.

Finally, feasibility also requires that

$$x_2, \tau_2, \widetilde{\tau}_2 \leq 1. \tag{17}$$

Again, as discussed above, there is no such constraint on $\widetilde{x}_2$, since a cash supplier can receive more than one unit of assets in exchange for one unit of cash.

An implication from (15) and (16) is

$$\frac{\widetilde{x}_2}{\widetilde{\tau}_2} = \frac{x_2}{\tau_2}.$$

As for period 1, feasibility implies that the expected price of cash in period 2 is the same for cash demanders and cash suppliers.

**Incentive-feasible mechanisms for period 2** An incentive-feasible mechanism for period 2 is a mechanism $(x_2, \tau_2, \widetilde{x}_2, \widetilde{\tau}_2)$ that satisfies the participation constraints (13) and (14) and the feasibility conditions (15) through (17).

As for period 1, there are many mechanisms that will satisfy these conditions. In what follows, we will choose an incentive-feasible mechanism that will get us as close to an efficient allocation as possible.

**An efficient incentive-feasible mechanism for period 2** As for period 1, efficiency in period 2 requires that no assets and no cash holdings are wasted.

Again, assets are wasted because of the liquidation costs of default. Hence, to attain efficiency we set $x_2 = 1$ which implies that $\widetilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}$. Furthermore, as in period 1 no cash should be wasted. That is, all the cash should rest with the liquid banks, since they have a need for it in period 3. This can be attained by setting $\widetilde{\tau}_2 = 0$ which implies from (15) that $\tau_2 = 0$. Hence, consider the mechanism

$$(x_2, \tau_2, \widetilde{x}_2, \widetilde{\tau}_2) = (1, 0, \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}, 0). \tag{18}$$

It is easy to verify that by construction, the mechanism (18) satisfies (15) through (17).

## 4.3 Efficiency in period 1 and 2

The efficient incentive-feasible mechanism (12) and (18) satisfies (9) through (11) and (15) through (17). Moreover, it satisfies participation constraints (7), (8), (13) and (14). The mechanism is efficient, since no assets and no cash holdings are wasted.

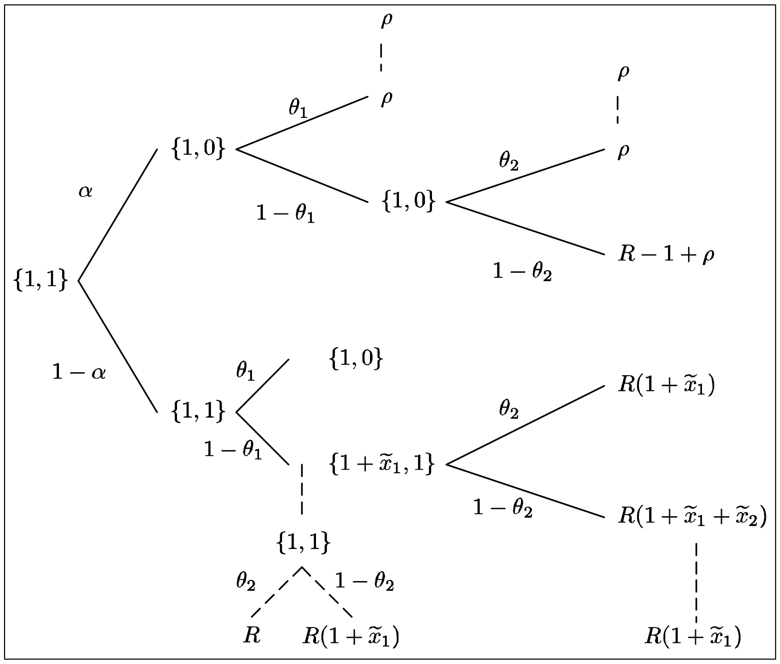

Figure 3: Final payoffs

Figure 3 displays the reduced game tree and the final payoffs given the mechanisms (12) and (18) for period 1 and 2. The reduced game tree in Figure 3 also includes all 'not trading' decisions indicated by the dashed lines. Note that $\tau_t, \widetilde{\tau}_t = 0$ simplifies the analysis of the game tree considerably.

**Proposition 3** *Given $\alpha$, the mechanism*

$$\left(x_t, \tau_t, \widetilde{x}_t, \widetilde{\tau}_t\right)_{t=1,2} = \left(1, 0, \frac{\alpha\theta_t}{(1-\alpha)(1-\theta_t)}, 0\right)_{t=1,2} \tag{19}$$

*is an efficient incentive-feasible mechanism for period 1 and 2.*

**Proof.** From the above derivations we know that the mechanisms for period 1 and 2 are feasible and efficient. In what follows, we will show that the proposed mechanism also satisfies the participation constraints (7), (8), (13) and (14).

Consider first, (7). With our mechanisms for the two periods

$$\begin{aligned} X_1^S &= \int_0^1 \left\{\theta_2 R\left(1+\widetilde{x}_1\right) + \left(1-\theta_2\right) R\left(1+\widetilde{x}_1+\widetilde{x}_2\right)\right\} f\left(\theta_2\right) d\theta_2 \\ Y_1^S &= \int_0^1 \left\{\theta_2 R + \left(1-\theta_2\right) R\left(1+\widetilde{x}_2\right)\right\} f\left(\theta_2\right) d\theta_2, \end{aligned}$$

where $X_1^S$ represents the expected utility of a cash supplier and $Y_1^S$ the expected utility of a hoarder. It is easy to see that (7) is always satisfied. Note that this is simply a statement that no hoarding occurs with our mechanism.

Second, consider (8). With our mechanism (19) for the two periods

$$\begin{aligned} X_1^D &= \tau_1 R(1-x_1) = 0 \\ Y_1^D &= 0, \end{aligned}$$

where $X_1^D$ represents the expected utility of an cash demander which trades and $Y_1^D$ represents the expected utility of the same bank which does not trade and consequently defaults. Given (19), the former group of banks has expected payoff $\tau_1 R(1-x_1) = 0$, since $\tau_1 = 0$ and $x_1 = 1$. Hence, also (8) is satisfied.[8]

Third, consider (13). With our mechanisms for the two periods

$$\begin{aligned} X_2^S &= R(1+\widetilde{x}_1+\widetilde{x}_2) \\ Y_2^S &= R(1+\widetilde{x}_1). \end{aligned}$$

Since (7) is satisfied, there are no hoarders. Hence, $X_2^S$ represents the expected payoff of a cash supplier without a liquidity shock in period 1 and 2 which decides to trade. $Y_2^S$ is the expected payoff of a cash supplier without a liquidity shock in period 1 and 2 which decides not to trade. It is straightforward to see that (13) is satisfied.

Finally, consider (14). With our mechanisms for the two periods

$$\begin{aligned} X_2^D &= \tau_2 R(1-x_2) = 0 \\ Y_2^D &= 0. \end{aligned}$$

Again, since (8) is satisfied, there are only illiquid banks with a shock in period 2 which demand cash. They can either accept the mechanism (19) or not and consequently default. Banks which accept have expected payoff $\tau_2 R(1-x_2) = 0$, since $\tau_2 = 0$ and $x_2 = 1$ given (19). Again, following the same argumentation from above, the participation constraint (14) is satisfied. This concludes the proof. ■

# 5 Portfolio Choice

Proposition 3 describes an efficient mechanism given $\alpha$. In what follows, we derive the portfolio choice of banks in period 0.

[8] If an illiquid bank does not trade, it defaults and its payoff is zero. Accordingly, the bank is indifferent between accepting the mechanism (19) and default. It is standard in game theory to assume that the illiquid bank is willing to accept this trade since we can always offer some additional marginally small utility.

In period 0, a fraction $\alpha$ of banks becomes illiquid and a fraction $(1-\alpha)$ of banks remains liquid. For $0 < \alpha < 1$, banks must be indifferent between the two alternatives. In what follows, we show under which conditions this must hold. In order to so, we compare the expected payoff of an illiquid bank and the expected payoff of a liquid bank under the efficient mechanism (19).

The expected utility of a liquid bank is

$$\begin{aligned}\Psi_{LI} \equiv & \int_0^1 \int_0^1 \{\theta_1 R + (1-\theta_1)\left[\theta_2 R(1+\widetilde{x}_1)\right. \\ & \left. +(1-\theta_2)R(1+\widetilde{x}_1+\widetilde{x}_2)\right]\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.\end{aligned}$$

With probability $\theta_1$, a liquid bank receives a liquidity shock in period 1. The bank can avoid default by using its own cash holdings and carry forward the remaining portfolio to period 3, where the return $R$ is realized. With probability $(1-\theta_1)\theta_2$, a liquid bank receives a liquidity shock in period 2. In period 1, $\widetilde{x}_1$ units of assets were acquired and given (19), no cash had to be delivered in exchange. Hence, the bank can avoid default by using its own cash holdings and carry forward the remaining portfolio to period 3, where the return $R(1+\widetilde{x}_1)$ is realized. Finally, with probability $(1-\theta_1)(1-\theta_2)$, a liquid bank receives no liquidity shock in period 1 and 2. In periods 1 and 2, $\widetilde{x}_1$ and $\widetilde{x}_2$ units of assets were acquired and given (19), no cash had to be delivered in exchange. In period 3, the bank can make the required repayment by using its own cash holdings and hence the return $R(1+\widetilde{x}_1+\widetilde{x}_2)$ is realized.

The expected utility of an illiquid bank is

$$\Psi_{IL} \equiv \rho + \int_0^1 \int_0^1 \{(1-\theta_1)(1-\theta_2)(R-1)\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \tag{20}$$

An illiquid bank receives $\rho$ from consuming its one unit of cash in period 0. In period 1 and 2, the surplus is zero so that only if no shock is experienced in both periods the bank receives payoff $(R-1)$.

The expected payoffs are equal if $\Psi_{IL} = \Psi_{LI}$.

**Proposition 4** *There exists a critical value $1 < \rho_0 < R$ such that if $\rho \leq \rho_0$, $\alpha = 0$ and if $\rho > \rho_0$, $\alpha \in (0,1)$.*

**Proof.** Using the expressions for $\widetilde{x}_1$ and $\widetilde{x}_2$ from (19) we can rewrite the expected utility of a liquid bank as follows

$$\Psi_{LI} = \int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1-\alpha)} R + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)} R \right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \tag{21}$$

The expected payoffs are equal if

$$\rho + \int_0^1 \int_0^1 \{(1-\theta_1)(1-\theta_2)(R-1)\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2 \tag{22}$$
$$= \int_0^1 \int_0^1 \left\{ R + \frac{\alpha\theta_1}{(1-\alpha)} R + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)} R \right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

Note that the left-hand side of (22) is independent of $\alpha$ and the right-hand side is increasing in $\alpha$, approaching $\infty$ as $\alpha \to 1$. Thus, for an interior solution, we need that the right-hand side of (22) at $\alpha = 0$ is smaller than the left-hand side of (22); i.e.,

$$\rho + \int_0^1 \int_0^1 \{(1-\theta_1)(1-\theta_2)(R-1)\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2 > R.$$

Accordingly, the critical value $\rho_0$ is defined by

$$\rho_0 = R - \int_0^1 \int_0^1 \{(1-\theta_1)(1-\theta_2)(R-1)\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2$$

Hence, if $\rho > \rho_0$, $0 < \alpha < 1$ must hold in equilibrium. Otherwise, if $\rho \leq \rho_0$, $\alpha = 0$ must hold in equilibrium, since banks prefer to remain liquid in period 0. ■

Note that an interior value for $\alpha$ is more likely if $\rho$ increases or $R$ decreases. Thus, as in GY, if $\rho$ is not too high relative to $R$, $\alpha \in (0, 1)$.

# 6 Competitive equilibrium

In the previous sections, we have shown that a lottery mechanism can restore efficiency in the GY model. Here, we study the question of whether the efficient lottery mechanism can be decentralized. In particular, we want to know whether the efficient allocation can be attained as the allocation of a competitive equilibrium. The answer is yes. To see why, notice that the hoarding inefficiency identified by GY will give rise to profit maximizing financial intermediaries that emerge endogenously to offer the efficient lotteries that are acquired by the GY-banks.[9]

We will show how, for a given pair of lotteries $\ell_2$ and $\tilde{\ell}_2$, one can derive aggregate demand, aggregate supply, and market clearing prices. So, essentially, we construct a competitive lottery equilibrium for a given pair of lotteries $\ell_2$ and $\tilde{\ell}_2$. This is the same approach as in GY except that in GY the lotteries $\ell_2$ and $\tilde{\ell}_2$ are degenerate; i.e., GY construct a competitive equilibrium by assuming that $\tau_2 = \tilde{\tau}_2 = 1$.

[9] This is similar as in Berentsen et al. (2007). They consider a model where agents are subject to trading shocks, which generates an inefficient allocation of cash. They show that this inefficiency creates a role for financial intermediaries that take deposits and make loans, and that these financial activities improve the allocation.

Throughout this section, we only analyze period 2 since once we know how to decentralize the allocation of period 2, the decentralization of the allocation of period 1 is straightforward. Furthermore, since we want to decentralize the efficient allocation, we assume for the first period that $\widetilde{\tau}_1 = \tau_1 = 0$.

Recall that $\ell_2$ denotes the lottery of receiving one unit of cash with probability $\tau_2$, and that $x_2$ is the price of lottery $\ell_2$; i.e., $x_2$ is the quantity of assets that has to be paid in order to receive one unit of cash with probability $\tau_2 \in [0, 1]$. This is the same notion of prices as in GY, except that in GY, $\tau_2 = 1$.[10] Similarly, recall that $\widetilde{\ell}_2$ denotes the lottery of having to deliver one unit of cash with probability $\widetilde{\tau}_2$, and that $\widetilde{x}_2$ is the price of lottery $\widetilde{\ell}_2$; i.e., $\widetilde{x}_2$ is the quantity of assets received for delivering one unit of cash with probability $\widetilde{\tau}_2$ (in GY, $\widetilde{\tau}_2 = 1$). Finally, recall that lotteries are indivisible and that a bank can acquire at most one unit of the lottery $\ell_2$ or one unit of the lottery $\widetilde{\ell}_2$.[11]

**Aggregate demand for lotteries** The demand for $\ell_2$ originates from the cash demanders (the illiquid banks without a shock in period 1 but with a shock in period 2) which have a pre-trade portfolio $\{1, 0\}$. To avoid default, they need to acquire cash in period 2. If they acquire one unit of lottery $\ell_2$, they have portfolio $\{1 - x_2, 0\}$ with probability $\tau_2$ and portfolio $\{0, 0\}$ with probability $(1 - \tau_2)$. Otherwise, they default and the portfolio is $\{0, 0\}$. Accordingly, for any $1 \geq \tau_2 > 0$, the aggregate demand $L_2$ for lottery $\ell_2$ satisfies

$$L_2 = \begin{cases} 0 & if \quad x_2 > 1 \\ \in [0, \alpha(1-\theta_1)\theta_2] & if \quad x_2 = 1 \\ \alpha(1-\theta_1)\theta_2 & if \quad x_2 < 1 \end{cases} . \tag{23}$$

For $x_2 > 1$, the lottery $\ell_2$ is not affordable since cash demanders hold no more than one unit of the asset in period 2. Consequently, the aggregate demand $L_2$ is zero. For $x_2 = 1$, cash demanders are indifferent between acquiring one unit of the lottery $\ell_2$ or not, since default yields a portfolio of $\{0, 0\}$. Accordingly, the aggregate demand $L_2$ is in the interval $[0, \alpha(1 - \theta_1)\theta_2]$. Finally, for $x_2 < 1$, the aggregate demand is $\alpha(1 - \theta_1)\theta_2$, since cash demanders receive a strictly positive expected surplus from acquiring lottery $\ell_2$.

For $\tau_2 = 0$, the aggregate demand for lottery $\ell_2$ satisfies $L_2 \in [0, \alpha(1 - \theta_1)\theta_2]$. In this case, cash demanders are indifferent between acquiring one unit of the lottery or not (and consequently default) since the payoff is equal to zero in either case. Throughout the analysis that follows, we assume that if a bank is indifferent between

[10] GY use the notation $p_2$ for the price of receiving one unit of cash with certainty.

[11] This latter assumption simplifies the analysis because it implies that at each point in time banks hold either one or zero units of cash. The same assumption is made in GY (p. 300), who assume that "at each date, bankers hold either one or zero units of cash in equilibrium." As in GY, it turns out that this is optimal in our model, too.

obtaining the lottery $\ell_2$ or not, it will acquire it. Consequently, for any $1 \geq \tau_2 \geq 0$, if $x_2 > 1$ aggregate demand is $L_2 = 0$ and if $x_2 \leq 1$, it is $L_2 = \alpha(1-\theta_1)\theta_2$.

The demand for $\tilde{\ell}_2$ originates from the cash suppliers (the liquid banks with no shock in period 1 and 2) which have pre-trade portfolio $\{1+\widetilde{x}_1, 1\}$. If they acquire one unit of lottery $\tilde{\ell}_2$, they have portfolio $\{1+\widetilde{x}_1+\widetilde{x}_2, 0\}$ with probability $\widetilde{\tau}_2$ and portfolio $\{1+\widetilde{x}_1+\widetilde{x}_2, 1\}$ with probability $(1-\widetilde{\tau}_2)$. For any $1 \geq \widetilde{\tau}_2 \geq 0$, the aggregate demand $\tilde{L}_2$ for $\tilde{\ell}_2$ satisfies

$$\tilde{L}_2 = \begin{cases} 0 & if \quad \widetilde{x}_2 < \widetilde{\tau}_2/R \\ \in [0, (1-\alpha)(1-\theta_1)(1-\theta_2)] & if \quad \widetilde{x}_2 = \widetilde{\tau}_2/R \\ (1-\alpha)(1-\theta_1)(1-\theta_2) & if \quad \widetilde{x}_2 > \widetilde{\tau}_2/R \end{cases} \tag{24}$$

The aggregate demand for lottery $\tilde{\ell}_2$ can be derived as follows. The expected payoff from acquiring lottery $\tilde{\ell}_2$ is $(1+\widetilde{x}_1+\widetilde{x}_2)R-\widetilde{\tau}_2$, and the expected payoff from not acquiring it is $(1+\widetilde{x}_1)R$. Accordingly, liquid banks with no shock in period 1 and 2 strictly prefer to buy if $\widetilde{x}_2 > \widetilde{\tau}_2/R$ in which case the aggregate supply of cash is $(1-\alpha)(1-\theta_1)(1-\theta_2)$. For $\widetilde{x}_2 = \widetilde{\tau}_2/R$, they are indifferent between buying and not buying and so the aggregate supply is in the interval $[0, (1-\alpha)(1-\theta_1)(1-\theta_2)]$. Finally, if $\widetilde{x}_2 < \widetilde{\tau}_2/R$ there is no demand for lottery $\tilde{\ell}_2$ and so the cash supply is zero. Again, if a bank is indifferent between acquiring lottery $\tilde{\ell}_2$ or not, we assume that banks will acquire the lottery. Consequently, for any $1 \geq \widetilde{\tau}_2 \geq 0$, if $\widetilde{x}_2 < \widetilde{\tau}_2/R$ aggregate demand is $\tilde{L}_2 = 0$ and if $\widetilde{x}_2 < \widetilde{\tau}_2/R$, it is $\tilde{L}_2 = (1-\alpha)(1-\theta_1)(1-\theta_2)$.

**Aggregate supply of lotteries** The lotteries $\ell$ and $\tilde{\ell}$ are offered by competitive financial intermediaries that operate at zero costs. Perfect competition avoids any strategic interaction (such as bargaining) that might arise from direct contacts of the financial intermediary and the GY-banks. To simplify matters, we assume that in equilibrium there is only one intermediary offering these lotteries, but because of the threat of entry (it is a contestable market), the financial intermediary makes zero profit.

The financial intermediary offers one unit of the lottery $\ell_2$ at price $x_2$. That is, it asks for $x_2$ units of the asset in exchange for one unit of cash delivered with probability $\tau_2$. The financial intermediary also offers lottery $\tilde{\ell}_2$ at price $\widetilde{x}_2$. In this case, it offers $\widetilde{x}_2$ units of assets in exchange for one unit of cash to be paid with probability $\widetilde{\tau}_2$. In practice, financial intermediaries offering similar types of financial services would be broker-dealers or market makers which buy and sell securities (in our case the assets) on behalf of their clients or for their own books. These financial intermediaries are either paid a fee, or make a return from bid-ask spreads, or both.[12]

[12] To fix ideas, assume that the intermediary operates ATM machines. They are programed to deliver one unit of lottery $\ell_2$ for $x_2$ units of the asset. That is, a cash demander inserts $x_2$ units of the asset into the ATM machine and the ATM machine ejects one unit of cash with probability $\tau_2$.

Note that the financial intermediary operates under similar feasibility conditions as banks do (see Section 4). In particular, the financial intermediary cannot sell quantities of the lotteries $\ell_2$ and $\tilde{\ell}$ such that more cash or more assets are delivered than received. It is feasible, though, that the financial intermediary retains some of the cash and/or some of the assets and hence makes a profit. This, however, is excluded by the zero profit condition, which implies that all assets and all cash received by the financial intermediary must be paid out. This immediately implies that under the zero profit condition, the feasibility conditions (15) and (16) hold.

Note that (15) and (16) do not pin down a unique competitive lottery equilibrium, since there exists a continuum of combinations of lotteries $\ell_2$ and $\tilde{\ell}_2$ that satisfy (15) and (16). One of them is the efficient lottery mechanism (18). Under (18), the aggregate demand for the efficient lottery $\ell_2$ is $L_2 = \alpha(1-\theta_1)\theta_2$, and its price is $x_2 = 1$. The aggregate demand for efficient lottery $\tilde{\ell}_2$ is $\tilde{L}_2 = \alpha(1-\theta_1)\theta_2$ and its price is $\widetilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}$.

Note that under the efficient lottery mechanism the prices for the lotteries are finite, namely, $x_2 = 1$ and $\widetilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}$. However, the price of cash in terms of assets is infinite under the efficient lottery mechanism, but this is simply an artifact of the GY-assumption that, in case of default, the liquidation costs consume all remaining assets. Because of this assumption, an illiquid GY-bank is indifferent between not trading and acquiring a lottery that yields the unit of cash with probability zero. We can easily change this GY-assumption and assume that, for example, a defaulting bank can keep $1 > \epsilon > 0$ assets. In this case, in order to make the illiquid bank indifferent between trading and not trading, the lottery must give the unit of cash with a strictly positive probability. Consequently, the price of cash in terms of assets would be finite.

Finally, the competitive lottery equilibria that can be constructed can be Pareto ranked (not all of them are efficient). We can imagine an initial period, before portfolio choices are made (before $\alpha$ is determined), where financial intermediaries compete by suggesting lottery allocations. The GY-banks then choose the one that maximizes ex-ante utility. The efficient allocation described in (19) maximizes ex-ante welfare and will be chosen by the GY-banks.

**Aggregate demand and aggregate supply of cash** After having derived the aggregate demand and the aggregate supply of lotteries $\ell_2$ and $\tilde{\ell}_2$, we now derive the implied demand and supply of cash. Further, we also discuss the multiplicity of competitive equilibrium outcomes and discuss the role of the probabilities $\tau$ and $\widetilde{\tau}_2$.

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The ATM machine also delivers lottery $\tilde{\ell}_2$ at price $\widetilde{x}_2$. In this case, the cash supplier inserts one unit of cash into the ATM and the ATM machine ejects $\widetilde{x}_2$ units of assets and returns the unit of cash with probability $1-\widetilde{\tau}_2$. Throughout the paper we assume that one unit of cash can only be used once. That is, if a cash supplier receives the unit of cash back, he cannot use it in order to purchase a second lottery.

Finally, we show the set of feasible allocations and discuss efficiency.

The demand for cash originates from the illiquid banks without a shock in period 1 but with a shock in period 2 that demand lottery $\ell_2$. From (23), for any $\tau_2 > 0$, the aggregate demand for cash satisfies

$$D_2 = \begin{cases} 0 & if \quad x_2 > 1 \\ \in [0, \tau_2 \alpha (1-\theta_1)\theta_2] & if \quad x_2 = 1 \\ \tau_2 \alpha (1-\theta_1)\theta_2 & if \quad x_2 < 1 \end{cases}.$$

Notice that the aggregate demand for lottery $\ell_2$, $L_2$, is similar to the aggregate demand for cash, $D_2$, but for the term $\tau_2$. The term $\tau_2$ reflects the fact that the 'effective' demand for cash is lower than the aggregate demand for lottery $\ell_2$, since cash demanders receive the cash with probability $\tau_2$, only.

The supply of cash originates from liquid banks without a shock in period 1 and 2 that purchase lottery $\tilde{\ell}_2$. From (24), for any $\tilde{\tau}_2 > 0$, the aggregate supply of cash satisfies

$$S_2 = \begin{cases} 0 & if \quad \tilde{x}_2 < \tilde{\tau}_2/R \\ \in [0, \tilde{\tau}_2 (1-\alpha)(1-\theta_1)(1-\theta_2)] & if \quad \tilde{x}_2 = \tilde{\tau}_2/R \\ \tilde{\tau}_2 (1-\alpha)(1-\theta_1)(1-\theta_2) & if \quad \tilde{x}_2 > \tilde{\tau}_2/R \end{cases}.$$

Notice that the aggregate demand for lottery $\tilde{\ell}_2$, $\tilde{L}_2$, is similar to the aggregate supply of cash, $S_2$, but for the term $\tilde{\tau}_2$. The term $\tilde{\tau}_2$ reflects the fact that the 'effective' supply of cash is lower than the aggregate demand for lottery $\tilde{L}_2$, since the cash suppliers deliver the cash with probability $\tilde{\tau}_2$ only.

Aggregate demand and aggregate supply of cash in period 2 are drawn in Figure 4. The GY analysis is a special case of it with $\tilde{\tau}_2 = \tau_2 = 1$ (in the absence of hoarding in period 1).

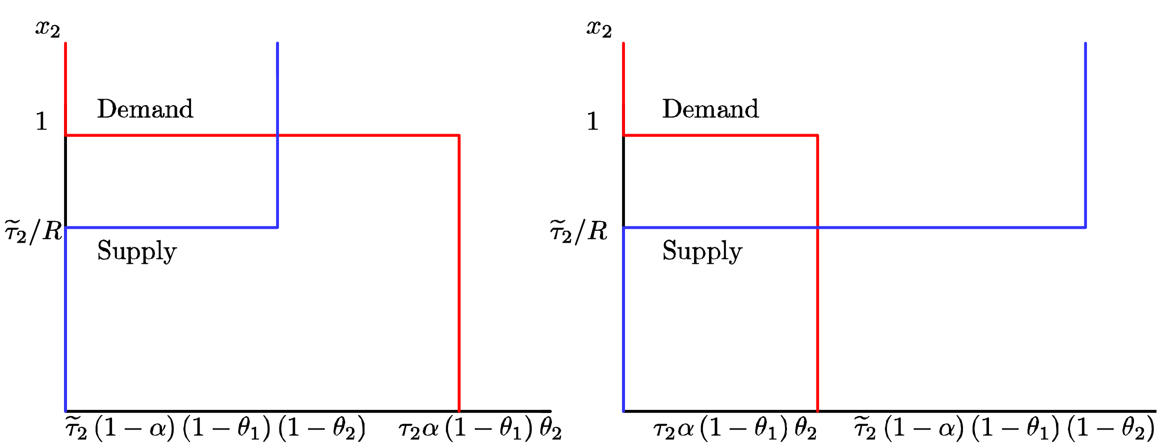


Figure 4: Aggregate demand and supply of cash in $t = 2$

**Changing the probabilities** Here we shortly discuss, how changing $\tau_2$ and $\widetilde{\tau}_2$ affect the aggregate demand and the aggregate supply of cash. The effects of such changes are displayed in Figure 5.

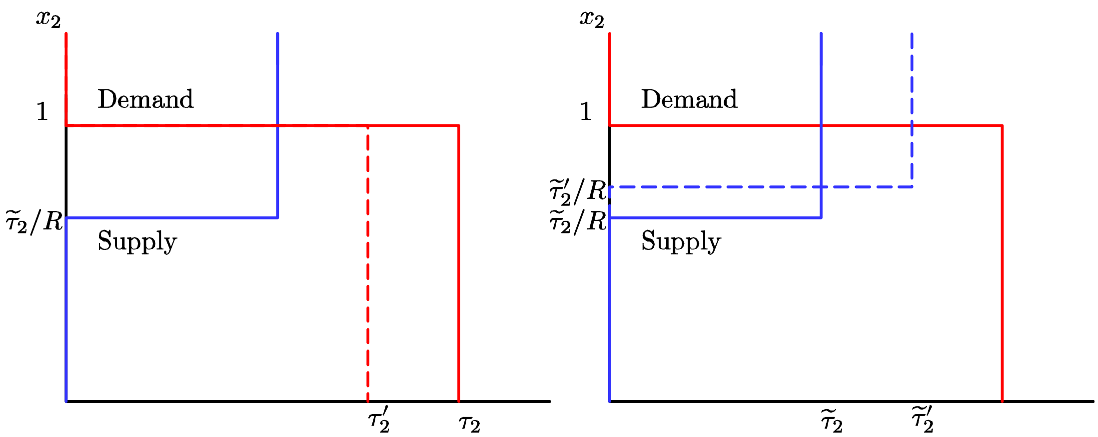


Figure 5: Impact of changing $\tau_2$ and $\widetilde{\tau}_2$

It is interesting to note that an increase of $\widetilde{\tau}_2$ moves the horizontal part of the aggregate supply curve up and shifts the vertical part of the curve to the right. In contrast, a change in $\tau_2$ only shifts the vertical part of the aggregate demand curve. This simply follows from the fact that a cash demander's alternative to not trading is default and so the horizontal part is not affected by a change in $\tau_2$ for $\tau_2 > 0$.

**Feasibility requirements** As discussed above, equilibrium in period 2 requires that the measure of banks which receive one unit of cash is equivalent to the measure of banks which deliver one unit of cash. That is, equilibrium requires that the feasibility condition (15) holds, which we replicate below for easier reference

$$(1-\alpha)(1-\theta_1)(1-\theta_2)\widetilde{\tau}_2 = \alpha(1-\theta_1)\theta_2\tau_2. \tag{25}$$

This immediately implies that the vertical parts of the demand and supply curves overlap as can be seen from Figure 6.

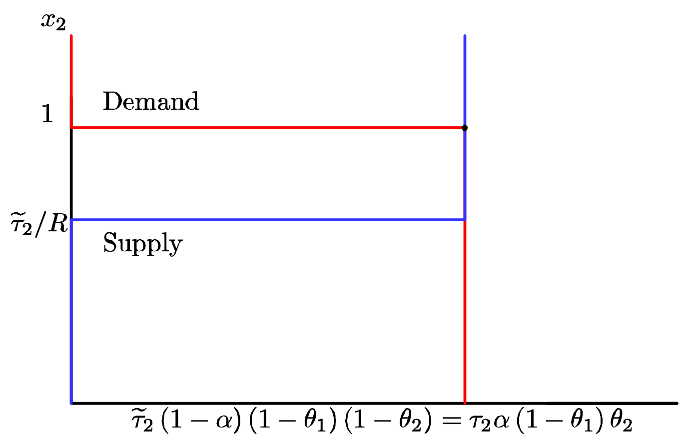

Figure 6: Feasibility

Feasibility also requires that the quantity of assets exchanged are equal. That is, we need to verify that $(1-\alpha)(1-\theta_2)\widetilde{x}_2 = \alpha\theta_2 x_2$. In particular, our mechanism assumes that $x_2 = 1$ and so

$$\widetilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}.$$

From (23), the aggregate demand for lottery $\ell_2$ is $L_2 \in [0, \alpha(1-\theta_1)\theta_2]$. For efficiency reasons, we always assume that $L_2 = \alpha(1-\theta_1)\theta_2$. That is, we assume that all illiquid banks purchase the lottery. From (24), the aggregate demand for the lottery $\tilde{\ell}_2$ is $\tilde{L}_2 = (1-\alpha)\,(1-\theta_1)\,(1-\theta_2)$ if $\frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)} > \widetilde{\tau}_2/R$. Since, again for efficiency reasons, we will choose a mechanism with a low $\widetilde{\tau}$, this condition is satisfied.

**Efficiency** The above analysis shows that lotteries can be part of a competitive equilibrium. They also help to implement a more efficient allocation. Consider the case $(1-\alpha)\,(1-\theta_2) < \alpha\theta_2$ with lottery prices $x_2 = 1$ and $\widetilde{x}_2 = \frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)}$. In GY, because of the absence of lotteries, market clearing requires that $(1-\alpha)\,(1-\theta_2) = \eta_2\alpha\theta_2$, where $\eta_2$ is the fraction of buyers that receive one unit of cash for one unit of the asset. The fraction $1-\eta_2$ receives no cash and defaults. This is clearly inefficient since the liquidation cost consumes the remaining value of the portfolio. That is, $1-\eta_2$ assets get destroyed. With lotteries, we can have $\eta_2 = 1$ and $(1-\alpha)\,(1-\theta_2) = \tau_2\alpha\theta_2$. That is all cash demanders deliver one unit of the asset in order to receive one unit of cash with probability $\tau_2$. Thus, with this lottery, all assets are saved. This can be graphically seen from Figure 7.

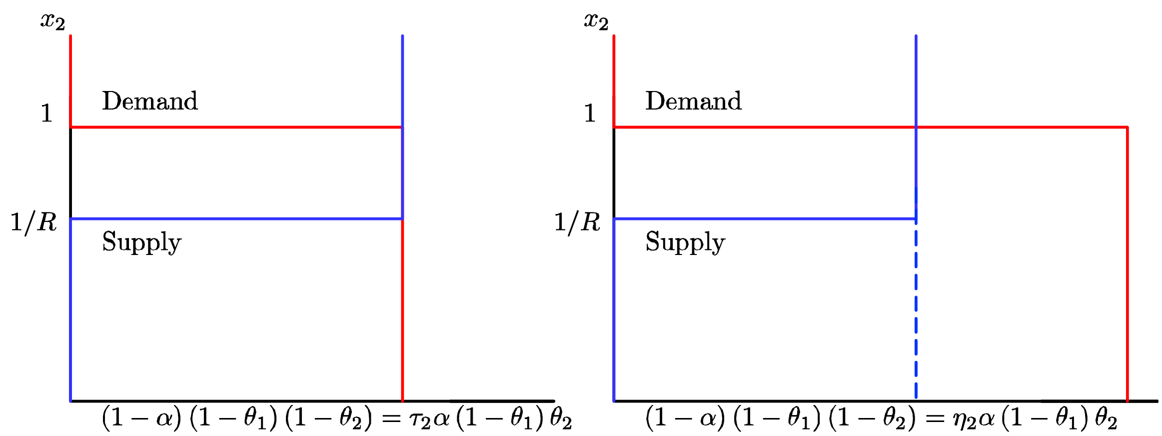


Figure 7: Efficiency

The efficient mechanism described in (19) does even more. Since cash is needed in future periods, efficiency improves when less cash ends up in the hands of illiquid banks after trading. The mechanism, therefore, attempts to set $\tau_2$ as low as possible, while still inducing the cash demanders to pay $x_2 = 1$ to acquire the lottery $\ell_2$. Since, as explained above, changing $\tau_2$ does not affect the demand for lottery $\ell_2$, we let $\tau_2$ approach zero. It is important to notice that the price for the lottery $\ell_2$ is equal to $x_2 = 1$ for any $\tau_2$.

From (25), since efficiency requires that $\tau_2$ is small, market clearing for cash requires that $\widetilde{\tau}_2$ is small. This of course helps to induce cash suppliers to supply their cash. That is, it helps to satisfy $\frac{\alpha\theta_2}{(1-\alpha)(1-\theta_2)} > \widetilde{\tau}_2/R$.

# 7 Discussion

In this section, our results from above are compared with the findings of GY.

## 7.1 Fire-sales, liquidity hoarding and efficiency

In GY, fire-sales are the source of liquidity hoarding. Liquid banks acquire assets in period 1 by selling cash. If these banks receive a liquidity shock in period 2, they are able to offer more assets in exchange for one unit of cash than was feasible in period 1. Hence, they will drive down asset prices so that assets are traded at fire-sale prices. As a result, if the expected asset price is lower in period 2 than in period 1, liquid banks react by hoarding cash in period 1.[13]

[13] On page 293, GY explain the connection between fire-sales and hoarding as follows: "Asset-price volatility results from the use of the asset market as a source of liquidity. When liquid bankers first

Our results are very different. With our mechanism, no hoarding occurs since all liquid banks trade in period 1. Furthermore, our mechanism is efficient, since no assets and no cash is wasted. The origin of these contradictory results is the absence of lotteries in the pricing of assets in GY. To see this, define the price of assets as follows:

$$q_t \equiv \frac{\widetilde{\tau}_t}{\widetilde{x}_t} = \frac{\tau_t}{x_t}. \tag{26}$$

Following (6), we argued that the ratios $\widetilde{x}_t/\widetilde{\tau}_t$ and $x_t/\tau_t$ can be interpreted as the expected price of cash. Accordingly, $q_t$ is the price for assets. In period 1, the asset price in GY is restricted to $q_1 \geq 1$. To see this, set $\widetilde{\tau}_t = \tau_t = 1$ in (26) and note that from (9) through (11) it follows that $\widetilde{x}_t = x_t \leq 1$. In contrast, the expected asset price in period 2 can be smaller than one; i.e., $E(q_2) < q_1$. In this case, liquid banks hoard liquidity in anticipation for a lower asset price in period 2. With our mechanism, the asset price in period 1 can fall below one, since $\widetilde{\tau}_1$ and $\tau_1$ can be smaller than one. If assets are sold at a fire-sale price in period 1, there is no reason to wait for a fire-sale price in period 2.

Interestingly, our results imply that fire-sales are efficient. They are needed to allocate all assets from the illiquid banks to the liquid banks. It is clear that our mechanism allows for a fire-sale asset price in period 1 ($q_1 \geq 0$), while a fire-sale price in GY is not feasible ($q_1 \geq 1$). In contrast, fire-sale prices are possible in GY and in our model in period 2 ($q_2 \geq 0$). Table 1 summarizes this relation of GY and our solution (BM).

| | $t=1$ | $t=2$ | Efficient |
|---|---|---|---|
| GY | Liquidity hoarding ($q_1 \geq 1$) | Fire-sale prices ($q_2 \geq 0$) | No |
| BM | Fire-sale prices ($q_1 \geq 0$) | Fire-sale prices ($q_2 \geq 0$) | Yes |

Table 1: Comparison of GY and BM

## 7.2 Welfare and market liquidity

We define welfare $W(\alpha)$ to be the expected utility of a bank at the beginning of period 0. Since the fraction of illiquid banks is $\alpha$ and the fraction of liquid banks is $(1-\alpha)$, we have

$$W(\alpha) = \alpha\Psi_{IL} + (1-\alpha)\Psi_{LI}, \tag{27}$$

where $\Psi_{IL}$ and $\Psi_{LI}$ are defined in (20) and (21), respectively.[14]

supply cash in exchange for assets, they create an imbalance in the system. They are increasing their holdings of illiquid assets and reducing their holding of liquid assets. If these large, illiquid bankers are subsequently hit by a liquidity shock, they have even more assets to dump on the market, producing a greater fire sale and reducing asset prices further. A laisser-faire equilibrium is inefficient bcause the incentive to hoard are simply too high."

[14] It can be verified that for $\alpha \in (0,1)$, $W(\alpha) = \Psi_{IL} = \Psi_{LI}$.

In what follows, we will discuss welfare and market liquidity. Given the mechanism (19), two important questions arise: First, is our mechanism welfare maximizing and second, what is the welfare maximizing level of liquidity in this economy?

Simplifying (27) yields

$$W(\alpha) = R + \alpha\rho - \alpha \int_0^1 \int_0^1 (1-\theta_1)(1-\theta_2) f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \tag{28}$$

$W(\alpha)$ is increasing in $\alpha$. Hence, it is optimal to set $\alpha$ as close as possible to one, which is equivalent to say that the entire stock of cash should be consumed in the initial period.[15] For $\alpha = 1$, we get

$$W(1) = R + \rho - \int_0^1 \int_0^1 (1-\theta_1)(1-\theta_2) f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

Note that this expression is identical to the planner's solution $W^P$ defined in (2), but for the term

$$\int_0^1 \int_0^1 (1-\theta_1)(1-\theta_2) f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

This term reflects that some illiquid banks experience no shock in period 1 and 2, but have to make a payment of one unit of cash in period 3. They can do so from the realized return of their asset holdings.

The planner avoids this payment by redistributing the asset holdings from illiquid banks with no shock in period 1 and 2 to those banks that already experienced a liquidity shock and let the former banks default. Our mechanism cannot do this because there is no interbank market in period 3.

Again, this reveals two interesting aspects. First, in GY, the market solution implies that the aggregate level of cash in the economy is too low compared to their constrained-efficient planner solution, whereas our aggregate level of liquidity is too high compared to the planner solution. Second, in addition to the usual feasibility constraints, the planner in GY operates under the constraint that he cannot transfer assets between banks.[16] Note that lotteries allow transferring assets exactly in the way that GY restrict the planner's allocation.

## 7.3 Welfare without liquidity shocks in period 3

Here, we maintain all assumption, but we assume that in period 3, there are no costs of maintaining the portfolio. Hence, with this assumption banks that had no liquidity

[15] When we set $\alpha = 1$, we effectively mean $\alpha \to 1$, since for the mechanism to work, the stock of cash has to be strictly positive, but arbitrarily small.

[16] See Footnote 5.

shock in period 1 or 2 receive the return of their asset holdings and don't need to repay one unit of cash in period 3. In this case, the expected utility of an illiquid bank is

$$\rho + \int_0^1 \int_0^1 \left\{(1-\theta_1)(1-\theta_2)R\right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2,$$

and the expected utility of a liquid bank is

$$\int_0^1 \int_0^1 \left\{R + \frac{\alpha\theta_1}{(1-\alpha)}R + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)}R + (1-\theta_1)(1-\theta_2)\right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

Hence, the welfare function is represented by the following expression

$$\begin{aligned} W(\alpha) &= \alpha\left(\rho + \int_0^1 \int_0^1 \left\{(1-\theta_1)(1-\theta_2)R\right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2\right) \\ &\quad + (1-\alpha)\left(\int_0^1 \int_0^1 \left\{R + \frac{\alpha\theta_1}{(1-\alpha)}R\right.\right. \\ &\quad \left.\left. + (1-\theta_1)\frac{\alpha\theta_2}{(1-\alpha)}R + (1-\theta_1)(1-\theta_2)\right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2\right), \end{aligned}$$

which can be simplified to yield

$$W(\alpha) = \alpha\rho + R + \int_0^1 \int_0^1 \left\{(1-\alpha)(1-\theta_1)(1-\theta_2)\right\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

It is easy to see that if $\alpha = 1$,

$$W(1) = \rho + R,$$

which equals the welfare level that the planner can achieve (see (2)).

## 7.4 Adding back creditors into the welfare criterion

In an earlier version of the paper (GY, 2011), GY characterized the planner solution under the assumption that the initial cash holdings had to be borrowed from a creditor. In that case, the liquidity shock was modelled as the random demand of the creditor for repayment of a callable bond. Creditors, as in Diamond and Dybvig (1983), are uncertain about their time preferences, but they want to consume at precisely one of the dates $t = 1, 2, 3$. With probability $\theta_1$, the creditor wants to consume in period 1, with probability $(1-\theta_1)\theta_2$ in period 2 and with probability $(1-\theta_1)(1-\theta_2)$ in period 3. The creditor's expected utility function is given by

$$V(c_1, c_2, c_3) = \theta_1 c_1 + (1-\theta_1)\theta_2 c_1 + (1-\theta_1)(1-\theta_2)c_{3.}$$

If the creditors enter the planner's objective function, welfare under the first-best allocation satisfies

$$W^P_{Creditors} = R + \rho - 1. \tag{29}$$

Here, the term $\rho - 1$ reflects the fact that consuming one unit of cash in the initial period, but not paying it back to the creditor, yields the net surplus of $\rho - 1$ to society (without creditors, the net surplus is just $\rho$; see (2)).

Note that adding back creditors affect neither the pricing nor the demand and supply in period 1 and 2, respectively. Hence, our analysis continues to hold. However, the welfare calculation needs some adjustments. In particular, with our market solution $\Delta \equiv \alpha \int_0^1 \int_0^1 [\theta_1 + (1-\theta_1)\theta_2] f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2$ banks default and they don't repay one unit of cash to the creditor, and so, welfare is reduced by $-\Delta$. Accordingly, the welfare function (28) needs to be adjusted as follows:

$$W(\alpha) = R + \alpha\rho - \alpha \int_0^1 \int_0^1 \{(1-\theta_1)(1-\theta_2) - [\theta_1 + (1-\theta_1)\theta_2]\} f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

This expression can be simplified to

$$W(\alpha) = \alpha(\rho - 1) + R,$$

which, for $\alpha = 1$, satisfies

$$W(1) = \rho + R - 1,$$

which equals the welfare level with creditors that the planner can achieve (see (29)).

# 8 Conclusion

We generalize the interbank market model of GY which features indivisible cash and divisible assets by introducing randomized trading (lotteries). We derive an efficient mechanism under which no hoarding occurs. Rather, the economy is characterized by fire-sale asset prices. Counterintuitive, fire-sales ensure efficiency in the GY framework. We also find that with our efficient mechanism, too much liquidity is provided by the market compared to the planner solution, while the market liquidity in GY is insufficient compared to the social optimum.

During the recent financial crisis, markets for liquidity were subject to severe stress which heavily impaired the ability of banks to transform illiquid portfolios into liquid portfolios. As a result, a number of banks became illiquid or even insolvent. Our results suggest that the mechanism proposed by GY might not be at the origin of these phenomena. Rather, we speculate that private information problems generated severe counterparty risks that made liquidity hoarding an optimal choice. However, our believe needs to be subject to further research in order to improve our understanding of the markets for liquidity.

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