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Abstract

We consider a repeated stochastic coordination game with imperfect public monitoring. In the game any pattern of coordinated play is a perfect Bayesian Nash equilibrium. Moreover, standard equilibrium selection arguments either have no bite or they select an equilibrium that is not observed in actual plays of the game. We give experimental evidence for a unique equilibrium selection and explain this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and over-confidence. Our results have interesting applications in finance because the observed equilibrium exhibits momentum, reversal and excess volatility. Moreover, the results may help to explain why technical analysis is a commonly observed investment style.

Keywords: Coordination games, behavioral equilibrium selection, experimental asset markets, behavioral finance, investor sentiment, technical analysis.
JEL-Classification: C72, C91, C92, D83, G12.

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1 Introduction

Ninety percent of what we do is based on perception. It doesn't matter if that perception is right or wrong or real. It only matters that other people in the market believe it. I may know it's crazy, I may think it's wrong. But I lose my shirt by ignoring it.

"Making Book on the Buck"
Wall Street Journal, Sept. 23, 1988, p. 17

We consider a repeated stochastic coordination game with imperfect public monitoring: A finite number of strategic players and a die interact in two rounds, each with a finite number of periods. In the stage game every player chooses between two actions, say "u" and "d". The outcome of the stage game is given by the majority rule. If more players choose "u" than "d" the outcome is "u", otherwise the outcome is "d". Each choice of a strategic player has weight one while the die distributes a weight equal to the number of strategic players plus one according to a uniform distribution between "u" and "d". In every period the strategic players receive a positive payoff if their choice matches the majority, otherwise the payoff is zero. Each player only observes her own payoff and the choice of the majority. The individual actions of the other strategic players and that of the die remain hidden. The payoff of the repeated game is the sum of the payoffs of the stage games. In this game any pattern of coordinated play is a perfect Bayesian Nash equilibrium of the repeated game. Nevertheless, we give robust experimental evidence for a particular equilibrium selection.

The game we consider is an example of a stochastic coordination game with imperfect public monitoring. Such games were first studied by Green and Porter (1984) in the context of an oligopoly model with stochastic demand. Their paper initiated a whole line of research analyzing the set of equilibria for this interesting class of games (see, for example, Abreu, Pearce, and Stacchetti, 1990, and the paper by Lehrer, 1990, 1992a, 1992b). One of the main results is the proof of a Folk Theorem by Fudenberg, Levine, and Maskin (1994). According to this theorem, if the signal distribution satisfies certain rank conditions and if the discount factor is sufficiently large, then any feasible, individually rational payoff vector of the stage game can be supported as a perfect equilibrium of the infinitely repeated game.

Our game does not satisfy the rank conditions of Fudenberg, Levine, and Maskin's (1994) Folk Theorem but still it has a large set of equilibria even if it is only repeated finitely many times. In particular, any pattern of coordinated play is a Nash equilibrium of the repeated game. Our main concern in this paper is with the question of equilibrium selection. Dynamic models of learning, adaptive play or evolution[1] have no bite, in general, because they belong to the class of so-called uncoupled dynamics, in which a player's dynamic does not depend on the payoff functions of the other players. For this class of dynamics Hart and Mas-Colell (2003) have recently shown that they do not converge to a Nash equilibrium of the stage game, let alone select between different equilibria. Some dynamics select a unique equilibrium for specific games[2], but they require players to observe the history of play and hence cannot be applied under imperfect monitoring.

For our game the tracing procedure of Harsanyi and Selten (1988) selects the inefficient equilibrium with random behavior (in each period all players mix between "u" and "d" with probability 0.5). This equilibrium is, however, not observed in actual plays of the game. Indeed we are able to give experimental evidence for a different unique equilibrium selection. In this equilibrium the strategic players simply choose the previous period's outcome as their action. We call this equilibrium the "switch" equilibrium because actual play is coordinated on "u" or "d" unless the die breaks the coordination and everybody switches to the other action. We explain this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and over-confidence.

While a contribution to behavioral equilibrium selection may itself be of interest for game theory, we are interested in this simple game for a second reason. The game has a nice interpretation in terms of a financial market. We used this financial market interpretation for the framing of the laboratory experiment.

[1]For an overview see the books by Weibull (1995), Vega-Redondo (1996), Samuelson (1997), Young (1993) and Fudenberg and Levine (1998).

[2]For example, the adaptive play dynamics of Young (1993) and Kandori, Mailath, and Rob (1993) select the risk-dominant equilibrium in a 2×2 symmetric coordination game.

Imagine that, when a player chooses "u" ("d"), he initiates an order to buy (sell) one unit of an asset. Then the strategic players may be interpreted as managers of mutual funds, pension funds or hedge funds, while the die is representing noise traders. Indeed the payoff function of our game matches the reward function of those managers. In every period they are assessed in terms of the gains/losses resulting from the actions they have taken in that period. If the manager decides to buy (sell) and prices go up (down) in this period, she will get a positive reward. Otherwise she will get a lower reward. The actual price movement reflected in the majority rule then is a simple version of the law of demand and supply.

Of course this is a quite stylized model of a financial market, but the coordination game structure underlying a financial market has been put forward ever since Keynes' (1936) classical description of stock markets. Starting from the observation that very few investors hold stocks for ever, Keynes pointed out that for most investors the selling value of their stock will be more important than the dividends. Hence, beliefs about the fundamental value of a stock may be less important than higher order beliefs, i.e. beliefs about the other investors' beliefs about the asset price. As an analogy he compared stock markets to newspaper beauty contests in which the reader, whose choice coincides with the average pick, receives a prize. Thus, in the short run, guessing the average opinion on the stock market price is much more important than guessing the correct fundamental value. As a result, stock market prices may deviate from their fundamental values. According to Keynes they may even become an almost arbitrary social convention. While Keynes' analogy of the beauty contest does not contain a prediction about the degree of the deviation from the fundamental value, it has nevertheless made clear that in the short run stock markets exhibit the structure of coordination games, as it is also documented by the initial quote from a trader cited above. The coordination game structure of stock markets has recently also been emphasized in the behavioral finance literature. Shleifer (2000), for example, points out the importance of "noise trader risk," which is also called "market risk": All investment strategies based on fundamental values run the risk that the average investor does not follow the fundamental view. Even though the fundamental investor will eventually benefit from his strategy, in the short run he will lose and may even be deprived of his wealth before the long-term development of the asset prices turns to his favor. Or as Keynes has put it: “Markets can remain irrational longer than you can remain solvent.”

Keeping up this analogy to a coordination game, our paper shows that some of the main mechanisms underlying excess volatility, short-term momentum and long-term reversal [3] can be explained as the outcome of a repeated beauty contest with noise. Indeed the “switch” equilibrium has all these features. The volatility (measured in terms of variance) of prices is higher than that of the exogenous noise (given by the die), there are phases in which prices continue in the same direction and every now and then prices change their direction and they revert to the long-term average. Note that our model explains the upwards and the downwards trends of asset markets, commonly called “investor” sentiment, as a perfect Bayesian Nash equilibrium. That is to say, we give a rational explanation of investor sentiment. In particular, in contrast to Barberis, Shleifer, and Vishny (1998), the traders in our model have no misperceptions about the statistical distribution of the exogenous random process.

Recently, Hommes, Sonnemans, Tuinstra, and van de Velden (2005) have studied expectation formation in a similar asset market game. In their experiment agents also have to predict the future price of an asset, but they have no information about how the equilibrium price is determined given the agents' forecasts, which makes it difficult to compare the experimental results with some rational benchmark. In accordance with our result Hommes et al. find that subjects coordinate on a common prediction strategy within each group. However, the observed prediction strategy varies widely across groups. Hommes et al. do not explain their findings by behavioral principles as, for example, focal points, probability matching or overconfidence.

The results of our simple game may help to explain why technical analysis is a commonly observed investment style. In contrast to the efficient market hypothesis put forward by Fama (1970), in the game considered here prices have

[3] Empirical evidence has shown that stock prices often deviate substantially from their fundamental values and are more volatile than the dividends (Shiller, 1981). Moreover, short-term momentum and long-term reversal of stock market prices are empirically robust stock price anomalies (see, for example, Jegadeesh, 1990, De Bondt and Thaler, 1985, Lo and MacKinlay, 1999, Campbell, 2000, and Hirshleifer, 2001).

information content as they are a signal for the achieved coordination in the market. Indeed, in the switch equilibrium the players understand this signal and they base their trading on past prices.

In the next section we give a formal description of the game considered in this paper. Thereafter, in section 3, we present the results from a laboratory experiment. Section 4 explains the equilibrium selection observed in the experiment and section 5 concludes.

2 The Game

In the following we present the stochastic coordination game that we studied in a laboratory experiment.

2.1 The Stage Game

There are five players $i = 1, \ldots, 5$, who simultaneously choose between two actions "u" (up) and "d" (down).[4] There is exogenous noise which we model by a random variable X that is uniformly distributed on $\{0, 1, \ldots, 6\}$. We interpret X as the number of non-strategic players choosing action "u". Correspondingly, $6-X$ is the number of non-strategic players choosing action "d." The realization of X is not observed by the players. For any strategy profile $s = (s_1, \ldots, s_5)$ with $s_i \in \{u, d\}$ for all i, let $R(s)$ be the random variable defined by

$$R(s) = \begin{cases} u \text{ , if } |\{i \,|\, s_i = u\}| + X > |\{i \,|\, s_i = d\}| + 6 - X \\ d \text{ , else} \end{cases} .^5$$

Hence, $R(s) = u$, respectively $R(s) = d$, if the players' *sentiment*

$$|\{i \,|\, s_i = u\}| - |\{i \,|\, s_i = d\}|$$

plus the sentiment of the exogenous noise

$$X - (6 - X)$$

[4]A more general version of the game involves an arbitrary odd number of players. Since there were five players in our laboratory experiment w.l.o.g. we focus our discussion on this case.

[5]By $|A|$ we denote the cardinality of a set A.

is positive, respectively negative. Hence, R is called the *sentiment variable*. Observe that the exogenous noise can overrule the strategic players: If all players choose "u" and $X = 0$, then $R(s) = d$. Similarly, $R(s) = u$ if all players choose "d" and $X = 6$.

Player i's payoff is $G > 0$, if she correctly predicted whether the overall sentiment will be u or d, and it is zero, otherwise. More precisely, i's payoff at strategy profile s is a random variable $\Pi^i(s)$ given by

$$\Pi^i(s) = \begin{cases} G \text{ , if } s_i = R(s) \\ 0 \text{ , else} \end{cases} .$$

As we have argued in the introduction, this game can be motivated by trading decisions on asset markets. We can interpret the players in our game as agents acting on behalf of some principals of an investment fund. The agents' actions are given by buying ("u") or selling ("d") one unit of an asset. At the end of a period the principals reward their agents according to the success of their action taken at the beginning of that period. Clearly, buying one extra unit of an asset at the beginning of a period is optimal if and only if prices increase during that period. Similarly, selling is optimal if and only if prices decrease. If the agent's action was optimal she receives a positive reward $G > 0$ from the principal. Otherwise, she receives 0. In addition to the strategic traders there are agents who are trading for other reasons, for example in order to satisfy certain liquidity needs. The behavior of these "noise traders" can be modelled by an exogenous random variable X. The actual asset price movement R is determined by the actions of all agents, strategic and noise traders. If the market's sentiment is positive, i.e. if there are more buying than selling orders, then prices will go up. Conversely, if the market's sentiment is negative, prices will go down. Observe that in this stylized model of an asset market the size of the sentiment does not play any role. Prices go up (down) by one tick only, independent of whether the sentiment was strongly or weakly positive (negative). Also, traders can only buy or sell one unit of the asset.

The stage game is a symmetric coordination game in expected payoffs. It is immediate to see that it has two Nash equilibria in pure strategies, namely s^U

with $s_i^U = u$ for all $i = 1, \ldots, 5$, and s^D with $s_i^D = d$ for all $i = 1, \ldots, 5$. The expected payoff of agent i at these equilibria is $(6/7)G$.

In Appendix A we show that the stage game also has a unique Nash equilibrium in mixed strategies, where each player chooses "u" with probability 0.5. Observe that the expected payoff of a player in the mixed Nash equilibrium is smaller than her expected payoff in a pure strategy Nash equilibrium.

2.2 The Repeated Game

Consider now a finite repetition of the stage game introduced in the previous section. Let T be the number of repetitions. It is immediate to see that any sequence of pure strategy Nash equilibria of the stage game, i.e. any sequence of play that leads to coordination in all periods, is a pure strategy Nash equilibrium of the repeated game and vice versa. Among these there are two stationary pure strategy Nash equilibria, where all players play "u" or all players play "d" in all periods. We call this *stolid up*, resp. *stolid down* behavior. If we let R_t be the random variable that gives the realized sentiment in period t, then under stolid behavior the sentiment process $(R_t)_t$ is i.i.d. with

$$\text{Prob}(R_t = u) = \frac{6}{7}, \quad \text{Prob}(R_t = d) = \frac{1}{7}, \quad t = 1, \ldots, T,$$

for stolid up and

$$\text{Prob}(R_t = d) = \frac{6}{7}, \quad \text{Prob}(R_t = u) = \frac{1}{7}, \quad t = 1, \ldots, T,$$

for stolid down.

There are many other equilibria in pure strategies. Obviously any pure strategy Nash equilibrium is payoff equivalent to a Nash equilibrium in pure strategies which depend on public information only, i.e. only on past realizations of the sentiment variable R_τ and not on past actions taken by the player herself. One particularly simple Nash equilibrium in nontrivial public strategies is such that in all periods $t \geq 2$ all players choose

$$\begin{cases} u \text{ , if } R_{t-1} = u \\ d \text{ , else} \end{cases} .$$

Here, the realization of the sentiment variable in the last period is taken as a signal on which players coordinate their action. We call this *switch behavior* since the players' sentiment changes from an extreme "up" to an extreme "down" mood if and only if the noise has overruled them in the last period. Under switch behavior $(R_t)_t$ is a stationary Markov process with

$$\begin{aligned} \text{Prob}(R_{t+1} = u \,|\, R_t = u) &= \frac{6}{7}, \\ \text{and} \quad \text{Prob}(R_{t+1} = u \,|\, R_t = d) &= \frac{1}{7}, \end{aligned}$$

for all $t = 1, \ldots, T$.

The repeated game also has many (perfect Bayesian) Nash equilibria in mixed strategies. For example, any sequence of pure and mixed strategy Nash equilibria of the stage game gives rise to a mixed strategy Nash equilibrium of the repeated game. In particular, there is the stationary and symmetric mixed strategy Nash equilibrium, where in all periods all players choose "u" with probability 0.5. We call this *random behavior*. In this case $(R_t)_t$ is a random walk with

$$\text{Prob}(R_t = u) = 0.5$$

for all $t = 1, \ldots, T$. It is easy to see that any Nash equilibrium in strategies which depend on public information only must be given by a sequence of Nash equilibria (pure or mixed) of the stage game. In addition there is a plethora of (perfect Bayesian) Nash equilibria that depend on private information.

Summarizing we see that the repeated game has a large number of Nash equilibria, even if we restrict to pure strategy equilibria which are all strict. Moreover, the stochastic properties of the sentiment process $(R_t)_t$, which is the price process in our asset market interpretation of the game, critically depend on the equilibrium that is being played.

2.3 Equilibrium Selection

As we have seen in the last section the predictive power of Nash equilibrium is very limited so that the question of equilibrium selection arises. To our knowledge

Harsanyi and Selten (1988) is the only equilibrium selection theory that gives a reasonably narrow (and even unique) prediction for the game we are studying. Their procedure selects the inefficient equilibrium with random behavior (in each period all players mix between “u” and “d” with probability 0.5). This is due to symmetry reasons and the fact that the Harsanyi-Selten procedure always selects a unique equilibrium. Since our game is symmetric with respect to the actions “u” and “d” and since the selection must not depend on the labelling of these actions, there is only one equilibrium for which there does not exist a different equilibrium with the role of the actions “u” and “d” just reversed: the equilibrium with random behavior. This gives a testable hypothesis since the observable sentiment process $(R_t)_t$ is a random walk under random behavior. Hence, coming back to the financial market interpretation of our game, where the sentiment variable corresponds to the asset price change, we see that random behavior is inconsistent with stock price phenomena like momentum, mean reversion and excess volatility which we observe on real stock markets.

A different approach to equilibrium selection in the stage game is by dynamic models of learning, adaptive play or evolution.[6] In general, as it was shown by Hart and Mas-Colell (2003) so-called uncoupled dynamics, where a player's dynamic does not depend on the payoff functions of the other players, do not converge to a Nash equilibrium of the stage game, let alone select between different equilibria.[7] Only for special classes of games some dynamics have been shown to converge to the set of Nash equilibria and in even more special cases they have been shown to converge to a unique equilibrium. For example, the adaptive play dynamics of Young (1993) and the evolutionary dynamics of Kandori, Mailath, and Rob (1993) select the risk-dominant equilibrium in a 2×2 symmetric coordination game. However, these dynamics require players to observe the history of play, in particular the actions chosen by the other players in the past, and hence cannot be applied to our game of imperfect public monitoring. Moreover, the two pure strategy Nash equilibria of our stage game generate the same payoffs,

[6]For an overview see the books by Weibull (1995), Vega-Redondo (1996), Samuelson (1997), Young (1993) and Fudenberg and Levine (1998).

[7]Basically all adaptive dynamics considered in the literature, like best-reply, fictitious play, regret-based dynamics, reinforcement learning, replicator dynamics etc. are uncoupled.

so no unbiased dynamics will select between the two pure strategy equilibria of our stage game.

If there is no dynamic model that selects a unique equilibrium of our stage game, the next question is whether there are dynamics that induce behavior which is consistent with some non-stationary equilibrium of the repeated game. It should come at no surprise that the answer is positive. Consider, for example, the adaptive dynamic, where in each period, any player chooses a best-reply to a maximum-likelihood estimation of the other players' action in the last period. This dynamic induces switching behavior as in the switch equilibrium we described in section 2.2: If the sentiment variable is R_t in period t, then, independently of player i's action in t, it is most likely that all players $j \neq i$ have chosen action R_t. Hence, if i myopically expects all players to retain their action in period $t+1$, then it is a best-reply to play R_t in period $t+1$. Since this is true for all i, agents will play according to the switch equilibrium in all periods $t \geq 2$. We do not want to argue whether this is a reasonable behavioral model or not.[8] Instead we would like to point out that it is straightforward to construct similar adaptive models, which support other equilibria of the repeated game. So no equilibrium seems to be particularly prominent in this respect.[9]

3 The Experiment

In this section we present the results of an experiment where the game was played in a computer laboratory. Given our theoretical analysis and the negative results concerning equilibrium selection presented in the previous section we would expect to observe a wide variety of play across groups, even if the subjects within one group learn to coordinate on one equilibrium. Surprisingly, however, we find that after an initial learning phase most groups in our experiment played according to the switch equilibrium.

[8]It is clearly no learning model since players never learn the true strategies used by their opponents.

[9]For example, any player may choose a best-reply to a maximum-likelihood estimation of the other players' actions in the last three periods, assuming that the others choose the same action in all periods, etc.

3.1 Experimental Design

We framed the experiment according to the financial market interpretation of the coordination game. In our view this is the most interesting economic application of our game and the framing allows us to compare the experimental results with stylized facts about stock prices that are observed on real markets. Accordingly, participants in the experiment were asked to bet on an increasing, u(p), or decreasing, d(own), asset price movement and they gained whenever their prediction matched the actual asset price movement.

Each group consisting of five participants played the game in two rounds with 100 periods each, where the first round was intended to be a learning phase. The details of the experimental procedure are presented below.

3.2 Experimental Procedure and Subject Pool

The computerized experiment was conducted in the computer laboratories of the University of Zurich in November and December 2001.[10] It lasted for approximately 90 minutes with the first 20 minutes consisting of orientation and instructions.[11] A total of 50 students from the University of Zurich participated in the experiment. They were recruited by announcements in the university promising a monetary reward contingent on performance in a group decision making experiment. The average payoff of a participant was 40 CHF (approximately $25, at that time).[12]

We had 5 sessions with 2 groups each. Participants were assigned randomly to a group and played the game via computer terminals. The computer terminals were completely separated from one another preventing any communication between the participants. After the first round with 100 periods the experiment was restarted and the same groups played a second round with 100 periods. Subjects were not allowed to communicate at any time during the experiment and they were monitored to make sure that they observe this rule. In particular, subjects

[10] The experiment was programmed and conducted with the software z-Tree (Fischbacher, 1999).

[11] An English translation of the instructions can be found in Appendix B.

[12] The payoff was the sum of a show-up fee and of the payoffs for the single plays and the strategy game, which are described in the following.

were not allowed to communicate during the short break between the two rounds of play. The participants' payoffs were given in ECU (experimental currency units), where 100 ECU corresponded to 0.25 CHF (approximately \$0.15).

The realization of the noise variable was determined by a 10-sided die, the throw of which was repeated until a number between 0 and 6 was realized. The realized number was invisible to the subjects. Both groups in each session played with the same die so that the exogenous noise was the same for these groups. The gain for a correct guess in any period was G= 20 ECU. In each period the price movement and the subject's gain in the previous period were displayed on her computer terminal. The price changes in the last seven periods were also visible. Subjects could see the whole history of the given round by scrolling down in the field, in which the last seven periods were displayed. At no time the subjects got any information about the previous actions of the other members of their group or about the previous realizations of the die.

After the two rounds of the game subjects played a strategy game. All participants had to define a strategy, which we coded in a computer program and with which the game was played afterwards.[13] For this play the participants were randomly matched to each other. In the strategy game the participants were asked to indicate when they change their strategy from u(p) to d(own) and when they change from d(own) to u(p). They were free to write down their strategy as they wanted, e.g. they could choose a free text description of their strategy. One play of the strategy game over 100 periods was paid per person. In the strategy game 100 ECU corresponded to 1 CHF, i.e. the payoff was four times the payoff in the single plays. Finally, we asked the subjects to fill out a questionnaire concerning their own behavior during the experiment and their beliefs about the strategies chosen by their opponents.[14]

After the experiment was completed the participants were paid separately in cash contingent on their performance in the single plays of the game and in the strategy game.

[13] An English translation of the instructions for the strategy game can be found in Appendix B.

[14] See Appendix C for an English translation of the questionnaire.

3.3 Predictions

From the theoretical analysis in sections 2.2 and 2.3 we deduce the following main hypotheses which we will test with our experiment.

Hypothesis 1: Subjects play according to some pure strategy Nash equilibrium of the repeated game.

Hypothesis 2: There is no unique equilibrium selection.

3.4 Results

In Appendix D we present some charts summarizing the data of our experiment. The first series of charts displays the frequency of "up" choices among subjects in both rounds. As we see, for the majority of groups the degree of coordination is quite low in the beginning of play in round 1 while it is very high during the second round of play, where at least 4 and in most periods 5 subjects choose the same action. Hence, the charts seem to confirm our first hypothesis, that subjects play according to some pure strategy Nash equilibrium of the repeated game. The following analysis will provide a rigorous test of this hypothesis.

The same charts, which display the subjects' degree of coordination, also display the price movement, so that we can analyze the strategies used by the subjects in the course of the game. Contradicting our hypothesis 2 we find much evidence for a unique equilibrium selection: with the exception of group 5, all groups seem to play according to the switch equilibrium in the majority of periods during round 2, i.e. in each period they bet on the price movement in the previous period (groups 6 and 9 show this behavior in its purest form). Below we will present a rigorous analysis of the data which will lead to a rejection of hypothesis 2.

Appendix D also presents another series of charts which display the cumulated change in the noise traders' sentiment as determined by the die and the cumulated change in the price movement.[15] A first inspection of these charts shows that the

[15]By definition the noise traders' sentiment is positive, if the die shows $4, 5$, or 6 points, it is negative, if the die shows $0, 1$, or 2 points, and it is zero, if the die shows exactly 3 points.

volatility (measured in terms of variance) of the price movement is much higher than the volatility of the exogenous noise. Moreover, in round 2, there are long phases in which prices move in the same direction. These phases are broken every now and then and the price reverts to its long-term average. In the introduction we explained that these phenomena of excess volatility, short-term momentum and long-term reversal we identify in our experiment can also be observed on real financial markets. The finance literature classifies these price properties as "anomalies" and explains them by irrational behavior on the part of investors. By contrast, our experiment shows that they can also be obtained as the result of a particular equilibrium selection in a game, which captures the coordination game structure of stock markets. Below we will present a detailed analysis of the price process in our experiment.

Coordination

For a more detailed analysis of hypothesis 1 we determine the degree of coordination for all groups throughout round 2. Table 3 shows the number of periods in round 2, where $3, 4$, or 5 subjects within one group chose the same action.[16] From the data in the table we see that the degree of coordination is indeed very high in round 2: no coordination, i.e. exactly 3 subjects choosing the same action, is observed in no period for 6 out of 10 groups, it is observed in 1 period for 2 groups and in 6 periods for 1 group. The only exception is group 5 which shows no coordinated play in 40 periods. Taking each group as an independent observation we can therefore reject the hypothesis that there is no coordinated play at the 1% level in a one-sided binomial test.

The question then is whether the degree of coordination is large enough such as to conclude that subjects play according to some pure strategy Nash equilibrium.[17] If we count the number of periods with full coordination, i.e. all subjects choosing the same action, we find that 4 out of 10 groups are fully coordinated in more than 90% of the periods, 7 groups are fully coordinated in more than

[16]Observe that there are always at least 3 players who choose the same action in any period. Hence, 3 subjects choosing the same action corresponds to no coordination.

[17]Recall from section 2.2 that a strategy profile is a pure strategy Nash equilibrium of the repeated game if and only if it leads to full coordination in all periods.

80% of the periods, and 9 groups are fully coordinated in at least 74% of the periods. Again group 5 is the only exception, showing full coordination in 14% of the periods only. Hence, 9 out of 10 groups are fully coordinated in a clear majority of periods in round 2 so that we can reject the hypotheses that subjects do not play according to a pure strategy Nash equilibrium of the repeated game at the 1% level in a one-sided binomial test.

Learning to Coordinate

In order to examine whether subjects learn to coordinate over time we determine the cumulative frequency of coordinated play by at most 3 and 4 players within each group and compare it across intervals with 50 periods in both rounds (see Table 4). If subjects learn to coordinate over time, then we should observe that the cumulative frequency distribution in periods $51 - 100$ of round 2 first order stochastically dominates (FOSD) the distribution in periods $1 - 50$ of round 2, which in turn FOSD the cumulative frequency distribution in periods $51 - 100$ in round 1, which in turn FOSD the distribution in periods $1 - 50$ in round 1. As can be seen from Table 4 this is indeed the case: For 5 out of 10 groups the cumulative frequency for coordinated play by 3 and 4 players is monotonically decreasing over time. For 4 groups there is only a slight violation of FOSD in round 2 between periods $1 - 50$ and $51 - 100$. Only for group 5 we do not find any monotonicity in the degree of coordination.

We apply a chi-square test in order to test the null hypothesis that the distribution of the degree of coordination, which has support $\{3, 4, 5\}$, is the same in the first 50 periods of round 1 and in the last 50 periods of round 2. For 6 groups (2, 3, 4, 6, 8, 10) we can reject this hypothesis at the 1% level, and for 2 groups (1 and 7) we can reject it at the 10% level (p-values are 0.068 for group 1 and 0.073 for group 7). For groups 5 and 9 the hypothesis cannot be rejected. These groups are either very well coordinated from the very beginning (group 9), so that learning in is not significant, or they do not show a high degree of coordination during the whole game (group 5).

Although almost all groups learn to coordinate over time, there is a large heterogeneity in the speed of learning across groups. For example, groups 1, 7, 9 and 10 are already well coordinated during the first 50 periods while group 2 does not show a high degree of coordination till the beginning of round 2 (see Table 4). In order to further analyze the speed of learning we define the *learning phase* as follows. In accordance with our analysis above, we say that a group is *coordinated* in some period τ if at least four members of the group choose the same action in τ.[18] The end of the learning phase then is the first period t in round 1 such that the group is coordinated in all periods τ thereafter, $t+1 \leq \tau \leq 100$.[19] Table 5 shows the length of the learning phase for all groups which are coordinated in round 2, which, as we have seen, are all groups except group 5. We see that there is a sizable learning phase (the mean is 50.11 and the median is 38 periods) and that there is a large heterogeneity in the speed of learning across groups. Groups 7, 9, and 10, can be classified as fast learners (coordination is achieved within the first third of the periods in round 1), groups 1, 3, 4, and 10, can be classified as moderate learners (coordination is achieved within the second third of the periods in round 1), and groups 2, 6, and 8, can be classified as slow learners (coordination is achieved only in the last third of the periods in round 1).

Equilibrium Selection

Given that we observe equilibrium play during round 2 the next question is whether all groups play according to the same equilibrium which would lead to a rejection of hypothesis 2. A first inspection of the charts in Appendix D has already shown that there is considerable evidence for the selection of the switch equilibrium. For a more detailed analysis we count for each subject the number of periods within the learning phase and within round 2, where the switch strategy was being played, i.e. where the action was identical to the observed price change in the previous period (see Table 6). We then determine the corresponding cumulative distribution over the relative frequency of play of the switch equilibrium. Figure 1 displays this cumulative distribution. We have excluded the members of group 5 from the subject pool since this group does not play according to an equilibrium in round 2, so that the question of equilibrium selection is irrelevant

[18] A group is *fully coordinated* in a period τ, if five members of the group choose the same action in τ.

[19] In case the length of the learning phase is close to 100 according to this definition we define it to be equal to 100.

for this group. Hence, there are 45 subjects left in the pool.

As can be seen in Figure 1 more than 90% of the subjects play the switch strategy in more than 90% of the periods in round 2. Already in the learning phase we observe a predominant play of the switch strategy: approximately 56% of the subjects choose the switch strategy in more than 80% of the periods within the learning phase and approximately 82% of the subjects choose the switch strategy in more than 70% of the periods within the learning phase. One may suspect that this predominant play of the switch strategies is mainly driven by subjects' betting on a trending price movement. Hence, we do the same analysis but restrict to those periods t that follow a change in the price movement, i.e. to those periods t, for which $R_{t-1} \neq R_{t-2}$ (see Table 6). Still we find that the switch strategy is chosen much more frequently than any other strategy (see Figure 2): more than 75% of the subjects use the switch strategy, i.e. follow the change in the price movement, in more than 90% of the periods where such a change occurred during round 2. During the learning phase these numbers are smaller but even there more than 50% of the subjects use the switch strategy in more than 60% of the periods where such a change occurred during the learning phase. And approximately 33% of the subjects follow the change in the price movement in more than 70% of the periods in which this event occurred during the learning phase.

For each subject we then test the hypothesis that he follows the price movement with probability 0.5 or lower. We can reject this hypothesis at the 1% level for all 45 subjects in a one-sided binomial test. Within the learning phase we can reject it at the 1% level for 31 subjects, at the 5% level for 3 subjects and at the 10% level for 5 subjects. For 6 subjects we cannot reject the hypothesis at a reasonable level of significance within the learning phase. If we restrict to those periods in round 2, which follow a change in the price movement, we find that for 30 out of 45 subjects we can reject the hypothesis at the 1% level; for 7 subjects we can reject it at the 5% level and for 3 subjects we can reject it at the 10% level. Only for 5 subjects the hypothesis cannot be rejected at a reasonable level of significance. For the learning phase we get a different picture now: Only for 5 subjects we can reject the hypothesis at the 1% level, for 4 subjects at the 5% level and for 3 subjects at the 10% level, while for 33 subjects we cannot reject the hypothesis. This result suggests that in the course of play subjects mainly learn which action to take after there was a change in the price movement.

The main conclusion from our analysis above is that subjects, who belong to groups showing coordinated play in round 2, play according to the switch equilibrium strategy in round 2. Hence, we can reject our hypothesis 2 that there is no unique equilibrium selection.

To give further evidence for the selection of the switch equilibrium we analyze the strategies specified by experienced subjects in the strategy game. As shown in Table 7 in the first period all subjects choose u(p). In later periods they were asked to indicate under which conditions they switch their action from u(p) to d(own) and vice versa. It turns out that for no person the decision to switch does depend on the direction of the switch (from u(p) to d(own) or from d(own) to u(p)). As can be seen in Table 8 there are five types of strategies which we can reduce to two main types. The first three are the switch strategy and some variants, i.e. switch after being wrong once, twice, or three times. 41 out of 50 subjects choose the switch strategy and 4 choose a variant of it. 5 subjects also switch after being right for a certain number of periods.[20] One explanation for the latter type of strategy is that these subjects fall into gamblers' fallacy and believe in a reversal for no good reason. Summarizing, the strategy game confirms the equilibrium selection we observed in the play of the game: the majority of subjects plays according to the switch equilibrium strategies.

Price Process

We close our analysis of the experimental data by studying some properties of the price process we observe in round 2. The first observation concerns the volatility of the exogenous noise and of the endogenous price movement. We take the volatility of the noise as a reference to determine excess volatility, because in our model the price movement would follow the exogenous noise if no strategic agents were present. For each group we compute the standard deviation of the cumulated price movement and the cumulated noise traders' sentiment. Here, we

[20]Group 5, which did not show equilibrium behavior, consisted mainly of players who selected strategies in the strategy game which where of this type.

define

$$P_t := |\{\tau \,|\, \tau \leq t, R_\tau = u\}| - |\{\tau \,|\, \tau \leq t, R_\tau = d\}|$$

to be the cumulated price movement

$$N_t := |\{\tau \,|\, \tau \leq t, X_\tau \geq 4\}| - |\{\tau \,|\, \tau \leq t, X_\tau \leq 2\}|$$

to be the cumulated noise-traders' sentiment in period $t, t \geq 1$, and we set $P_0 = N_0 = 0$. As we can see in Table 9, for all groups the standard deviation of the cumulated price movement is higher than the standard deviation of the cumulated noise traders' sentiment. Hence, the hypothesis that the price and the exogenous noise show the same volatility can be rejected on a 1% level in a one-sided binomial test.

Our next observation concerns the momentum of the cumulated price movement. As we can see in Table 10 for all groups the empirical frequency of $R_t = R_{t-1}$ is strictly larger than 0.5. For all groups we can reject the hypothesis that the price movement does not show any momentum, i.e. $\text{Prob}(R_t = R_{t-1}) \leq 0.5$, at the 1% level in a chi-square test. Taking each group as an independent observation we can therefore reject the hypothesis that the price movement does not show any momentum at the 1% level in a binomial test.

Finally, we test for mean-reversion in the cumulated price movement. A commonly used test is the *variance ratio test*, which exploits the fact that under the random walk hypothesis the variance of the qth differences $P_{t+q} - P_t$ is linear in q. Hence, if $\hat{\sigma}^2(q)$ denotes the sample variance for the qth differences, then the variance ratio

$$\text{VR}(q) = \frac{\frac{1}{q}\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)}$$

is close to 1 under the null hypothesis of a random walk.[21] If $\text{VR}(q)$ is significantly greater than 1 we can reject the random walk hypothesis in favor of trending behavior, while if $\text{VR}(q)$ is significantly smaller than 1 we can reject the random walk hypothesis in favor of mean reverting behavior. Table 11 reports the value

[21]Adjusting for finite samples and overlapping qth differences we define $\hat{\sigma}^2(q) := \frac{1}{(nq-q+1)(1-\frac{1}{n})}\sum_{k=q}^{nq}(P_k - P_{k-q} - q\hat{\mu})^2$, where $\hat{\mu} = \frac{1}{nq}(P_{nq} - P_0)$ is the sample mean and $nq = 100$ is the sample size.

of the variance-ratio test statistic for all groups in round 2. Due to the integer constraint we are restricted to $q \in \{10, 20, 25, 50\}$. As we see, we can clearly reject the random walk hypothesis at the 1% level. For all groups we observe highly significant trending behavior on a short horizon ($q = 10$). On longer horizons ($q = 25$) we observe mean reverting behavior for 6 groups, which, however, is significant for one group only.

4 Explaining the Equilibrium Selection

The main findings from our experiment are that

1. subjects learn to play according to a pure strategy Nash equilibrium of the repeated game,

2. the equilibrium selection is unique and is given by the switch equilibrium.

Both results are remarkable given that there is no communication and given that players do not monitor the actions of their opponents. As we have argued in section 2.3, Harsanyi and Selten (1988) is the only equilibrium selection theory which gives a clear prediction for the game we are considering. However, according to their theory the equilibrium with random behavior should be selected which is not confirmed by our experiment. Also, we have seen (see section 2.3) that there is a simple adaptive model which supports the switch equilibrium. According to this model, all players in each period play a best-reply to a maximum-likelihood estimation of the others' actions in the last period. While this adaptive model clearly can explain the selection of the switch equilibrium it does not seem to explain well the way the subjects in our experiment did understand what is going on in the game. In the questionnaire we conducted after the experiment we asked the subjects whether they think the die or the behavior of the other players in their group was responsible for a change in the price movement after long periods of increasing prices (see Appendix C, Questionnaire, Question 5). 42 out of 48 subjects, who gave a clear answer to this question, stated that they think the die was responsible for the change in the price movement. Maximum-likelihood estimation of the others' actions based on the last period only is not consistent with this answer, because it requires subjects to attribute any outcome to the other participants' choices.

In the following we will propose an explanation for the equilibrium selection, which is based on three behavioral arguments: focal-points, probability matching and over-confidence.

The first explanation relates to the maximum-likelihood estimation we discussed above. We have seen that the majority of subjects in our experiment attributes a change in the price movement to the exogenous noise and not to their opponents' behavior. As a consequence, if players believe that their opponents have the same understanding of the game, then there is no reason to switch the action, if the price movement changes from up to down or vice versa. Hence, we should observe stolid behavior. However, it is frequently found that subjects do not believe that others think or behave in the same way as they do. Instead, subjects often are over-confident, i.e. they believe to have understood a situation better than the average participant. In our case, an over-confident player has understood that the die was responsible for breaking a trend while she may believe that her opponents are naive in that they attribute the change in the price movement not to the noise but to the others' behavior, for example because they perform a naive maximum-likelihood estimation. If the player in such a situation actually switches, she reveals that she found it more likely that the average opponent did not understand the reason for the break in the trend. Hence, we can explain the selection of the switch equilibrium by over-confidence on the part of the players.

Our second explanation is based on a focal-point analysis. Games of pure coordination, like the one we are studying, have several equilibria which are indistinguishable from an abstract point of view, since they are all payoff equivalent. Nevertheless, we may observe a unique equilibrium selection in any actual play of the game. This situation was already illustrated by Schelling (1960) with his well known example about two strangers having to decide about a meeting point in New York without being able to communicate with each other. Schelling introduced the idea that persons coordinate on "focal points" (like the Grand Central

Station in New York) if they have to solve such a problem. Obviously, such a coordination requires the existence of a common "frame," in particular actions must be labelled in the same way for all players. If there is no common frame the players are in a state of complete ignorance about how their opponents perceive the game, so that mixing uniformly between all actions seems to be the only reasonable thing to do. Applied to our game, in the absence of a common frame we would expect to observe the mixed equilibrium we named "random behavior." However, since actions in our game are labelled "u" and "d," there is a common frame and the notion of a focal point can, in principle, be applied. Both actions, u(p) and d(own), could be focal leading to the stolid u(p), respectively stolid d(own) equilibria. One may suspect that the action u(p) is the focal one, which is also confirmed by the participants' choice in the first period of the strategy game (see Table 7). Nevertheless, in our experiment we neither observe the stolid u(p) nor the stolid d(own) equilibrium, which can be explained es follows.

Even if players think that stolid up behavior is the most reasonable strategy (it is focal and it is simple), they may doubt that their opponents have arrived at the same conclusion. Hence, having no idea about their opponents' behavior in the beginning of play, it seems reasonable to assume that others behave randomly, i.e. mix between u(p) and d(own) with probability 0.5. Consequently, players perceive the game as a single-person decision problem, namely the degenerate coordination game we obtain for $n = 1$. Clearly, given that the exogenous noise is unbiased, any strategy is optimal in this game. However, as it is known from many psychological studies (for a review see Fiorina, 1971, or Brackbill and Bravos, 1962) animals and human beings tend to perform probability matching in such single person decision situations. This kind of behavior was also regarded as important for decision making by Arrow (1958). In our case probability matching means that players select their strategy such that the frequency of u(p) choices is equal to the probability that the sentiment of the exogenous noise is positive or non-negative which is equal to 3/7 or 4/7. According to this reasoning we should observe switching between actions analogous to probability matching in the beginning of play. Looking at the data of our experiment we see that indeed there is considerable switching between u(p) and d(own) in the first 20 periods of round 1 before coordination is achieved in most groups (see Figure 3). The mean number of u(p) choices is 13.52 and hence is only slightly above $20 \times 4/7 = 11.43$, which is the number of u(p) choices we would expect to observe in a single person decision problem under probability matching. Probability matching behavior in the beginning of play rules out the occurrence of the stolid up (or stolid down) equilibrium.

Hence, given that there does not seem to be a "universal" focal action in our coordination game, the next question is whether in each period there exists a (potentially different) action, which will be considered as focal by all players. And indeed, the sentiment variable (price movement) is an endogenous and publicly observable signal, which can be used to label an action so that it becomes the focal one. In principle, any history of past realizations of the sentiment variable can be used as a signal but we will argue that the last period's sentiment is the prominent one. Firstly, using the realized sentiment in more than one period requires a sophisticated rule about how to translate this multidimensional signal into an action. Hence, one coordination problem is replaced by another, making the use of a multidimensional signal very unreasonable.[22] A different argument in favor of using a one-dimensional signal, i.e. the sentiment (or price movement) in a single period, relies on costs (cf. Binmore and Samuelson, 2004). If the observation and processing of a signal is costly, for example because it causes disutility to interpret complex signals, then the players' payoffs are maximized if they use a one-dimensional signal only.[23] Assuming that the cost of observing and processing a signal is not too high the players' payoff is higher when they use the signal than it is when they don't, since in the latter case they are unable to identify focal points and will most likely fail to coordinate.

Secondly, using the last period's sentiment as a signal seems to be more prominent than using the sentiment in any other previous period. The time scale induces a common frame which makes the last period's sentiment a focal signal. Summarizing, in the switch equilibrium players overcome the coordination problem by choosing in each period the action that is focal according to the publicly

[22] Of course, there is also not a unique way to translate the sentiment in one period into an action but choosing u(p) and not d(own) when the signal was "up" clearly is focal here.

[23] Provided, of course, they use the signal in the most efficient way, so that they achieve perfect coordination of their actions.

observed signal, namely last period's sentiment.

We have explained the selection of the switch equilibrium by over-confidence, probability matching and a focal-point property. These may be the reasons why the switch strategy is more frequently used than any other strategy already during the learning phase (see section 3.4). Given this bias towards switching behavior, during the learning phase subjects will necessarily learn that switching behavior is more successful than any other strategy, since switching is a best-reply to switching. As a consequence, subjects move from the rather uncoordinated behavior in the beginning of play to the coordinated play of the switch strategy.

5 Conclusion

We have studied a repeated stochastic coordination game with imperfect public monitoring. In this game any pattern of coordinated play is a perfect Bayesian Nash equilibrium. Moreover, standard equilibrium selection arguments either have no bite or they select an equilibrium that is not observed in actuals play of the game. We gave experimental evidence for a unique equilibrium selection in this game and explained this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and over-confidence.

Our results have interesting applications in finance because the observed equilibrium price process exhibits momentum, reversal and excess volatility. Indeed the behavioral arguments that we employed in the stylized asset market we considered are all well known from the behavioral finance literature (cf. Shefrin, 2000, Shiller, 1981, Shleifer, 2000, Barberis and Thaler, 2003). The behavioral finance literature has attributed large increases in investment activity on stock markets to the greater fool theory, i.e. to the idea that investors believe to find other investors that are still willing to buy their stock at a higher price. This is a clear sign of over-confidence since it is impossible for the majority of investors to be on the winning side of this speculation. Moreover, the large amount of trading activity exhibited by the average investor is attributed in this literature to probability matching, i.e. to the desire to employ a trading strategy that somehow matches the frequent ups and downs of the random process one is trying to master. As Shefrin (2000) has shown, for most traders a stolid buy&hold strategy would have obtained much better results for their investments. Finally, observed asset prices are a focal point in the search for signals revealing which information the other investors have at a time. Indeed, as Treynor and Ferguson (1985) have argued, technical analysis can be profitable due to the information it may reveal about the other traders' state of information.

Of course, a real stock market is much more complicated than the stylized asset market we considered in this paper. But on the other hand our much easier setting has the advantage of being able to demonstrate more cleanly the role of focal point analysis, probability matching and over-confidence in the coordination of investors' actions on asset markets, that is shown to lead to momentum, reversal and excess volatility in asset prices. Further research will have to enrich our simple model with more realistic features like a dividend process with asymmetric information on the part of the investors.

A Mixed Strategy Equilibrium

In the following we show that the unique mixed strategy Nash equilibrium of the game is such that all players choose "u" with probability 0.5.

Let $I = \{1, \ldots, 5\}$ and let α_i be the probability with which player i chooses "u." Then $\alpha = (\alpha_1, \ldots, \alpha_5)$ is a mixed strategy Nash equilibrium if and only if the following condition is satisfied: For all i, $\alpha_i = 1$ implies that

$$\begin{aligned}
&\text{Prob}(R = u \,|\, \alpha_{-i}, s_i = u) \geq \text{Prob}(R = d \,|\, \alpha_{-i}, s_i = d) \\
\Longleftrightarrow\ & \sum_{l=0}^{4} \frac{l+2}{7} \sum_{\substack{K \subset I \setminus \{i\} \\ |K| = l}} \prod_{k \in K} \alpha_k \prod_{\substack{k \notin K \\ k \neq i}} (1 - \alpha_k) \geq \sum_{l=0}^{4} \frac{l+2}{7} \sum_{\substack{K \subset I \setminus \{i\} \\ |K| = l}} \prod_{k \in K} (1 - \alpha_k) \prod_{\substack{k \notin K \\ k \neq i}} \alpha_k \\
\Longleftrightarrow\ & \sum_{l=0}^{4} \frac{2l-4}{7} \sum_{\substack{K \subset I \setminus \{i\} \\ |K| = l}} \prod_{k \in K} \alpha_k \prod_{\substack{k \notin K \\ k \neq i}} (1 - \alpha_k) \geq 0,
\end{aligned}$$

$\alpha_i = 0$ implies that the inequality holds with "$\leq$", and $\alpha_i \in (0,1)$ implies that the inequality is an equality "$=$".

It is immediate to see that α with $\alpha_i = 0.5$ for all i satisfies the condition above. In order to prove that this is the unique mixed strategy Nash equilibrium let i be such that $\alpha_i \in (0,1)$ and assume by way of contradiction that there exists $j \neq i$ such that $\alpha_j \neq \alpha_i$.[24] If $\alpha_j \in (0,1)$, then from the condition above it follows that

$$(\alpha_j - \alpha_i) \frac{2}{7} \sum_{l=0}^{3} \sum_{\substack{K \subset I \setminus \{i,j\} \\ |K| = l}} \prod_{k \in K} \alpha_k \prod_{\substack{k \notin K \\ k \notin \{i,j\}}} (1 - \alpha_k) = 0$$

which is impossible if $\alpha_i \neq \alpha_j$. Similarly, one can show that $\alpha_j = 1$ and $\alpha_j = 0$ lead to a contradiction. Hence, $\alpha_1 = \alpha_2 = \ldots = \alpha_5 =: \bar{\alpha} \in (0,1)$.

Assume by way of contradiction that $\bar{\alpha} \neq 0.5$. W.l.o.g. let $\bar{\alpha} > 0.5$. Then, from the condition above it follows that

$$0 = \sum_{l=0}^{4} \frac{2l-4}{7} \sum_{\substack{K \subset I \setminus \{i\} \\ |K| = l}} \bar{\alpha}^l (1 - \bar{\alpha})^{4-l} > \sum_{l=0}^{4} \frac{2l-4}{7} \binom{4}{l} (1 - \bar{\alpha})^4 = 0,$$

which is impossible. Hence, if α is a mixed strategy Nash equilibrium with $\alpha_i \in (0,1)$ for at least one i, then $\alpha_i = 0.5$ for all $i = 1, \ldots, 5$.

[24] If there exists no i with $\alpha_i \in (0,1)$, then the mixed strategy Nash equilibrium is in fact a pure strategy Nash equilibrium.

B Instructions

Following is an English translation of the instructions for the single game as well as for the strategy game as they were given to the participants of the experiment.

B.1 Instructions for the Single Game

Welcome! You are participating in a game about the development of security prices. Your payoff depends on your performance in the game.

Instructions

Participants

Altogether there are 5 players in your group.

Overview of the Game

The game is played for 100 periods. In each period you have to predict whether the price of a security goes up or down. You get a positive payoff if your prediction is correct, otherwise you do not get a payoff.

Your Endowment and Actions

In each period you get 1 point which you can place on any of the following alternatives:

A: the security price goes up
B: the security price goes down

Your Payoff

At the end of each period you receive a payoff of 20 ECU (Experimental Currency Units) if you correctly predicted the movement of the security price in that period. That is you get 20 ECU if either you put 1 point on A (the security price does up) and the security price went up, or if you put 1 point on B (the security price goes down) and the security price went down. Otherwise you get 0 ECU.

The Determination of the Security Price Movement

Whether the security price goes up or down in a period is a result of the decision of all players and of the throw of a fair die which has seven sides with $0, 1, ..., 5, 6,$ points. All sides are equally likely.

After all players have put their point on either A (the security price goes up) or B (the security price goes down) the die is thrown. Afterwards the total number of points on A and on B is determined. The points on the die are added to the sum of the points which the players placed on A. (6 - the points on the die) is added to the sum of the points which the players placed on B.

The security price goes up if the total number of points on A (the security price goes up) is larger than the total number of points on B (the security price goes down). Otherwise the security price goes down. Since the maximal sum of points for an alternative is 11, the security price goes up if the points for alternative A (the price goes up) are at least 6. The price goes down if the points for alternative B (the price goes down) are at least 6.

Your Information

At the end of each period you are informed about the movement of the security price and about your payoff in this period. You do not get any information about the decisions of the other players or about the result of the throw of the die. In addition the price movement in all previous periods is displayed.

Tables

Tables 1 and 2 summarize the determination of the security price movement depending on the decisions of all players and on the throw of the die.

Table 1: Total Number of Points on A (the security price goes up)

Points on the Die	Number of persons who choose A (the security price goes up)					
	0	1	2	3	4	5
	Number of persons who choose B (the security price goes down)					
	5	4	3	2	1	0
0	0 Points	1 Point	2 Points	3 Points	4 Points	5 Points
1	1 Point	2 Points	3 Points	4 Points	5 Points	6 Points
2	2 Points	3 Points	4 Points	5 Points	6 Points	7 Points
3	3 Points	4 Points	5 Points	6 Points	7 Points	8 Points
4	4 Points	5 Points	6 Points	7 Points	8 Points	9 Points
5	5 Points	6 Points	7 Points	8 Points	9 Points	10 Points
6	6 Points	7 Points	8 Points	9 Points	10 Points	11 Points

$\geq$ 6 Points: The security price goes up.

Table 2: Total Number of Points on B (the security price goes down)

Points on the Die	Number of persons who choose A (the security price goes up)					
	0	1	2	3	4	5
	Number of persons who choose B (the security price goes down)					
	5	4	3	2	1	0
0	11 Points	10 Points	9 Points	8 Points	7 Points	6 Points
1	10 Points	9 Points	8 Points	7 Points	6 Points	5 Points
2	9 Points	8 Points	7 Points	6 Points	5 Points	4 Points
3	8 Points	7 Points	6 Points	5 Points	4 Points	3 Points
4	7 Points	6 Points	5 Points	4 Points	3 Points	2 Points
5	6 Points	5 Points	4 Points	3 Points	2 Points	1 Point
6	5 Points	4 Points	3 Points	2 Points	1 Point	0 Points

$\geq$ 6 Points: The security price goes down.

Sequence of Actions during a Period

Every period (between 1 and 100) is identical:

1. You make your decision: 1 point on alternative A: the security price goes up or for alternative B: the security price goes down.

2. A die is thrown (Instead of a seven-sided die we take a ten-sided die. If the result is $7, 8, 9$ or 10, the throw is repeated, until a number between 0 and 6 is thrown): The points on the die are credited to alternative A. (6 - the points on the die) are credited to alternative B.

3. The security price movement is determined according to Tables 1 and 2.

4. You receive your payoff of 20 ECU* or 0 ECU.

* 100 ECU correspond to 0.25 CHF.

In the beginning we will ask you some questions about the game, which you have to answer before we start the game.

B.2 Instructions for the Strategy Game

In this game you are asked to indicate your choices in the game for all periods in advance. Decide what you choose in which situation. Your choice can, for example, depend on the number of the current period, or on $1, 2, 3 \ldots$ or arbitrarily many previous periods. Your decision may depend on the price movement in these previous periods and on whether your prediction in these periods was correct or false. To write down your strategy you can use the following sheets. But you can also write down your strategy as you want. We will play one game with your strategy. In this game 100 ECU correspond to 1 CHF (previous payoff times 4).

Period 1:

Please decide whether you put your point on A (the price goes up) or on B (the price goes down).

Ex Period 2:

Your decisions now may depend on the previous periods. Indicate, for which price histories you change your decision. Under (a) please indicate when you change from from B to A, under (b) please indicate when you change from A to B. In case your decision depends on whether your prediction in the previous periods was right or wrong, you can denote this by a "+" for "correct" and by a "−" for "false." It is not necessary that you write down the whole history of prices. Instead it suffices that you write down the decisive periods.

For example:

...	u	u	u:	Prices went up during the last 3 periods.
...	+	+	+:	Your prediction was correct during the last 3 periods.

Or:

...	d	d	d	d:	Prices went down during the last 4 periods.
...	−	+	+	+:	Your prediction was correct during the last 3 periods and wrong in the period before.

Or:

... udduuuudddduddduuuuu ...

Or:

... dduuuudddduuuuuuuudddddddddddududud ...

Indicate for which period your decisions should apply:

Valid for period:

(a) Indicate when you change from B to A.

(b) Indicate when you change from A to B.

C Questionnaire

The following questions were asked to the participants at the end of the experiment.

Final Questions

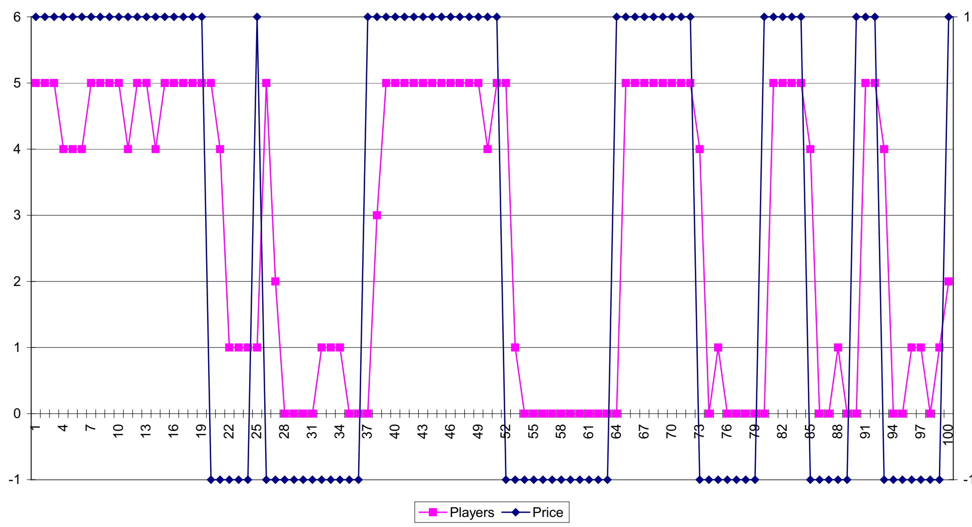
1. Did you always bet on the same price movement?
2. What do you think: How many of the 4 other players in your group did
   (a) always bet on the same price movement?
   (b) always bet on the last realized price movement?
   (c) followed more complex patterns of behavior?
3. Before taking your decision have you thought about what the other players will be doing?
4. Before taking your decision have you thought about what the other players believe about your own behavior?
5. Suppose prices have been increasing for several periods and the next period the price decreases. What do you think is the reason for the price decrease?
   (a) The behavior of the other players?
   (b) The die?
6. Suppose prices have been increasing for several periods and then decreasing for several periods. What do you think is the reason for the change in the price movement?
   (a) The behavior of the other players?
   (b) The die?

D Experimental Results

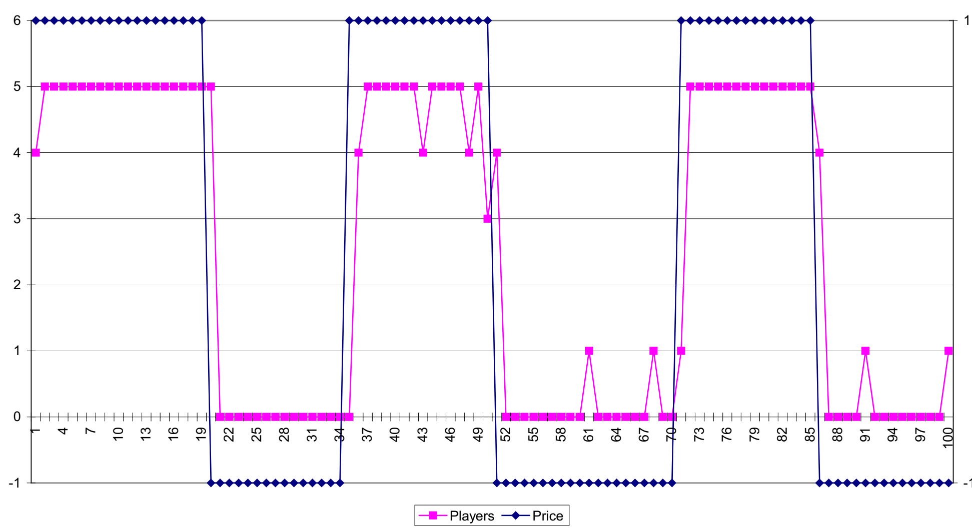
The following charts display the experimental results for all groups in both rounds. The first series of charts displays the frequency of "up" choices among the subjects (left scale from 0 to 5) and the price movement (right scale, "-1" stands for a downward price movement and "1" stands for an upward price movement).

The second series of charts displays the cumulated change in the noise traders' sentiment as determined by the die and the cumulated change in the price movement.

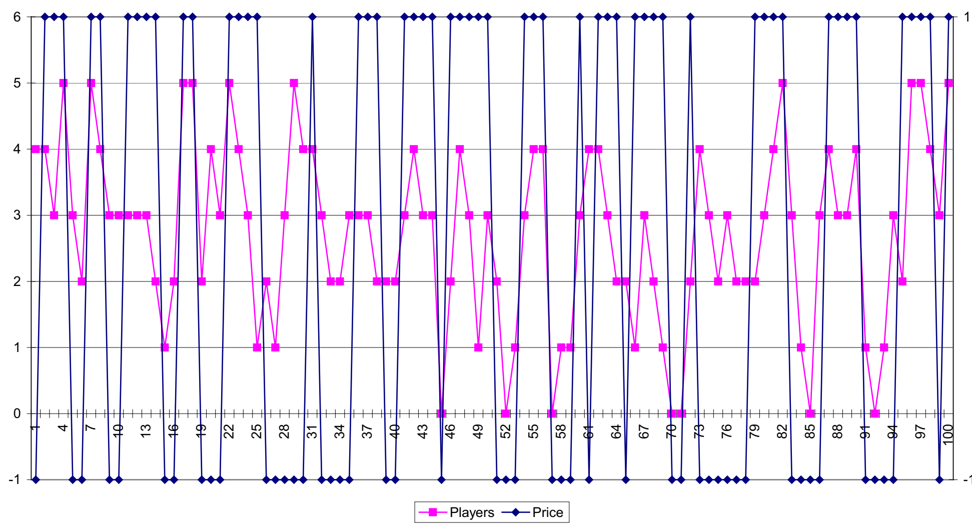
Group 1, Round 1:
Frequency of "up" choices and price movement



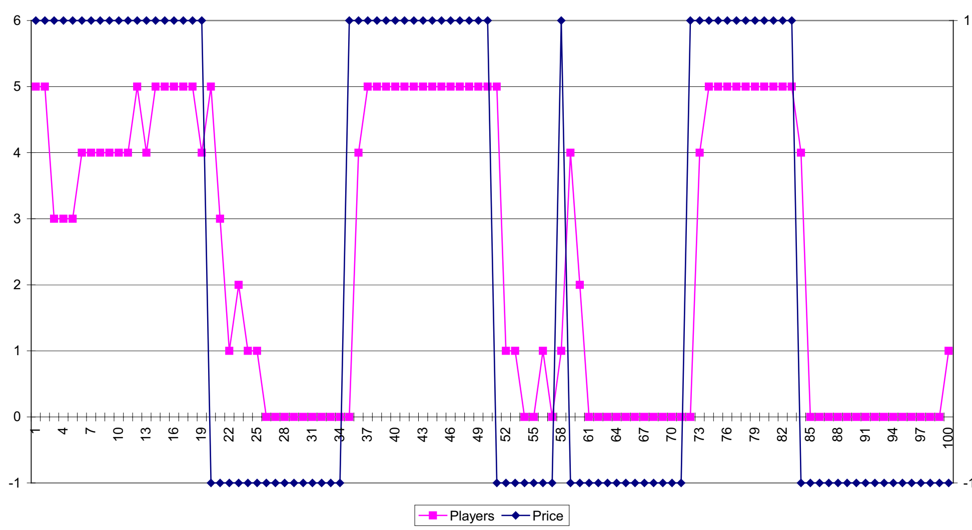
Group 1, Round 2:
Frequency of "up" choices and price movement



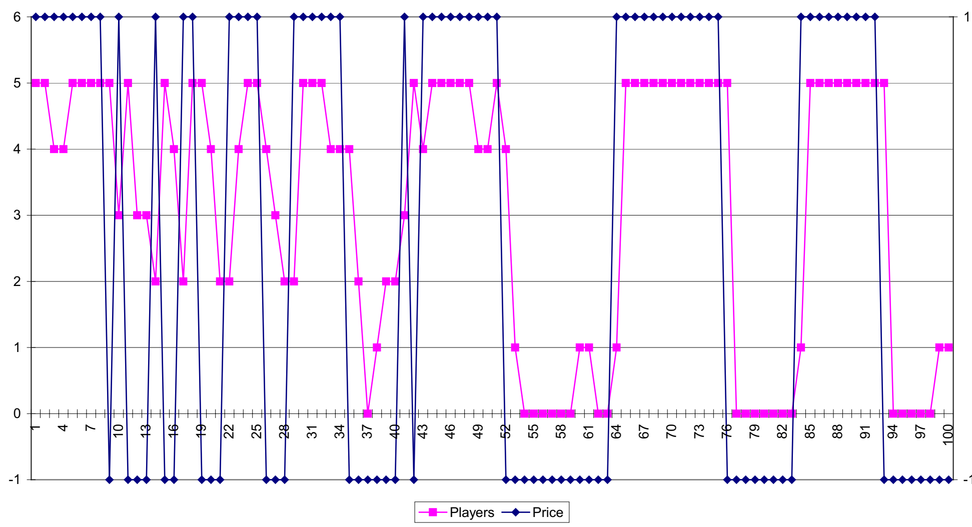
Group 2, Round 1:
Frequency of "up" choices and price movement



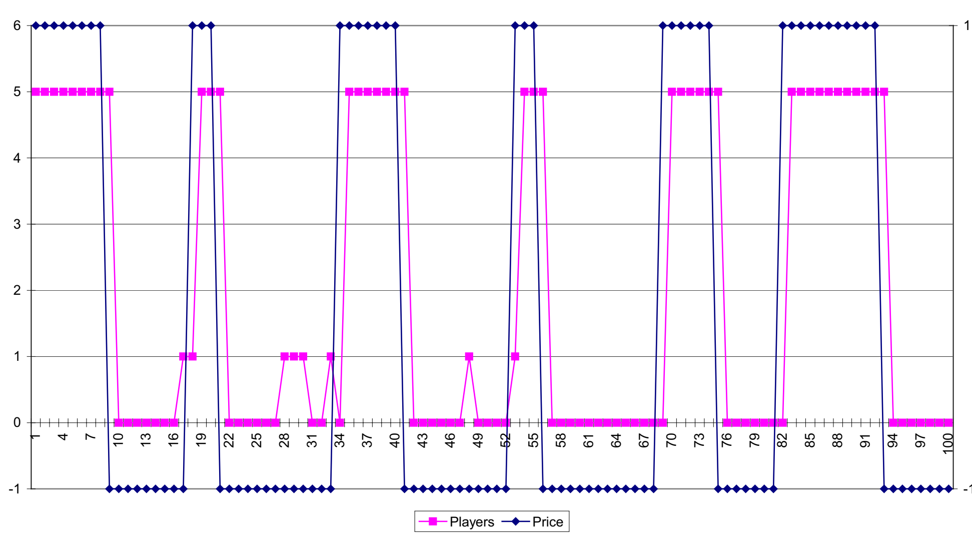
Group 2, Round 2:
Frequency of "up" choices and price movement



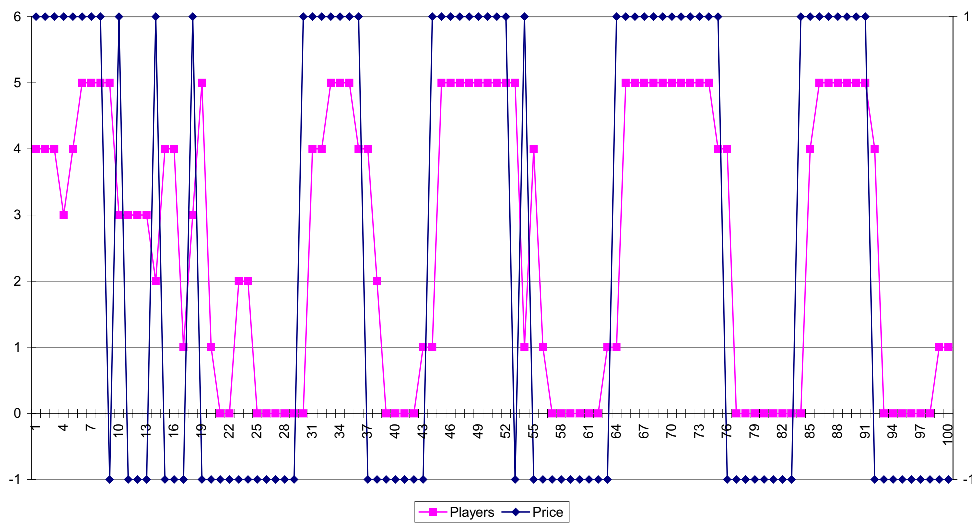
Group 3, Round 1:
Frequency of "up" choices and price movement



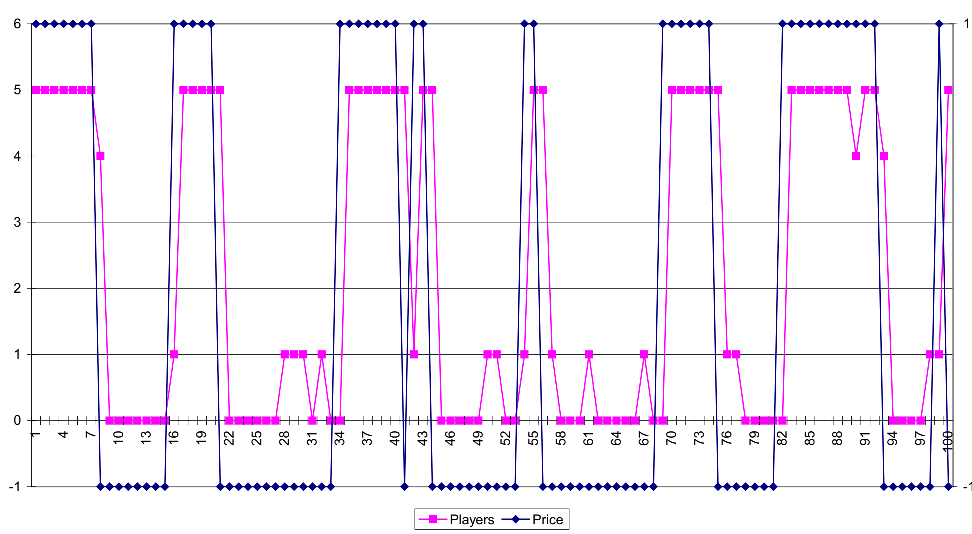
Group 3, Round 2:
Frequency of "up" choices and price movement



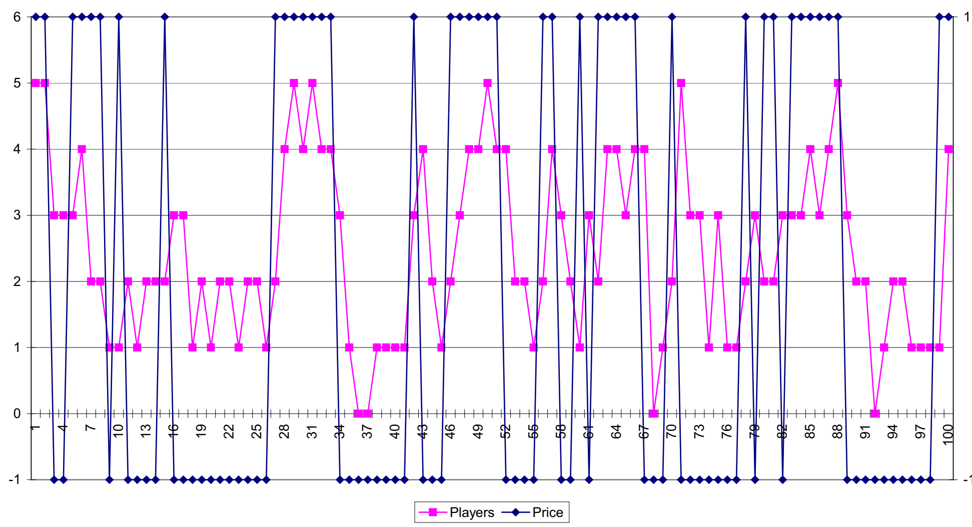
Group 4, Round 1:
Frequency of "up" choices and price movement



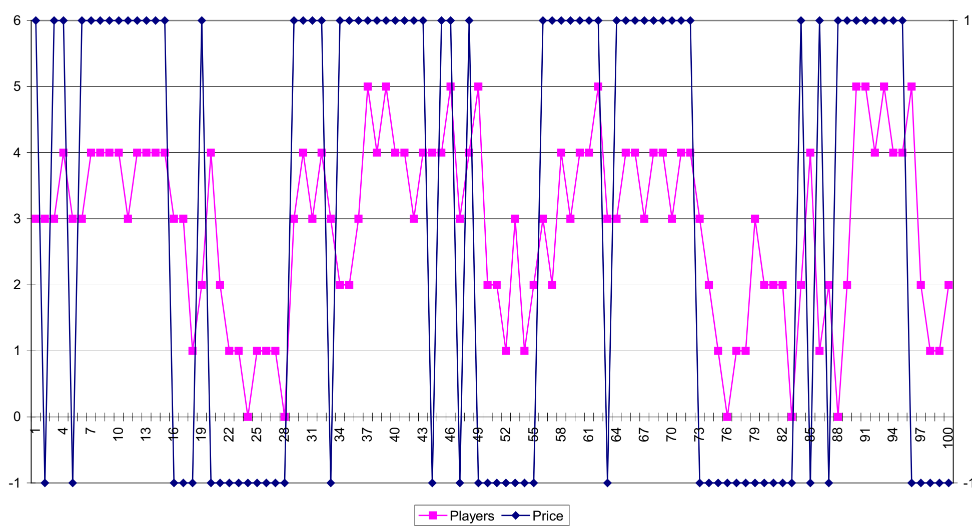
Group 4, Round 2:
Frequency of "up" choices and price movement



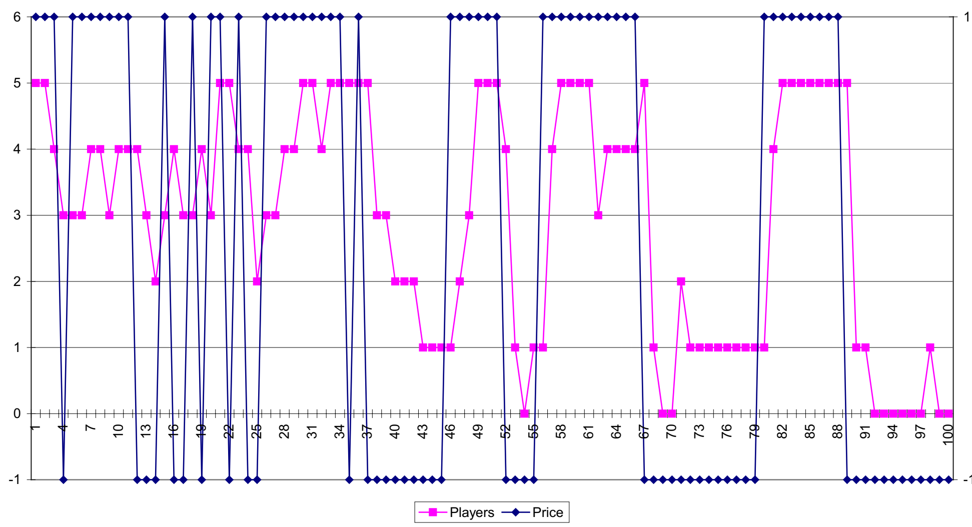
Group 5, Round 1:
Frequency of "up" choices and price movement



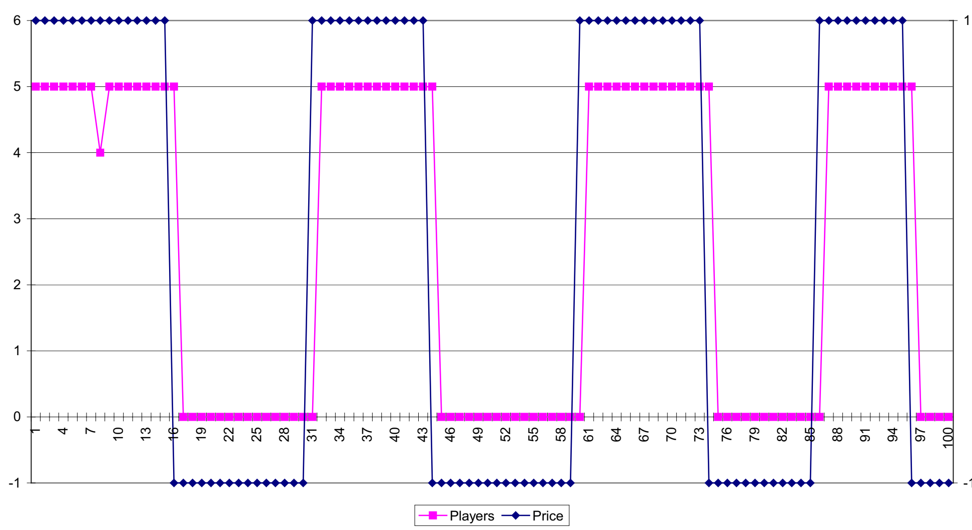
Group 5, Round 2:
Frequency of "up" choices and price movement



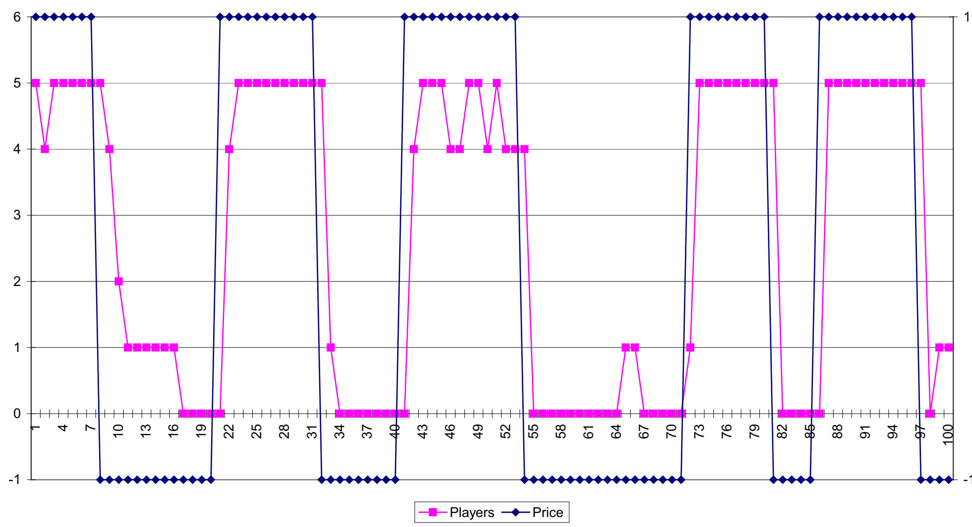
Group 6, Round 1:
Frequency of "up" choices and price movement



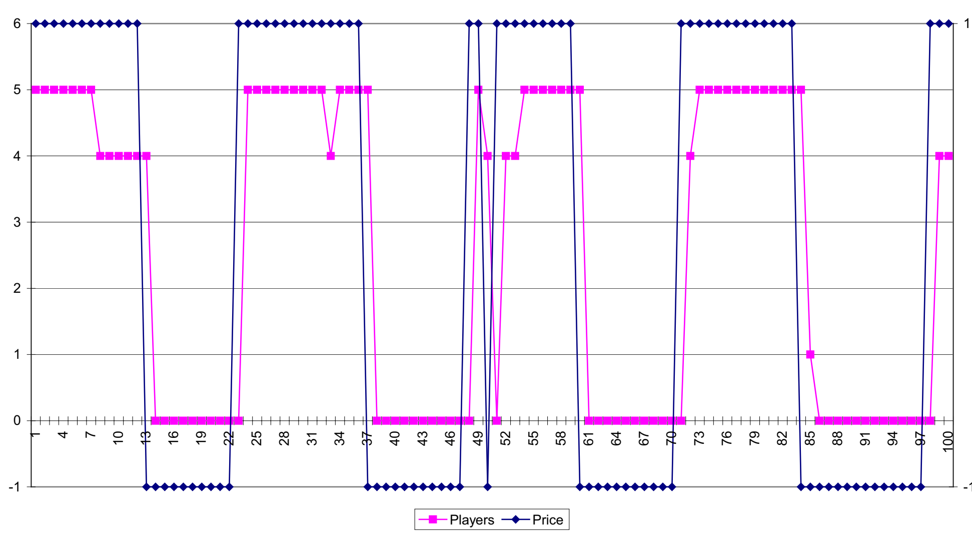
Group 6, Round 2:
Frequency of "up" choices and price movement



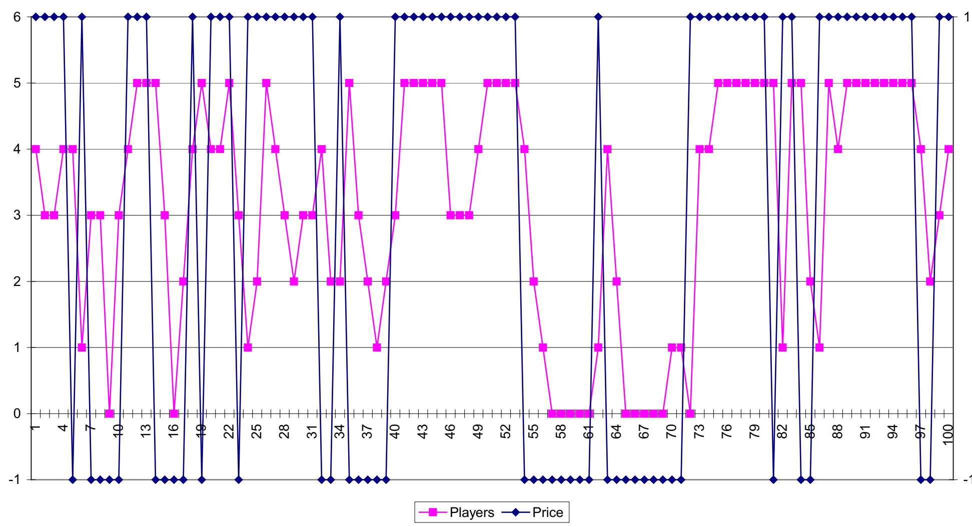
Group 7, Round 1:
Frequency of "up" choices and price movement



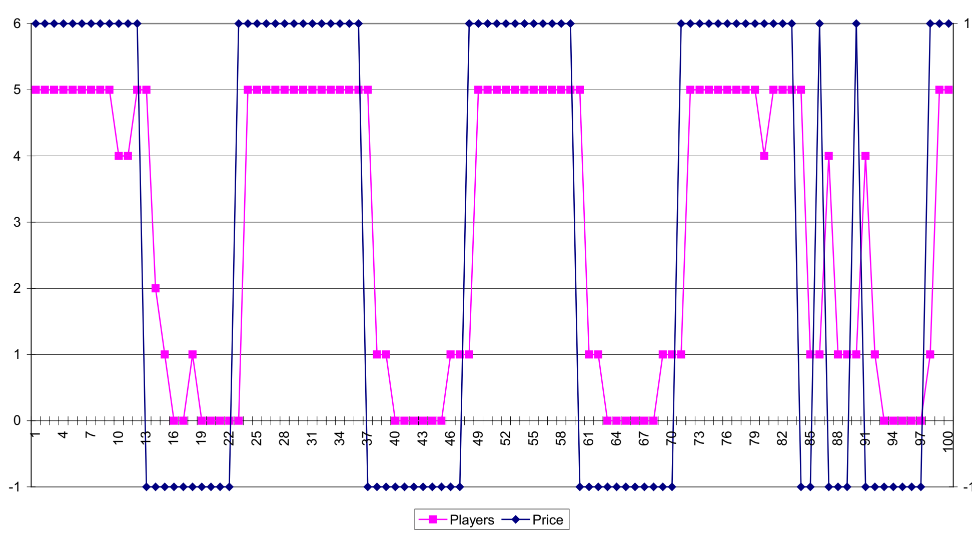
Group 7, Round 2:
Frequency of "up" choices and price movement



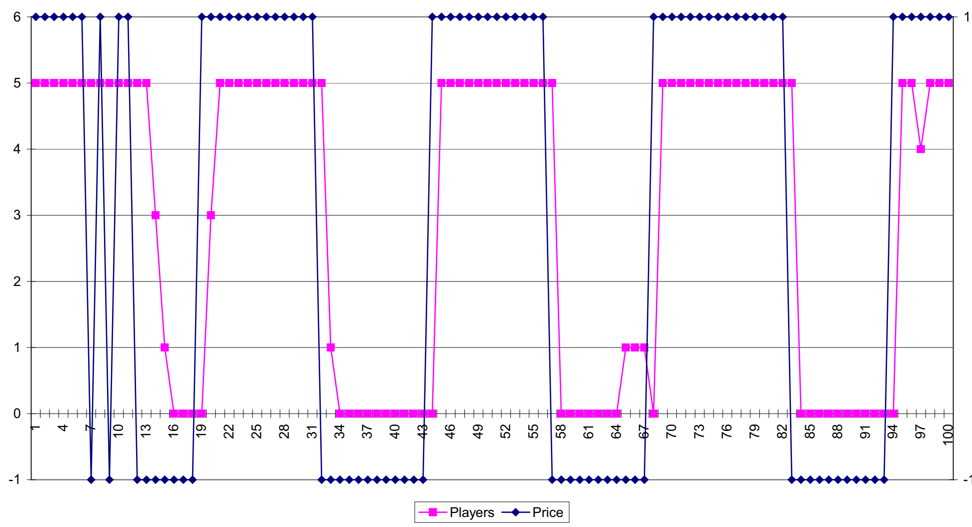
Group 8, Round 1:
Frequency of "up" choices and price movement



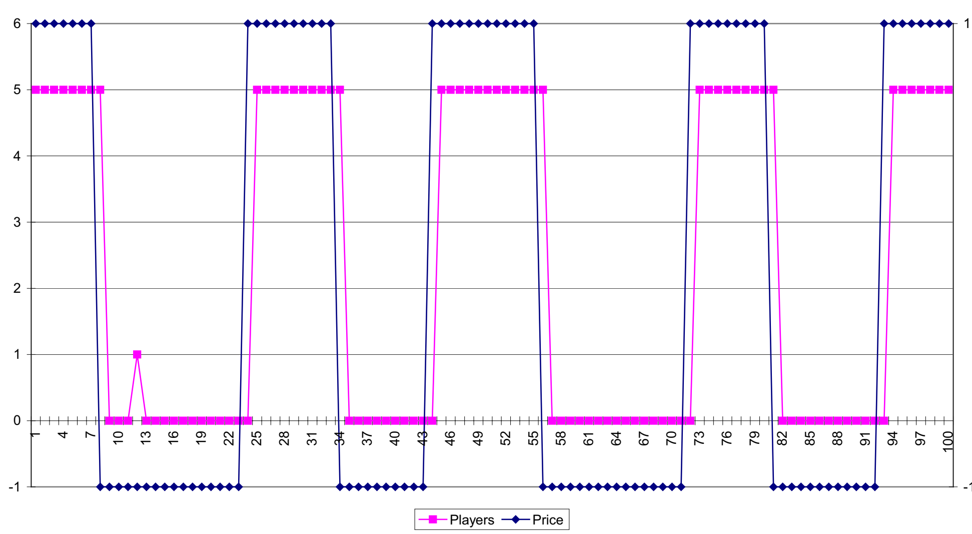
Group 8, Round 2:
Frequency of "up" choices and price movement



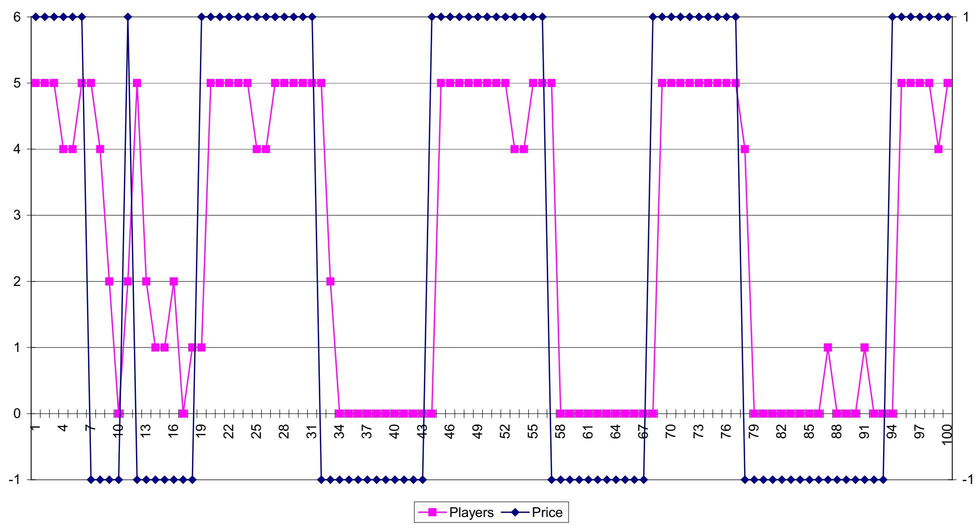
Group 9, Round 1:
Frequency of "up" choices and price movement



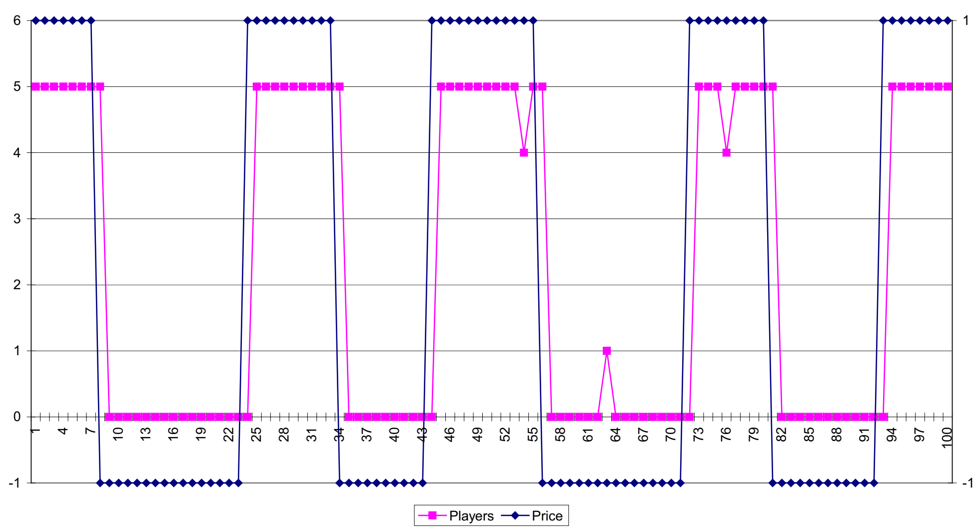
Group 9, Round 2:
Frequency of "up" choices and price movement

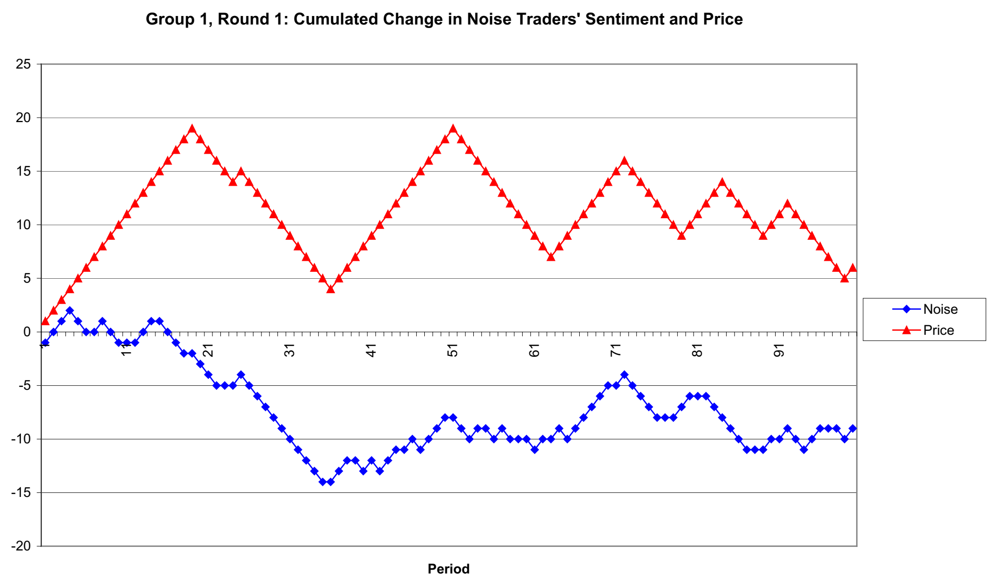


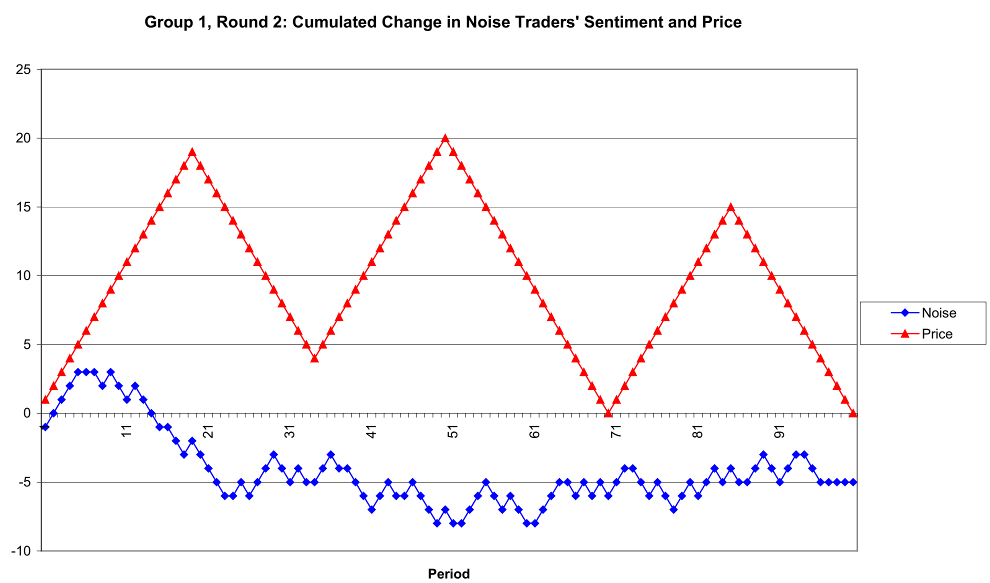
Group 10, Round 1:
Frequency of "up" choices and price movement



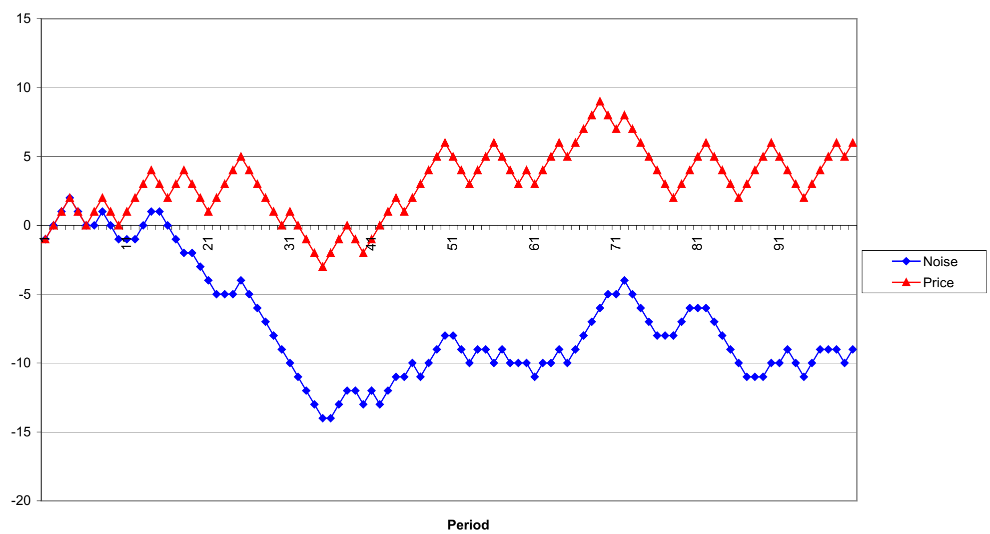
Group 10, Round 2:
Frequency of "up" choices and price movement



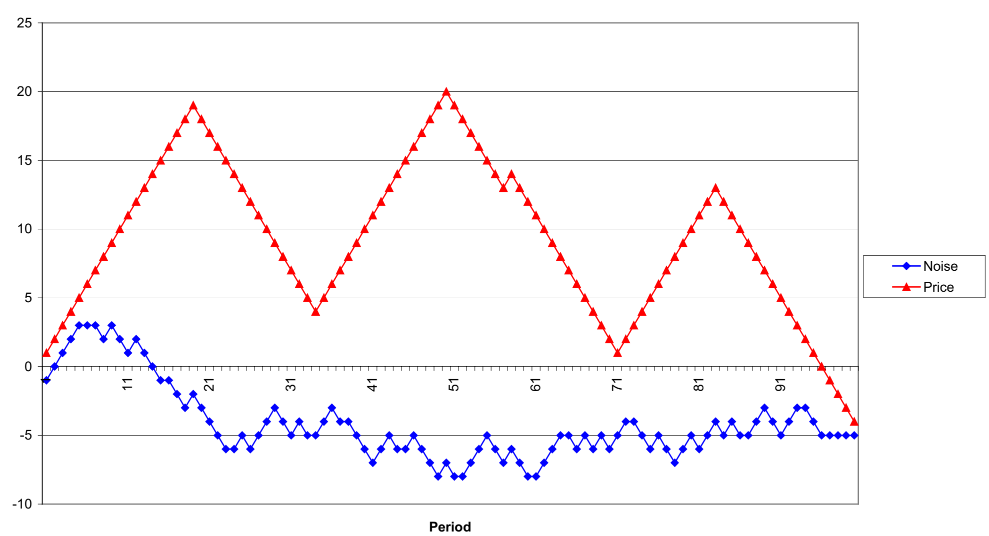

Group 1, Round 1: Cumulated Change in Noise Traders' Sentiment and Price
25
20
15
10
5
0
-5
-10
-15
-20
1
11
21
31
41
51
61
71
81
91
Period
Noise
Price

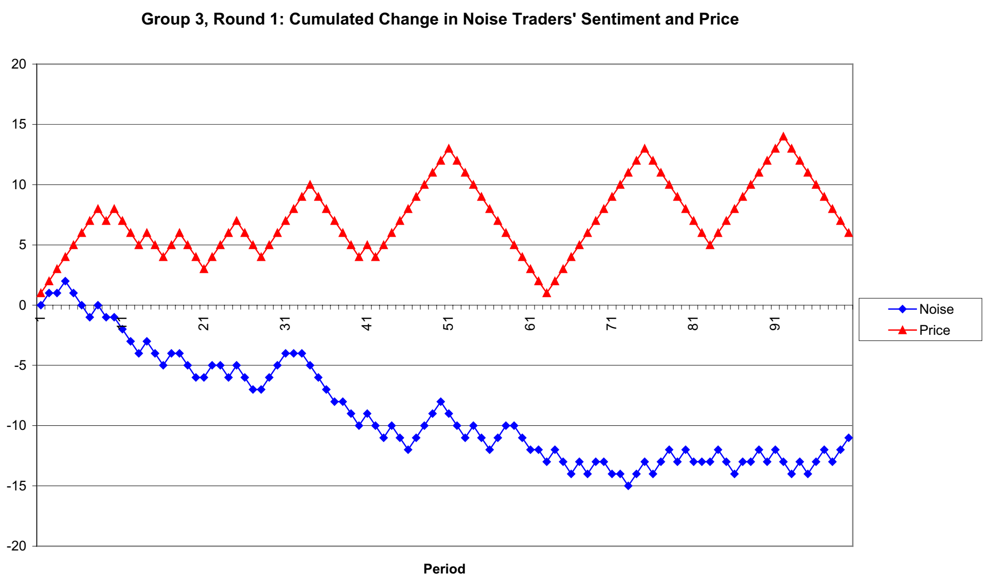

Group 1, Round 2: Cumulated Change in Noise Traders' Sentiment and Price
25
20
15
10
5
0
-5
-10
1
11
21
31
41
51
61
71
81
91
Period
Noise
Price

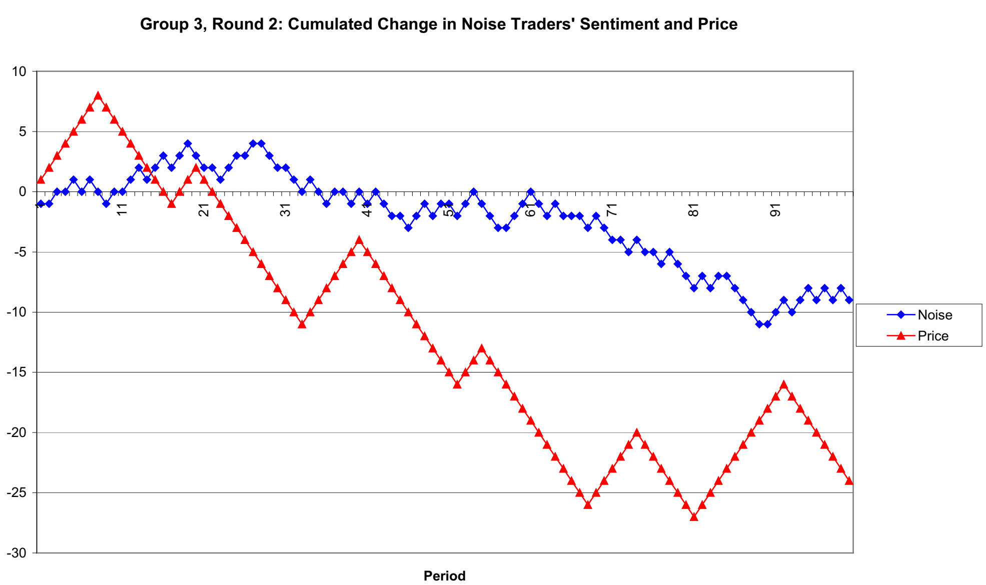
Group 2, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



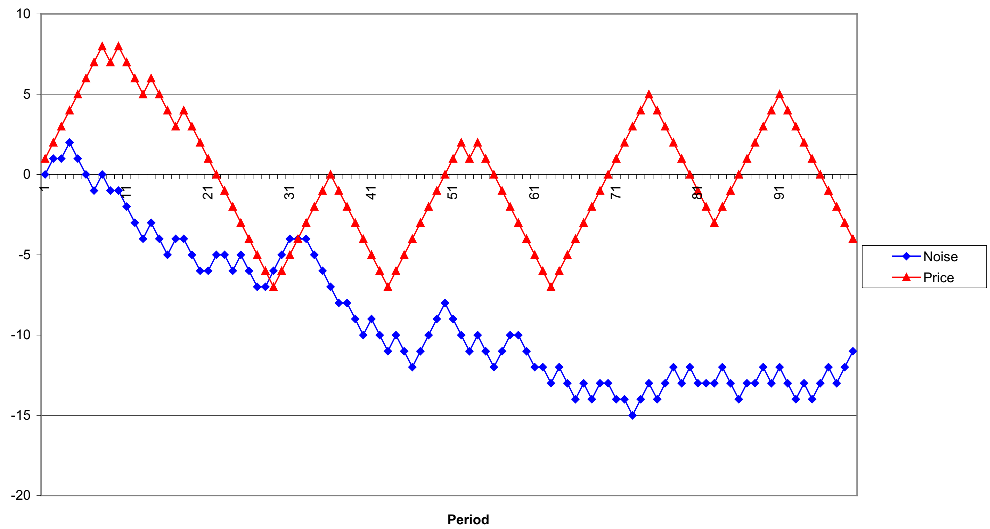
Group 2, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



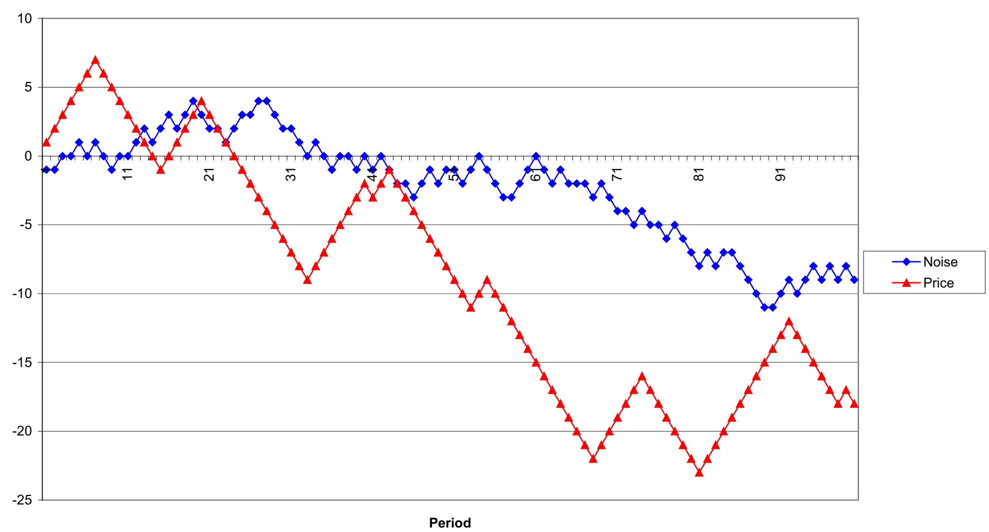

Group 3, Round 1: Cumulated Change in Noise Traders' Sentiment and Price
20
15
10
5
0
-5
-10
-15
-20
1
11
21
31
41
51
61
71
81
91
Period
Noise
Price


Group 3, Round 2: Cumulated Change in Noise Traders' Sentiment and Price
10
5
0
-5
-10
-15
-20
-25
-30
11
21
31
71
81
91
Period
Noise
Price

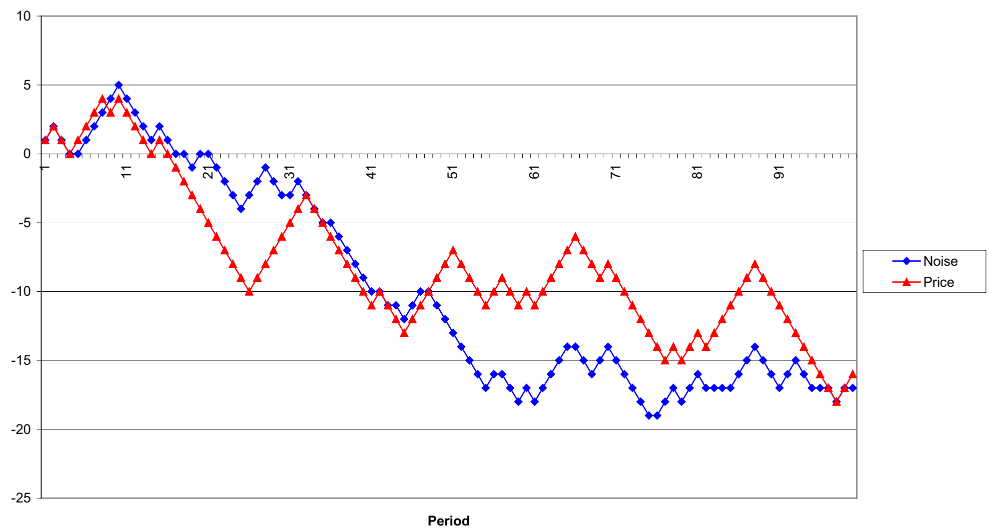
Group 4, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



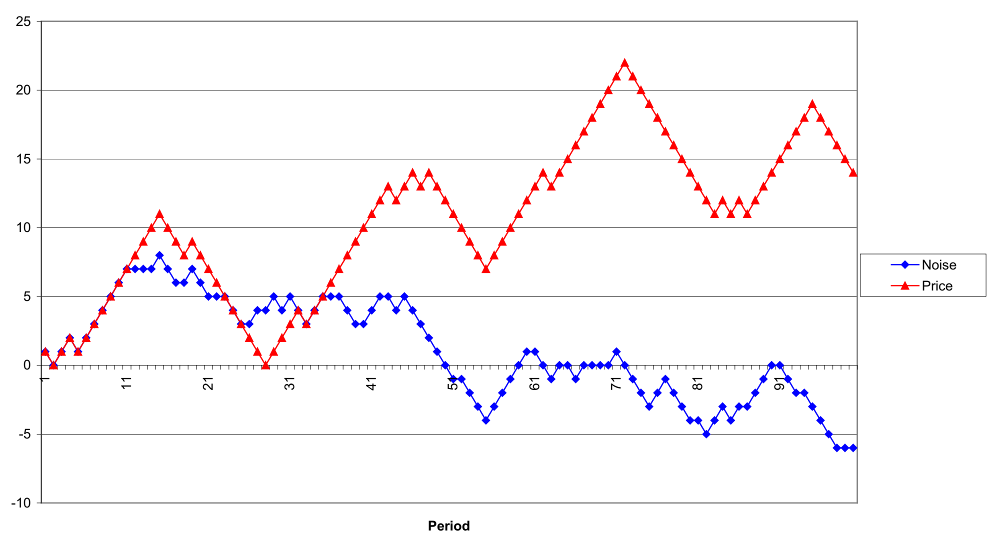
Group 4, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



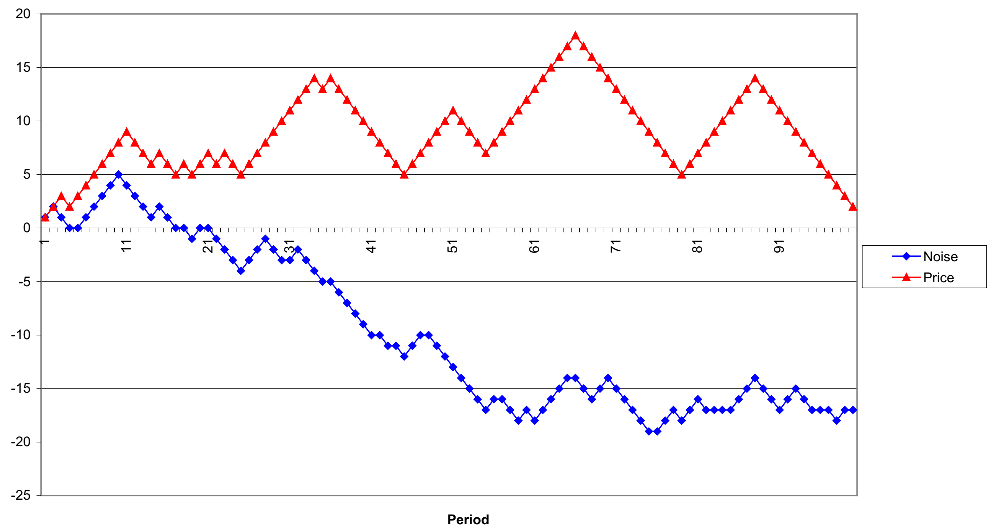
Group 5, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



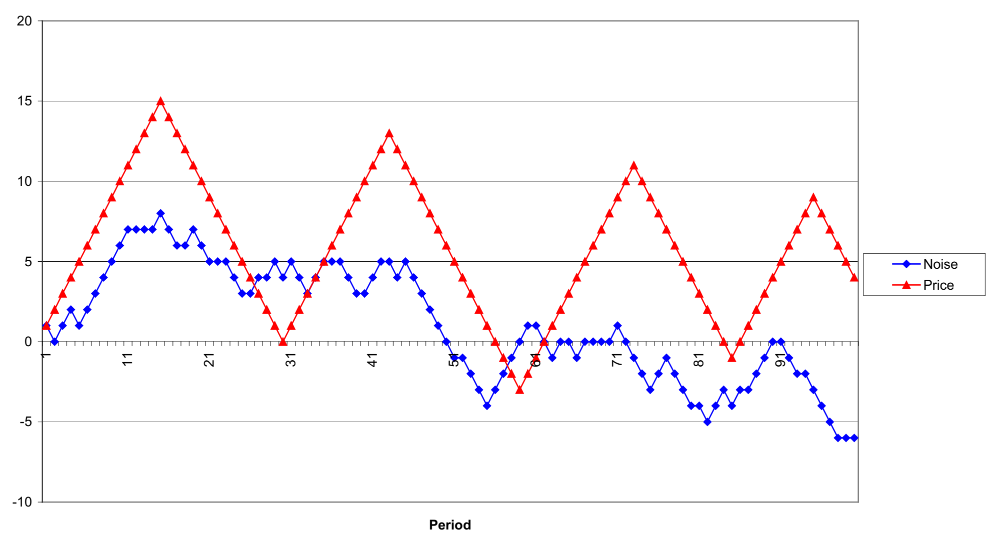
Group 5, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



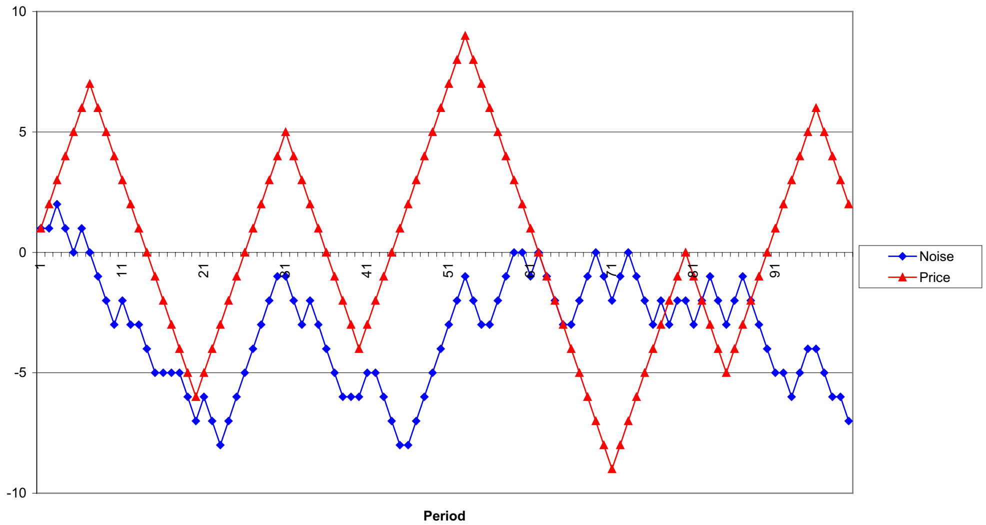
Group 6, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



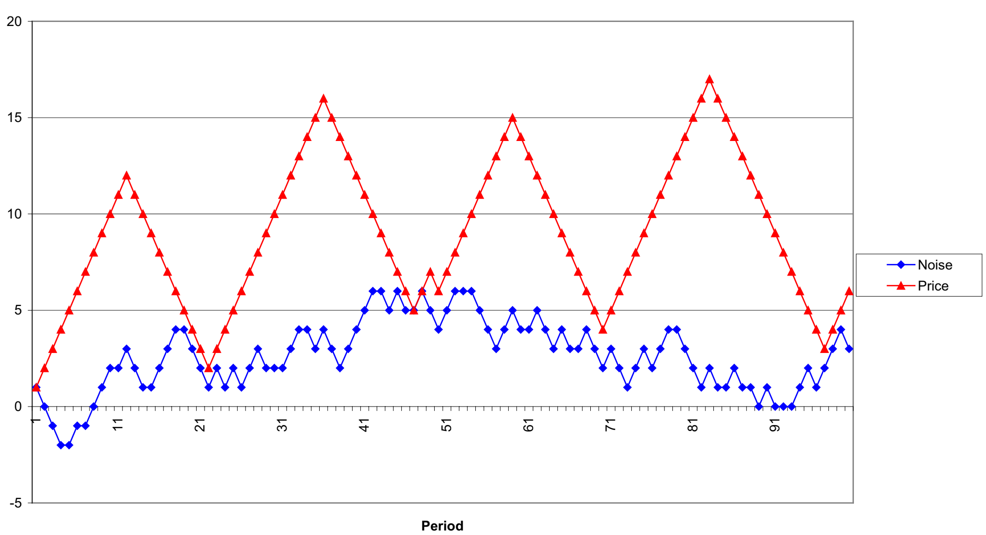
Group 6, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



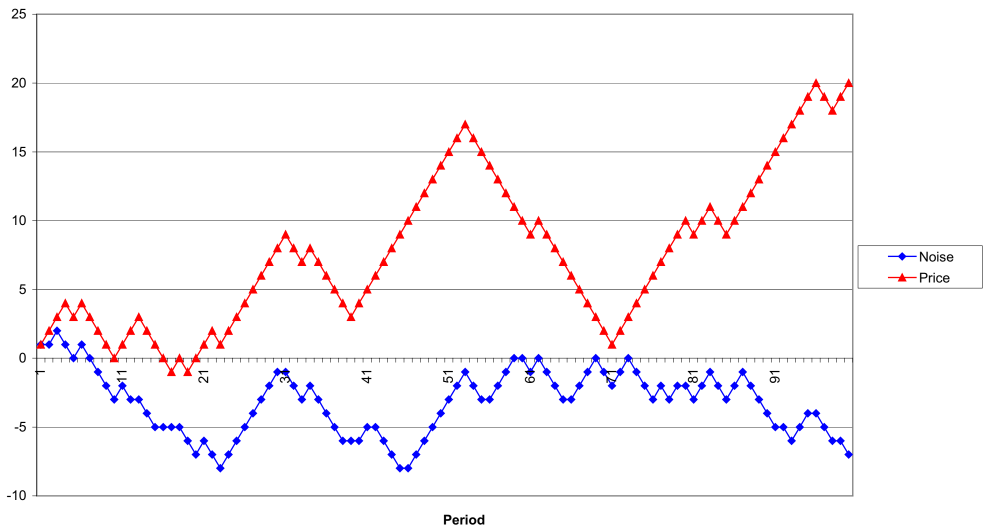
Group 7, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



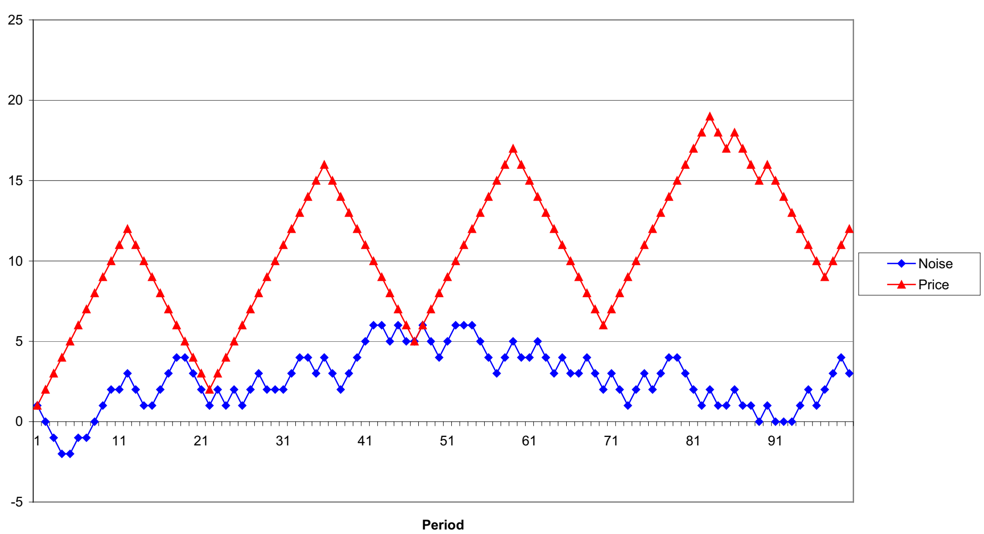
Group 7, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



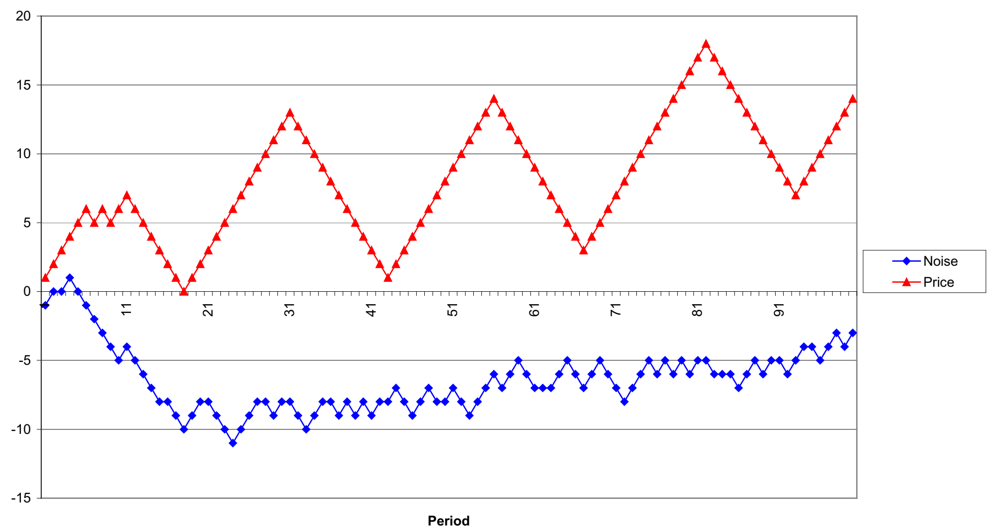
Group 8, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



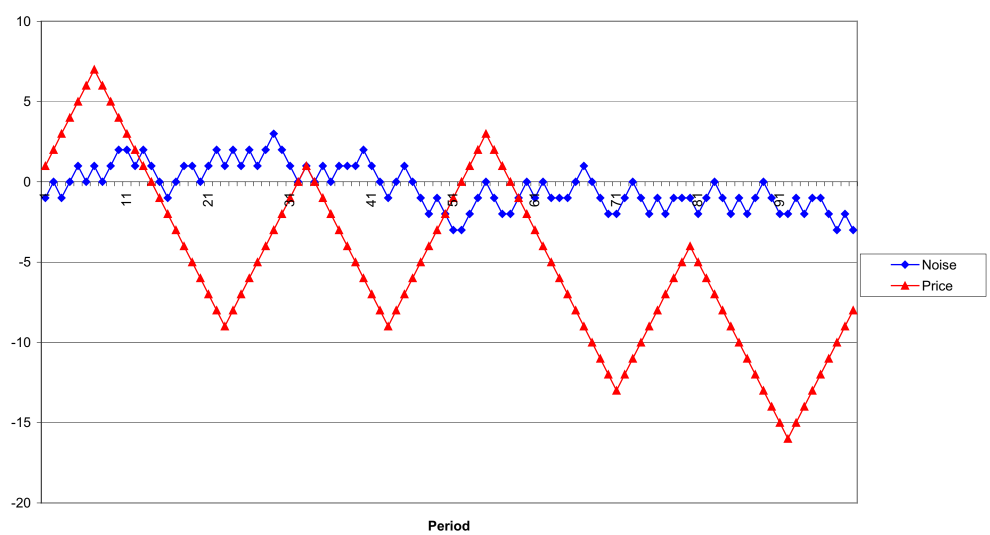
Group 8, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



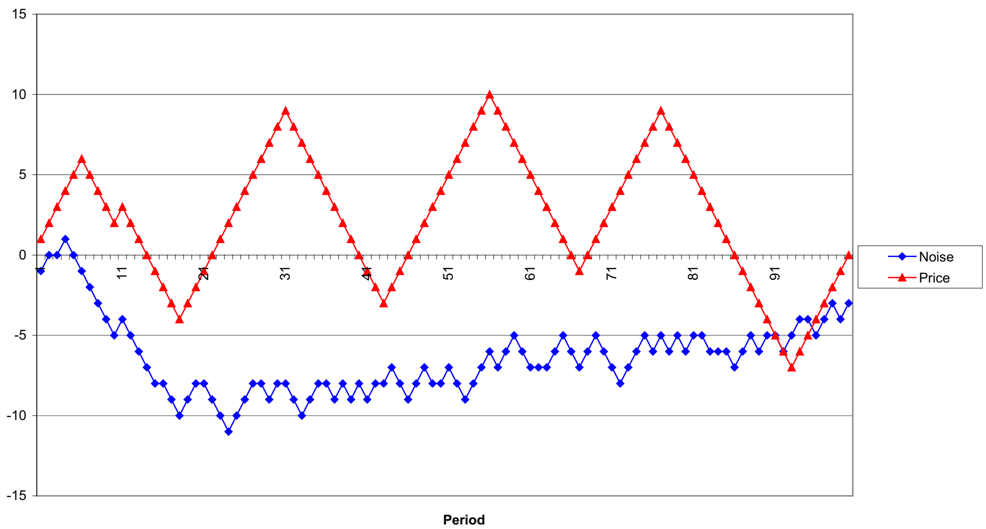
Group 9, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



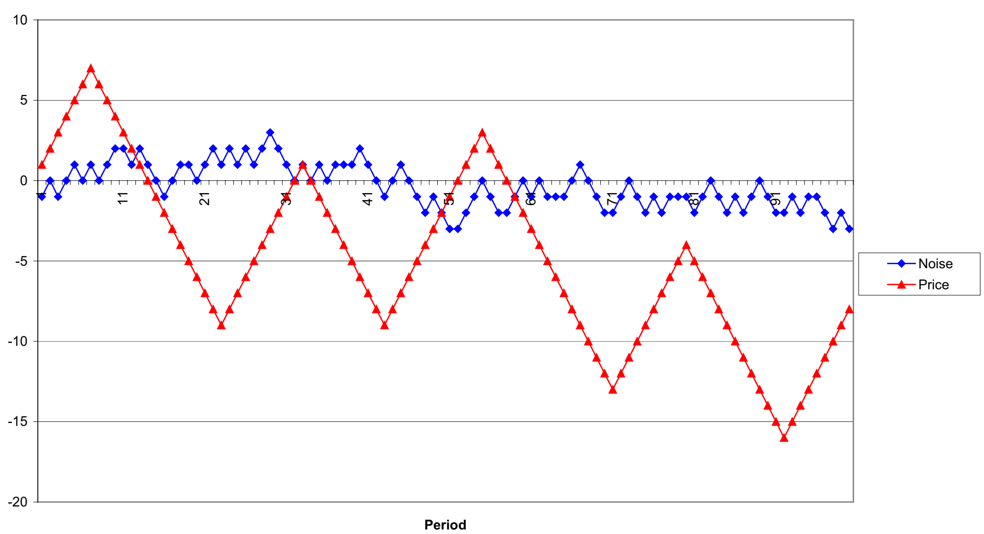
Group 9, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



Group 10, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



Group 10, Round 2: Cumulated Change in Noise Traders' Sentiment and Price



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Group	**3**	**4**	**5**
1	1	11	88
2	6	20	74
3	0	8	92
4	0	19	81
5	40	46	14
6	0	1	99
7	0	14	86
8	1	24	75
9	0	1	99
10	0	3	97

Table 3: Degree of coordination in round 2: number of periods in which $3, 4$, or 5 subjects choose the same action.

Group	Number of coordinated. subjects ≤	Round 1 Periods 1-50	Round 1 Periods 51-100	Round 2 Periods 1-50	Round 2 Periods 51-100
1	3	2	1	1	0
	4	16	10	5	7
2	3	30	23	5	1
	4	43	40	17	9
3	3	14	8	7	0
	4	27	8	7	1
4	3	10	0	0	0
	4	24	11	8	11
5	3	22	25	19	21
	4	43	46	44	42
6	3	20	2	0	0
	4	37	25	1	0
7	3	1	0	0	0
	4	15	8	8	6
8	3	22	5	1	0
	4	35	18	10	15
9	3	2	0	0	0
	4	4	4	1	0
10	3	5	0	0	0
	4	14	6	0	3

Table 4: Cumulative frequency distribution: number of periods in which at most 3 or 4 subjects choose the same action.

Group	**Period**
1	38
2	100
3	41
4	38
6	71
7	10
8	100
9	20
10	33
Mean	50.11
Median	38

Table 5: Length of the learning phase for all groups showing coordinated play in round 2.

Group	Subject	Learning Phase: Switch	Learning Phase: Switch after Price Change[a]	Round 2: Switch	Round 2: Switch after Price Change[b]
1	1	33	3 (4)	96	5 (5)
	2	35	2 (4)	96	5 (5)
	3	29	3 (4)	97	4 (5)
	4	34	2 (4)	99	5 (5)
	5	34	2 (4)	95	5 (5)
2	1	50	26 (34)	94	6 (7)
	2	49	19 (34)	81	3 (7)
	3	52	22 (34)	95	6 (7)
	4	67	26 (34)	95	5 (7)
	5	47	21 (34)	97	6 (7)
3	1	30	4 (11)	99	11 (11)
	2	34	7 (11)	99	11 (11)
	3	23	5 (11)	99	11 (11)
	4	39	10 (11)	99	11 (11)
	5	25	9 (11)	91	11 (11)
4	1	28	6 (9)	99	14 (14)
	2	27	6 (9)	80	11 (14)
	3	29	5 (9)	99	14 (14)
	4	35	7 (9)	99	14 (14)
	5	28	4 (9)	99	14 (14)
6	1	59	9 (19)	99	7 (7)
	2	56	12 (19)	98	7 (7)
	3	53	10 (19)	99	7 (7)
	4	60	12 (19)	99	7 (7)
	5	33	13 (19)	99	7 (7)
7	1	7	0 (1)	99	10 (10)
	2	8	0 (1)	98	9 (10)
	3	7	0 (1)	87	7 (10)
	4	9	1 (1)	98	10 (10)
	5	7	0 (1)	99	10 (10)
8	1	85	18 (24)	99	12 (12)
	2	74	18 (24)	97	10 (12)
	3	88	19 (24)	98	11 (12)
	4	75	10 (24)	80	6 (12)
	5	77	20 (24)	95	12 (12)
9	1	15	3 (6)	99	8 (8)
	2	14	2 (6)	99	8 (8)
	3	16	3 (6)	99	8 (8)
	4	14	2 (6)	99	8 (8)
	5	15	3 (6)	98	8 (8)
10	1	31	4 (5)	99	8 (8)
	2	30	4 (5)	97	8 (8)
	3	28	2 (5)	99	8 (8)
	4	21	2 (5)	98	8 (8)
	5	28	5 (5)	99	8 (8)

Table 6: Number of periods in which the switch strategy is played.

[a,b] Number of periods with a change in the price movement in brackets.

Action	Number of Subjects
u(p)	50
d(own)	0

Table 7: Strategy game: choices for period 1.

Strategies	Number of Subjects
Switch after being wrong once	41
Switch after being wrong twice	3
Switch after being wrong three times[a]	1
Switch after being wrong twice and being right seven (eight) times	3
Switch after being wrong twice and being right three times	2

Table 8: Strategy game: choices for periods ≥ 2.

[a]This person added a complicate estimation about the future development of the price to this rule.

Group	STDEV Price	STDEV Noise[a]
1	5.24	2.74
2	5.59	2.74
3	10.23	3.97
4	8.70	3.97
5	5.60	3.63
6	4.20	3.63
7	3.84	1.85
8	4.25	1.85
9	5.42	1.37
10	5.42	1.37

Table 9: Sample standard deviation of the cumulated price movement and the cumulated noise traders' sentiment.

[a]Observe that the standard deviation of the noise traders' sentiment is identical for groups 1 and 2, 3 and 4, 5 and 6, 7 and 8, 9 and 10, since these groups were in the same session and hence played with the same realization of the die.

Group	Empirical Frequency of $R_t = R_{t-1}$
1	0.95
2	0.93
3	0.89
4	0.86
5	0.75
6	0.93
7	0.9
8	0.88
9	0.92
10	0.82

Table 10: Momentum of the cumulated price movement.

	VR(q)[a]			
Group	**q** = 10	**q** = 20	**q** = 25	**q** = 50
1	7.01 (0.000)	5.26 (0.000)	2.71 (0.001)	4.50 (0.000)
2	6.59 (0.000)	4.68 (0.000)	2.41 (0.006)	4.56 (0.000)
3	2.98 (0.000)	2.39 (0.003)	2.77 (0.001)	2.50 (0.031)
4	2.98 (0.000)	2.40 (0.002)	2.61 (0.002)	2.20 (0.067)
5	3.82 (0.000)	1.96 (0.027)	0.92 (0.445)	0.78 (0.393)
6	5.76 (0.000)	2.64 (0.000)	0.96 (0.474)	2.27 (0.057)
7	5.07 (0.000)	1.07 (0.444)	0.35 (0.124)	0.43 (0.240)
8	4.76 (0.000)	0.94 (0.451)	0.19 (0.073)	0.25 (0.176)
9	4.57 (0.000)	1.57 (0.126)	0.97 (0.479)	0.24 (0.174)
10	4.57 (0.000)	1.57 (0.126)	0.97 (0.479)	0.24 (0.174)

Table 11: Variance ratio of the cumulated price movement for different qth differences (p-values in brackets).

[a]The test statistic $z(q) = \sqrt{nq}(\mathrm{VR}(q) - 1)\left(\frac{2(2q-1)(q-1)}{3q}\right)^{-1}$ is asymptotically $N(0,1)$ (see Lo and MacKinlay, 1988).

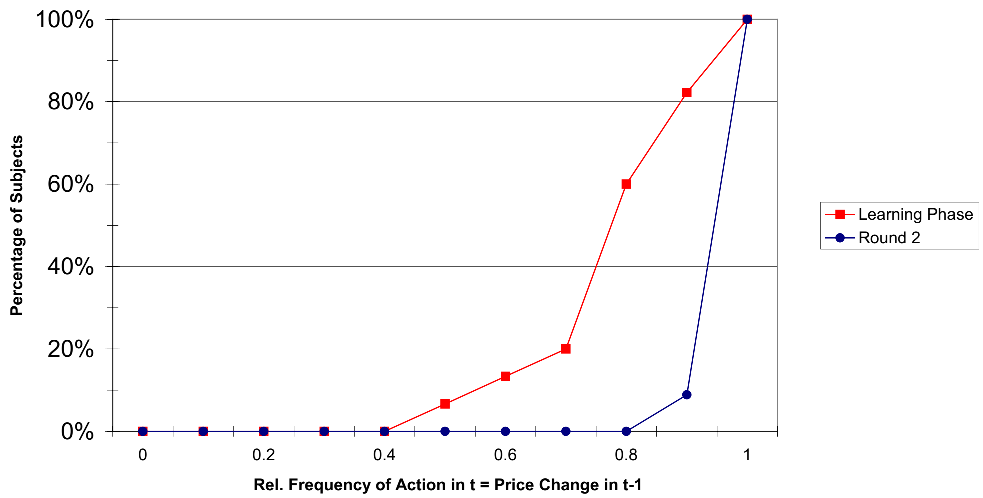

Figure 1: Cumulative distribution of the switch strategy.

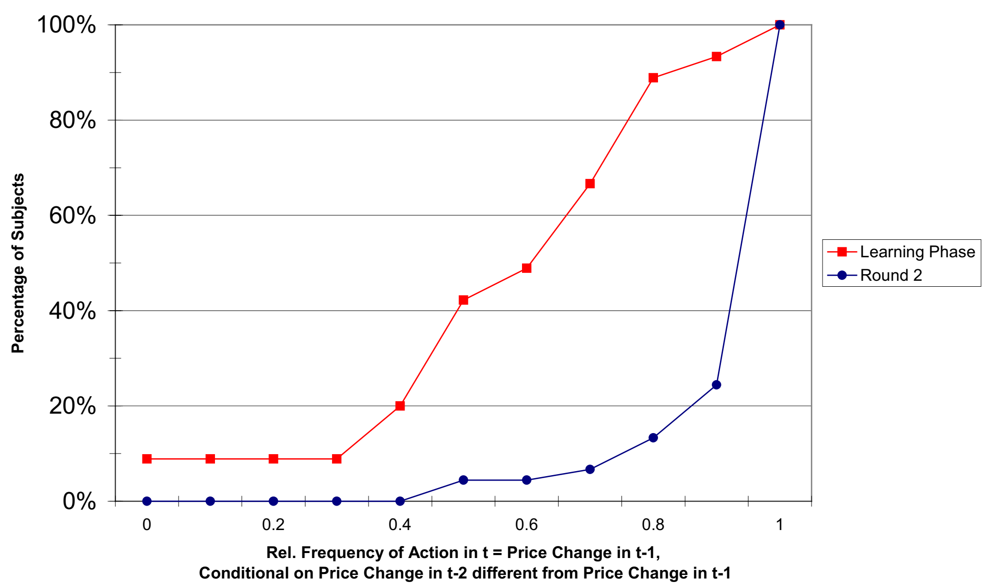

Figure 2: Cumulative distribution of the switch strategy in periods following a change in the price movement.

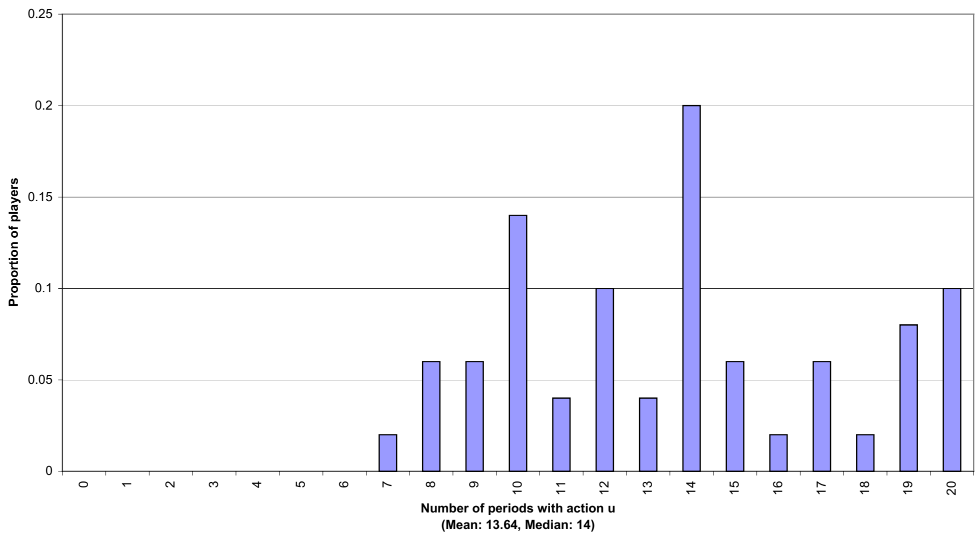

Figure 3: Distribution of the number of u(p) choices for all players in the first 20 periods of round 1.