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Racial Unemployment Gaps and the Disparate Impact of the Inflation Tax

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Racial Unemployment Gaps and the Disparate Impact of the Inflation Tax*

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Abstract

We study the nonlinearities present in a standard monetary labor search model modified to have two groups of workers facing exogenous differences in the job finding and separation rates. We use our setting to study the racial unemployment gap between Black and white workers in the US. A calibrated version of the model is able to replicate the difference between the two groups both in the level and volatility of unemployment. We show that the racial unemployment gap is counter-cyclical and that its reaction to shocks is state-dependent. In particular, following a negative productivity shock, when aggregate unemployment is above average the gap increases by 0.6pp more than when aggregate unemployment is below average. In terms of policy, we study the implications of different inflation regimes on the racial unemployment gap. Higher trend inflation increases both the level of the unemployment gap and the magnitude of its response to shocks.

Keywords: unemployment; discrimination; racial inequality; monetary policy; inflation.

JEL classification: E31, E32, E52, J64.

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1 Introduction

In macroeconomics, heterogeneity in general, and group differences in particular, continue to receive increasing attention from policymakers and academics. For example, Federal Reserve officials have shown increasing concern with the effects of monetary policy on historically disadvantaged groups, especially regarding labor market outcomes. In this paper, we seek to understand the cyclical behavior of the standard labor market search model (Pissarides, 2000) when different groups face different opportunities. In particular, we focus on the racial unemployment gap between Black and white workers in the US. With only minimal additional moments relative to the standard, single-group model, our calibrated model captures business cycle differences between the two groups remarkably well, and makes stark predictions deriving from the nonlinearities inherent to matching models.

The key mechanism of the paper rests on a simple observation about the basic job flow model modified to have two groups, say A and B. Suppose that θ summarizes labor market conditions, or more precisely tightness, and a function $f(\theta)$ describes the arrival rate for workers of meetings with firms. Further suppose that workers of each group differ in the probability that a meeting results in a productive match—with probability $\rho_A < \rho_B$ a meeting fails to result into a match, for instance due to discrimination. Likewise, workers differ in their rate of job destruction, $\delta_A < \delta_B$.

In such a model, the steady state unemployment rate for each group, $j \in \{A, B\}$, is given by

$$u_j = \frac{\delta_j}{\delta_j + (1-\rho_j)f(\theta)}.$$

If group A has a lower separation rate and a higher matching rate than group B, then the steady state unemployment of group A is less than that of group B. Now, consider the sensitivity of these unemployment rates to changes in labor market conditions, θ. This would usually be summarized by its elasticity. For each group, the elasticity of steady state unemployment with respect to θ is given by

$$\varepsilon_{u_j,\theta} \equiv \frac{\theta}{u_j}\frac{\partial u_j}{\partial \theta} = -(1-u_j)\varepsilon_{f,\theta}$$

where $\varepsilon_{f,\theta}$ is the elasticity of the meeting function. Since unemployment is lower for group A, the elasticity of unemployment for group A is *greater* in magnitude than the elasticity for group B. But it is not at all clear that the elasticity—the percentage change in unemployment for a percent change in labor market conditions—is indeed the welfare relevant measure. Instead, the more directly relevant question concerns the absolute change in unemployment. Hence, one calculates the semi-elasticity to find

$$\tilde{\varepsilon}_{u_j,\theta} \equiv \theta\frac{\partial u_j}{\partial \theta} = -u_j(1-u_j)\varepsilon_{f,\theta}.$$

Since unemployment is less than 1/2, the semi-elasticity for group A is *smaller* in magnitude than the semi-elasticity for group B. That is, a basic calculation shows that the disadvantaged group—the one with higher separation and lower job finding rates—should have a bigger absolute response to changes in labor market conditions, but a smaller proportional (or logarithmic) response. When considering the U.S. data for Black and white workers, this is exactly what one finds.

Table 1 presents the average and standard deviation of the unemployment rate for the overall US population as well as for white and Black workers over the period January 1972 to December 2019. The white unemployment rate was slightly below the population rate, while the Black unemployment rate is almost double it. The average racial unemployment gap (difference between Black and white unemployment rates) stands at 6.32 percentage points (pp). The average Black unemployment rate is 2.15 times the average white unemployment rate. In addition to the large difference in levels, a difference in terms of volatility stands out. The standard deviation of the unemployment rate for Black workers is about 1.65 times that for white workers while, in accordance with theory, the reverse ordering holds for the standard deviation of log unemployment.

Table 1: Labor market statistics (quarterly US data, 1972-2019)

	Population U rate	White U rate	Black U rate
Average	6.23%	5.48%	11.80%
Standard deviation, HP-cycle	0.77%	0.73%	1.20%
Standard deviation, log HP-cycle	11.32%	11.97%	9.63%

Notes: Statistics are computed using quarterly averages of monthly data. Cyclical unemployment is computed as (log-)deviations from an HP trend with $\lambda = 1600$.

In this paper, we propose an extension of the standard monetary labor search model of Berentsen *et al.* (2011) along the lines described above. Using a carefully calibrated version of our model, we show that the differential sensitivity to market conditions of unemployment for different groups combines with the inherent nonlinearities of the labor matching model to produce a strong differential impact of business cycles and policy. With regards to the cycle, a typical productivity shock is more harmful to Black workers when unemployment is already high than when it is low. More specifically, when unemployment is already above average, the unemployment gap increases by 0.62pp more after a one standard deviation shock than when aggregate unemployment is below average. We provide an analytical decomposition of the mechanisms leading to this state dependency.

With regards to policy, we consider the business cycle properties of the racial unemployment gap under different inflation regimes. In our calibrated model, we find that a substantial increase in trend inflation above the Federal Reserve's target from 2.5% to 5% would increase the average unemployment gap by about 0.71pp. It would increase unemployment volatility for Black workers by almost twice as much as for white workers, i.e. from 1.21% to 1.78% for Black unemployment compared to a move from 0.58% to 0.91% for white unemployment. In addition, we show that the reaction of Black unemployment to a typical negative productivity shock under a high trend inflation regime is on average 0.72pp higher than the reaction under a low trend inflation regime. For whites, this difference amounts to only about 0.36pp. Finally, we use the calibrated model to quantify the differential welfare effects of trend inflation. We find that an increase in trend inflation from the Friedman rule to a 10% inflation rate reduces welfare for Black workers by 1pp more than for white workers.

We should emphasize that, while we study a particular policy experiment concerning monetary policy, the mechanism we highlight is general. Anything that would affect labor productivity, such as taxes or investment incentives, would have an impact on the unemployment gap and its volatility.

2 Relationship to literature

Beginning with Freeman *et al.* (1973), a long and storied literature has explored various aspects of the racial unemployment gap and its behavior over the business cycle in the modern period. Here, we mention only a few recent studies. Cajner *et al.* (2017), employ CPS data to study labor market dynamics for various groups, finding, among other things, that Black workers have substantially higher and more cyclical unemployment rates than white workers, and that these differences are not well explained by observables. Forsythe and Wu (2021) consider heterogeneity in levels and cyclicality of unemployment across groups, and the sources of those differences, finding that group-differences in job finding rates most affects cyclicality while differences in separation rates are more important for persistent differences in the level of unemployment; this parallels our calibration with constant separation rates over the cycle, in line with Shimer (2012). Aaronson *et al.* (2019) build on existing evidence that the semi-elasticity of unemployment for less-advantaged groups is more cyclically sensitive and find that the semi-elasticity gap is smaller when the labor market is strong. Relatedly, Wilson (2015) compares the 90s with several less-robust expansions, showing that Black workers' employment and earnings may benefit from a high-pressure economy, while Fallick and Krolikowski (2019) provide evidence that the labor market benefits of a high-pressure economy may be short-lived.

A parallel literature studies labor market outcomes by gender, education, and other characteristics. For example, Jefferson (2008) analyzes cyclical responses of employment-population ratios by gender and education, and Albanesi and Şahin (2018) study the evolution of gender differences in unemployment from a long-run perspective and over the the business cycle, finding that the gender gap tends to close during periods of low unemployment.

Some theoretical explanations for the state-dependent behavior of unemployment-rate gaps follow the *last-in first-out* hypothesis. The hypothesis states that discriminated groups are the last to be hired during an expansion and the first to be fired during a contraction. For Black workers, however, the last-in first-out hypothesis is examined and largely rejected by Couch and Fairlie (2010). Kuhn and Chancı (2019) explain the racial unemployment gap by introducing discrimination in a Blanchard and Diamond (1994) urn-ball matching model. In the model, increased competition for jobs during a recession hurts Black workers. This can explain 70% of the extra business cycle volatility in the black unemployment rate. Our contribution is to show that with only a minor alteration to the standard Diamond (1982), Mortensen (1982) and Pissarides (1985) (DMP) framework, one can explain almost all the cyclicality of the unemployment gap. This alteration relies neither on the first-in first-out hypothesis, nor on rivalry among workers in the unemployment pool.

Our work contributes to the literature studying the cyclical properties of the DMP framework. Particularly, we explore mechanisms controlling the cyclicality of the racial unemployment gap, from both a theoretical and a quantitative perspective. This complements the existing literature, which focuses on the aggregate unemployment rate. Research on the cyclical properties of the DMP framework was spurred by Shimer (2005), who questions the ability of the framework to fit observed unemployment movements. Hagedorn and Manovskii (2008) provide a calibration which resolves Shimer's puzzle based on a high opportunity cost of labor. Hall and Milgrom (2008) explain the puzzle based on an alternate bargaining procedure. Ljungqvist and Sargent (2017, 2021) characterize resolutions of Shimer's puzzle as different manifestations of a common mechanism: a small fundamental labor surplus which leads to high unemployment volatility. We examine the importance of this mechanism in our exploration of the sources driving the cyclicality of the racial unemployment gap.

By employing a global method to solve our model, we follow Petrosky-Nadeau *et al.* (2018) and Petrosky-Nadeau and Zhang (2020). They show the importance of global methods to accurately characterize higher moments of the unemployment rate distribution. In the spirit of Bernstein *et al.* (2021), we also discuss higher moments of the unemployment-rate gap over the cycle and its determinants, such as nonlinearities in the matching function.

The question of the disparate impacts of monetary policy has also received study in the literature. Thorbecke (2001), using a VAR and the Romer dates, finds that contractionary shocks have approximately double the effect on unemployment for Black workers as for white workers. Carpenter and Rodgers (2004), building on Thorbecke, show that this is mostly due to changes in labor demand. More recently, Bartscher *et al.* (2021) employ an instrumental variable local projections method to study the effects of monetary shocks on asset markets and the Black-white unemployment gap over five-year horizons. They find that the unemployment rate for Black workers falls by 0.2pp more than for white workers after a 100bp monetary shock. Other VAR analyses include De *et al.* (2021), Bennani (2022), and Zavodny and Zha (2000). Lee *et al.* (2021) and Bergman *et al.* (2022) provide New Keynsian modelling of the racial unemployment gap and its response to monetary policy, considering the possibility of targeting Black unemployment and the recent move to average inflation targeting, respectively. Our study is distinguished from these by exploring the underlying theoretical mechanism for this differential impact, and in our use of a New Monetarist rather than a New Keynsian framework.

Regarding the New Monetarist framework, Berentsen *et al.* (2011) unify the DMP framework and the Lagos and Wright (2005) money-search framework to study the effects of monetary policy on unemployment in the long run. Among others, Gomis-Porqueras *et al.* (2013), Rocheteau and Rodriguez-Lopez (2014), Bethune *et al.* (2015), Bethune and Rocheteau (2017), Dong and Xiao (2019), Ait Lahcen (2020), Gomis-Porqueras *et al.* (2020), He and Zhang (2020), Jung and Pyun (2020), Gu *et al.* (2021), and Gabrovski *et al.* (2023) have further investigated this long-run relationship. We employed the framework of Berentsen *et al.* (2011) in related work (Ait Lahcen *et al.*, 2022) in order to study the effects of trend inflation on the cyclical behavior of unemployment and output. In the current paper, we build on our previous work to study how the cyclical behavior of the racial unemployment gap depends on trend inflation.

3 Model

Time, denoted with $t \geq 0$, is discrete and continues forever. Perfectly divisible *fiat money*, issued by a government, as well as two perfectly divisible goods—the *numeraire good* and the *special good*—are traded in the economy. We express all real prices and quantities in terms of the numeraire good. The economy is inhabited by a large mass of potential *firms* and a unit mass of infinitely lived *households*. A mass $1-\lambda$ of households belong to group A and a mass λ belong to group B. Group membership is permanent and observable on contact. Each household consists of two members—a *worker* and a *buyer*—and has preferences described by the utility function

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[h_t + \mathrm{u}\left(x_t\right)\right], \tag{1}$$

where h_t is the time-t net consumption of the numeraire good and x_t the time-t consumption of the special good. Firms have preferences described by the utility function

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t h_t, \tag{2}$$

with h_t again denoting the time-t net consumption of the numeraire good.

At time t, three markets convene sequentially. First, a frictional *labor market* (LM) in which unemployed workers and firms are matched to produce the numeraire good. Second, a *decentralized goods market* (DM) in which fiat money is essential and productive firms can use the numeraire good to produce the special good and sell it to buyers. Third, a frictionless *centralized market* (CM) in which firms pay wages to workers and sell the numeraire goods left after DM production, and in which the households re-adjust their money holdings.

Matching, production, separation, and discrimination in the LM. At each point in time, each worker is either *unemployed* or *employed*. When the LM convenes, a firm is either *inactive*, *posting a vacancy*, or already *matched* to a worker. In the LM, pairwise meetings take place between unemployed workers and vacancy-posting firms. This process is random and governed by a matching function—if there is a mass v_t of vacancies and a mass $u_{t-1} = \mu^A_{t-1} + \mu^B_{t-1}$ of unemployed workers, where μ^A_{t-1} (μ^B_{t-1}) is the mass of unemployed A-workers (resp. B-workers), a mass $\mathcal{M}(v_t, u_{t-1})$ of meetings take place. Here, $\mathcal{M}$ is a constant returns to scale matching function satisfying the standard assumptions. Writing $\theta_t = v_t/u_{t-1}$ for *market tightness*, the meeting probability for an unemployed household is

$$\mathcal{M}(\theta_t, 1) \equiv f(\theta_t) \tag{3}$$

and the meeting probability for a vacancy-posting firm is

$$\mathcal{M}(1, 1/\theta_t) \equiv q(\theta_t). \tag{4}$$

A key feature of our model is that some LM meetings fail to result in a job match due to discrimination—a fraction $\rho_j \in [0, 1)$ of randomly selected meetings between firms and workers from group $j \in \{A, B\}$ fail to result in a match. We normalize $\rho_A = 0$ and write $\rho = \rho_B \in (0, 1)$ to save on notation. The job finding probability for unemployed A-workers is then $f(\theta_t)$ and that for the unemployed B-workers is $(1 - \rho)f(\theta_t)$. Defining

$$\gamma_{t-1} = \frac{\mu^B_{t-1}}{\mu^A_{t-1} + \mu^B_{t-1}} \tag{5}$$

as the proportion of B-workers in the unemployed pool, a vacancy is filled by an A-worker with probability $(1 - \gamma_{t-1})q(\theta_t)$ and by a B-worker with probability $\gamma_{t-1}(1 - \rho)q(\theta_t)$. The overall probability of a vacancy being filled is thus $(1 - \rho\gamma_{t-1})q(\theta_t)$. Discrimination also affects the destruction of matches—in the LM, existing matches between firms and A-workers are destroyed with probability $\delta^A \in (0, 1)$, whereas those between firms and B-workers are destroyed with probability $\delta^B \in (\delta^A, 1)$.

Workers who are separated from their jobs at time t start searching for vacancies at time $t + 1$.

The mass of unemployed workers from group A and B thus evolve according to:

$$\mu_t^A = \delta^A(1-\lambda) + [1 - \delta^A - f(\theta_t)]\mu_{t-1}^A, \tag{6}$$

$$\mu_t^B = \delta^B\lambda + [1 - \delta^B - (1-\rho)f(\theta_t)]\mu_{t-1}^B. \tag{7}$$

Summing equations (6) and (7) and using our definition of γ_{t-1}, we find that (u_t, γ_t) satisfies the two-dimensional dynamic system:

$$u_t = \delta^A(1-\lambda) + \delta^B\lambda + \left[1 - (1-\gamma_{t-1})\delta^A - \gamma_{t-1}\delta^B - (1-\rho\gamma_{t-1})f(\theta_t)\right] u_{t-1}, \tag{8}$$

$$\gamma_t = \frac{\delta^B\lambda + [1 - \delta^B - (1-\rho)f(\theta_t)]\gamma_{t-1}u_{t-1}}{\delta^A(1-\lambda) + \delta^B\lambda + [1 - (1-\gamma_{t-1})\delta^A - \gamma_{t-1}\delta^B - (1-\rho\gamma_{t-1})f(\theta_t)]\, u_{t-1}}. \tag{9}$$

A matched worker-firm pair produces y_t numeraire goods in the LM, which the firm can use for special goods production in the DM or which the firm can sell in the CM. We therefore refer to matched firms as *productive*.

Monetary exchange in the DM. In the DM, the worker and buyer belonging to a household are spatially separated, and there is a mass $1 - u_t$ of productive firms seeking matches with a unit mass of buyers.[1] Matching is random and the mass of realized matches is governed by a CRS matching function $\mathcal{N}(1-u_t, 1)$. The CRS property implies that each buyer is matched to a firm with probability $\alpha(1-u_t)$ and that each firm is matched to a buyer with probability $\alpha(1-u_t)/(1-u_t)$, where $\mathcal{N}(1-u,1) \equiv \alpha(1-u)$, $\alpha' > 0$, $\alpha'' < 0$, and $\alpha(1-u) \leq 1-u$.

Buyers in the DM want to consume the special good, and the firms can produce x_t of it by using $c(x_t)$ numeraire goods as input. Terms of trade in DM matches are determined by proportional bargaining à la Kalai (1977), which we describe below. In the spirit of Kocherlakota (1998), information and commitment frictions rule out credit arrangements in the DM. To facilitate trade, therefore, agents need a medium of exchange, and this role is served exclusively by fiat money.

Settlement and re-balancing in the CM. The CM is a Walrasian market for money and numeraire goods. Productive firms sell their remaining inventories and pay out wages, whereas unproductive firms can open up a vacancy for the next LM. Households adjust their money holdings and pay lump-sum taxes (receive subsidies) collected (resp. distributed) by the government.

Lump-sum taxes T_t (subsidies when negative) are used to augment the supply of fiat money. We think of monetary policy in terms of the *nominal interest rate*, defined as $\iota_t = (1+\pi_t)/\beta - 1$, where π_t is the inflation rate for the price of numeraire goods. Our definition of the nominal rate implies that it exactly compensates a household for inflation and discounting. So in this sense,

[1] The mass of productive firms equals the mass of employed workers since every firm is matched to at most one worker.

it captures the opportunity cost of holding fiat money. When we refer to the Friedman rule we mean a monetary policy that eliminates this opportunity cost by setting $\iota = 0$.

Opening up a vacancy costs κ numeraire goods for firms. The wages paid out to the workers by productive firms are group specific and determined by Nash bargaining in the previous LM. Unemployed workers receive an exogenous amount b of the numeraire good in the CM, which represents an amalgamation of unemployment benefits, the value of leisure, and home production.

4 Stochastic Equilibrium

We focus on an environment in which the stochastic exogenous variables of our model—the production of the numeraire y_t and the nominal rate ι_t—follow first-order Markov processes. We therefore define the equilibrium in a recursive, Markovian fashion and use the superscript $-$ $(+)$ to denote variables at time $t-1$ (resp. $t+1$). The aggregate state of the economy at time t is sufficiently described by $\Omega = (u^-, \gamma^-, \iota, y)$. Here, u^- is the aggregate unemployment at the end of time $t-1$, γ^- the proportion of B-workers in the unemployment pool at the end of time $t-1$, ι the time-t nominal rate, and y the time-t numeraire production by a productive firm.

The aggregate state of the economy is taken as given at the beginning of a time period, with unemployment and the composition of the unemployment pool determined endogenously in the preceding LM. We assume that the current period's ι and y are already known in the CM of the preceding period. Hence, in the CM, in which households adjust their money holdings and firms post vacancies, the aggregate state Ω^+ is part of agents' information set.

4.1 CM value functions

A household with employment status $i \in \{0, 1\}$, that belongs to group $j \in \{A, B\}$, that holds real money balances z, and that negotiated a real wage w in the LM, faces the CM value function

$$V_{CM}^{i,j}(z, w, \Omega^+) = \max_{h,z^+} \left\{ h + \beta V_{LM}^{i,j}(z^+, \Omega^+) \right\} \tag{10}$$

$$\text{s.t.} \quad h + (1 + \pi^+)z^+ + T(\Omega^+) \leq z + iw + (1 - i)b \quad \text{and} \quad z^+ \geq 0, \tag{11}$$

where z^+ denotes the real money balances carried into the next period. These money balances are expressed in terms of next period's numeraire goods, which is why inflation shows up in the budget constraint. Using our definition of the nominal rate and that the budget constraint is binding, we can write

$$V_{CM}^{i,j}(z, w, \Omega^+) = z + iw + (1 - i)b + \overline{V}_{CM}^{i,j}(\Omega^+), \tag{12}$$

where

$$\overline{V}^{i,j}_{CM}(\Omega^+) = \beta \max_{z^+ \geq 0} \left\{ V^{i,j}_{LM}(z^+, \Omega^+) - (1 + \iota^+) z^+ \right\} - T(\Omega^+). \tag{13}$$

A productive firm which is matched to a worker from group $j \in \{A, B\}$, negotiated a wage w in the LM, earned money worth r numeraire goods in the DM, and has a remaining inventory o of numeraire goods, faces the CM value function

$$J^{1,j}_{CM}(r, o, w, \Omega^+) = r + o - w + \beta J^{1,j}_{LM}(\Omega^+), \tag{14}$$

where superscript 1 denotes a matched/productive firm. Both inactive firms and firms that were separated from a worker in the LM can open up a vacancy at cost κ. Their CM value function is

$$J^0_{CM}(\Omega^+) = \max \left\{ 0, -\kappa + \beta J^0_{LM}(\Omega^+) \right\}, \tag{15}$$

with superscript 0 denoting an unmatched firm in the CM and a vacancy-posting firm in the LM.[2] Because there is a large mass of firms, a free-entry condition imposes that $J^0_{CM}(\Omega^+) = 0$.

4.2 DM bargaining and value functions

Consider a match between, on the one hand, a buyer belonging to a household with employment status i from group j, carrying real money balances z, and, on the other hand, a productive firm, matched to a worker from group j' which has been promised a wage w'. Terms of trade (x, p), with x denoting special goods sold to the buyer and p the real payment by the buyer, are chosen to maximize the match surplus

$$\begin{aligned} &\mathrm{u}(x) + \mathbb{E}\left\{ V^{i,j}_{CM}(z - p, w, \Omega^+) - V^{i,j}_{CM}(z, w, \Omega^+) \right\} \\ &\qquad + \mathbb{E}\left\{ J^{1,j'}_{CM}(p, y - \mathrm{c}(x), w', \Omega^+) - J^{1,j'}_{CM}(0, y, w', \Omega^+) \right\}, \end{aligned} \tag{16}$$

subject to the liquidity constraint $p \leq z$, the production constraint $\mathrm{c}(x) \leq y$ and the sharing rule

$$\begin{aligned} &(1 - \varphi) \left[\mathrm{u}(x) + \mathbb{E}\left\{ V^{i,j}_{CM}(z - p, w, \Omega^+) - V^{i,j}_{CM}(z, w, \Omega^+) \right\} \right] \\ &\qquad = \varphi \mathbb{E}\left\{ J^{1,j'}_{CM}(p, y - \mathrm{c}(x), w', \Omega^+) - J^{1,j'}_{CM}(0, y, w', \Omega^+) \right\}. \end{aligned} \tag{17}$$

[2] Straightforward arguments imply that the value of remaining inactive in the LM, i.e. neither having a vacancy posted nor having a match with a worker, is zero.

In what follows, we assume that the production constraint is always slack. Because $V_{CM}^{i,j}$ and $J_{CM}^{1,j'}$ are affine in their first arguments, the bargaining problem simplifies to

$$\max_{x,p}\{\mathrm{u}(x)-\mathrm{c}(x)\}\quad \text{s.t.}\quad (1-\varphi)\left[\mathrm{u}(x)-p\right]=\varphi\left[p-\mathrm{c}(x)\right]\quad \text{and}\quad p\leq z. \tag{18}$$

We characterize the solution by defining the pricing protocol $\mathrm{g}: x \mapsto p$, mapping the traded quantity of special goods into a required payment by the buyer. The sharing rule implies

$$\mathrm{g}(x)=(1-\varphi)\mathrm{u}(x)+\varphi\mathrm{c}(x). \tag{19}$$

Defining x^* as the solution to $\mathrm{u}'(x)=\mathrm{c}'(x)$, terms of trade are

$$x=\min\{x^*,\mathrm{g}^{-1}(z)\}\quad \text{and}\quad p=\min\{\mathrm{g}(x^*),z\}. \tag{20}$$

Using the properties of $V_{CM}^{i,j}$ and the household's understanding of the dependency of u on Ω, the DM value function for the household is

$$\begin{aligned} V_{DM}^{i,j}(z,w,\Omega)=\alpha(1-u(\Omega))\varphi\left[\mathrm{u}\left(\min\{x^*,\mathrm{g}^{-1}(z)\}\right)-\mathrm{c}\left(\min\{x^*,\mathrm{g}^{-1}(z)\}\right)\right]& \\ +z+iw+(1-i)b+\mathbb{E}\left\{\overline{V}_{CM}^{i,j}(\Omega^+)\right\}.& \end{aligned} \tag{21}$$

Similarly, firms' understanding of the dependency of u and z on Ω implies that the DM value function for the matched firms is

$$J_{DM}^{1,j}(w,\Omega)=\frac{\alpha(1-u(\Omega))}{1-u(\Omega)}(1-\varphi)\left[\mathrm{u}\left(x(\Omega)\right)-\mathrm{c}\left(x(\Omega)\right)\right]+y-w+\beta\mathbb{E}\left\{J_{LM}^{1,j}(\Omega^+)\right\}. \tag{22}$$

For the unmatched and inactive firms, we have as DM value function

$$J_{DM}^{0}(\Omega)=\mathbb{E}\left\{J_{CM}^{0}(\Omega^+)\right\}=0. \tag{23}$$

4.3 LM value functions and wage bargaining

Worker-firm pairs negotiate wages in the LM, with wages dependent on the worker's group identity. The agents anticipate the dependency of market tightness θ and the group-specific wage w^j on Ω. The value functions for the households are therefore

$$V_{LM}^{1,j}(z,\Omega)=(1-\delta^j)V_{DM}^{1,j}(z,w^j(\Omega),\Omega)+\delta^jV_{DM}^{0,j}(z,0,\Omega), \tag{24}$$

$$\begin{aligned} V_{LM}^{0,j}(z,\Omega)=&(1-\rho\mathbf{1}_{\{j=B\}})f\left(\theta(\Omega)\right)V_{DM}^{1,j}(z,w^j(\Omega),\Omega)\\ &+\left[1-(1-\rho\mathbf{1}_{\{j=B\}})f\left(\theta(\Omega)\right)\right]V_{DM}^{0,j}(z,0,\Omega), \end{aligned} \tag{25}$$

and using that $J^0_{DM}(\Omega) = 0$, the value functions for the firms are

$$J^{1,j}_{LM}(\Omega) = (1-\delta^j) J^{1,j}_{DM}(w^j(\Omega), \Omega), \tag{26}$$
$$J^0_{LM}(\Omega) = (1-\gamma^-) q\left(\theta(\Omega)\right) J^{1,A}_{DM}(w^A(\Omega), \Omega) + \gamma^-(1-\rho) q\left(\theta(\Omega)\right) J^{1,B}_{DM}(w^B(\Omega), \Omega). \tag{27}$$

Forward iterating on (24) and (25), for a worker, the surplus of being matched to a firm at wage w is

$$\begin{aligned} S^{1,j}_{DM}(w, \Omega) &\equiv V^{1,j}_{DM}(z, w, \Omega) - V^{0,j}_{DM}(z, 0, \Omega) \\ &= w - b + \beta \mathbb{E}\left\{[1-\delta^j - (1-\rho \mathbf{1}_{\{j=B\}}) f(\theta(\Omega^+))] S^{1,j}_{DM}(w^j(\Omega^+), \Omega^+)\right\}. \end{aligned} \tag{28}$$

For a firm, the surplus of being matched to a worker from group j at wage w is $J^{1,j}_{DM}(w, \Omega)$ since $J^0_{DM} = 0$. Forward iterating on (26), we find

$$J^{1,j}_{DM}(w, \Omega) = \mathcal{O}(\Omega) - w + \beta(1-\delta^j)\mathbb{E}\left\{J^{1,j}_{DM}(w^j(\Omega^+), \Omega^+)\right\} \tag{29}$$

where

$$\mathcal{O}(\Omega) = y + \frac{\alpha(1-u(\Omega))}{1-u(\Omega)}(1-\varphi)\left[\mathrm{u}\left(x(\Omega)\right) - \mathrm{c}\left(x(\Omega)\right)\right] \tag{30}$$

is the expected flow output of the productive firm. Let $\mathcal{S}^j(\Omega) = S^{1,j}_{DM}(w, \Omega) + J^{1,j}_{DM}(w, \Omega)$ denote the surplus from a worker-firm match, with j denoting the worker's group identity and $\mathcal{S}^j(\Omega)$ independent of the wage since it cancels out in the calculation of match surplus. The wage is set by Nash bargaining with bargaining power ξ for the workers, so

$$S^{1,j}_{DM}(w^j(\Omega), \Omega) = \xi \mathcal{S}^j(\Omega). \tag{31}$$

Summing (28) and (29), we obtain the surplus of a worker-firm match:

$$\mathcal{S}^j(\Omega) = \mathcal{O}(\Omega) - b + \beta \mathbb{E}\left\{\left[1-\delta^j - \xi(1-\rho \mathbf{1}_{\{j=B\}}) f\left(\theta(\Omega^+)\right)\right] \mathcal{S}^j(\Omega^+)\right\}. \tag{32}$$

Finally, using that market tightness is $\theta(\Omega)$, we can solve for u and γ:

$$u(\Omega) = \delta^A(1-\lambda) + \delta^B \lambda + \left[1 - (1-\gamma^-)\delta^A - \delta^B \gamma^- - (1-\rho\gamma^-) f(\theta(\Omega))\right] u^-, \tag{33}$$
$$\gamma(\Omega) = \frac{\delta^B \lambda + [1 - \delta^B - (1-\rho) f(\theta(\Omega))]\gamma^- u^-}{\delta^A(1-\lambda) + \delta^B \lambda + [1 - (1-\gamma^-)\delta^A - \delta^B \gamma^- - (1-\rho\gamma^-) f(\theta(\Omega))] u^-}. \tag{34}$$

4.4 Real balances

With the derivations above, we can write $V^{i,j}_{LM}(z,\Omega)$ as

$$\begin{aligned} V^{i,j}_{LM}(z,\Omega) = {} & \xi\mathcal{S}\left(\Omega\right)\left[(1-\delta^{j})\mathbf{1}_{\{i=1\}} + (1-\rho\mathbf{1}_{\{j=B\}})f(\theta(\Omega))\mathbf{1}_{\{i=0\}}\right] \\ & + \alpha\left(1-u(\Omega)\right)\varphi\left[\mathrm{u}\left(\min\{x^{*},\mathrm{g}^{-1}(z)\}\right) - \mathrm{c}\left(\min\{x^{*},\mathrm{g}^{-1}(z)\}\right)\right] \\ & + z + b + \mathbb{E}\left\{\overline{V}^{0,j}_{CM}\left(\Omega^{+}\right)\right\}. \end{aligned} \tag{35}$$

From (13) it follows that real money balances are chosen to maximize

$$-\iota z + \alpha(1-u(\Omega))\varphi\left[\mathrm{u}\left(\min\{x^{*},\mathrm{g}^{-1}(z)\}\right) - \mathrm{c}\left(\min\{x^{*},\mathrm{g}^{-1}(z)\}\right)\right]. \tag{36}$$

The amount of special goods traded within DM matches therefore satisfies

$$\mathrm{u}'(x) \leq \left(1 + \frac{\iota}{\alpha(1-u(\Omega))}\right)\left[(1-\varphi)\mathrm{u}'(x) + \varphi\mathrm{c}'(x)\right], \quad \text{with " = " if } x > 0. \tag{37}$$

4.5 Vacancy creation

The free-entry condition requires that $\kappa = \beta J^{0}_{LM}(\Omega)$. Using the characterization of $J^{0}_{LM}(\Omega)$, the free entry condition implies that vacancy creation in the CM is such that

$$\kappa = \beta q(\theta(\Omega))(1-\xi)\left[(1-\gamma^{-})\mathcal{S}^{A}(\Omega) + (1-\rho)\gamma^{-}\mathcal{S}^{B}(\Omega)\right]. \tag{38}$$

4.6 Recursive Equilibrium

The recursive equilibrium consists of functions $x(\Omega)$, $\mathcal{O}(\Omega)$, $\mathcal{S}^{A}(\Omega)$, $\mathcal{S}^{B}(\Omega)$, $\theta(\Omega)$, $u(\Omega)$, and $\gamma(\Omega)$ satisfying the expected flow output equation (30), the two Bellman equations for the match surplus (32), the law of motion for unemployment (33) and its composition (34), the optimal choice of real balances (37), and the condition for vacancy creation (38).

5 Calibration and Quantitative Results

5.1 Calibration

We calibrate the model to a monthly frequency. Most of our data covers the period from January 1972 to December 2019.[3] In our calibration strategy we proceed in two steps. First, a set of parameters is calibrated externally. This set includes the discount factor β, the job separation rates δ^{A} and δ^{B}, the discrimination parameter ρ, the measure λ and the exogenous process for

[3] We start from 1972, the first year data on Black unemployment is available.

Table 2: Directly calibrated parameters

Parameter	Description	Value
β	Discount factor	0.997
λ	Measure of Blacks in the labor force	0.117
ρ	Degree of hiring discrimination	0.301
δ^A	Whites job separation rate	0.023
δ^B	Blacks job separation rate	0.045
$\bar{y}$	Average labor productivity	1.00
$\rho_{\hat{\imath}}$	Autocorr. of interest-rate shocks	0.939
$\varepsilon_{\hat{\imath}}$	SD of interest-rate shocks	0.0002

the interest rate ι. Second, the remaining set of parameters is calibrated jointly to match a set of monthly and quarterly empirical moments using a Simulated Method of Moments procedure.[4] We focus on matching moments on both the labor market and monetary data. In our simulations, the model economy is subject to both productivity and nominal interest-rate shocks.

Labor market parameters. The stochastic process for y_t follows the AR(1) process

$$\log y_t = (1 - \rho_y) \log \bar{y} + \rho_y \log y_{t-1} + \sigma_y \varepsilon_{y,t} \tag{39}$$

where $\varepsilon_y \sim \mathcal{N}(0, 1)$ and $\bar{y}$ is normalized to 1. We calibrate σ_y and ρ_y such that real output per worker in the model, which includes the endogenous quantity traded in the DM, matches the volatility and persistence of the observed real output per worker in the data. For the latter, we use real output per worker in the non-farm business sector as measured by the Bureau of Labor Statistics' (BLS) following Shimer (2005). As is standard, we use the HP-filtered cyclical component of the logarithm of the quarterly observations for both empirical and simulated data.

The measure λ is set to 0.117 in order to match the average ratio of Blacks to whites among civilian labor force participants in the Current Population Survey (CPS) data. This ratio averaged 0.133 over the period from January 1972 to December 2019.

The flow value of unemployment b is internally calibrated to match the empirical volatility of aggregate unemployment. The latter is measured as the standard deviation of the quarterly log-deviations of unemployment from its HP-filtered trend.

The cost of posting vacancies κ is calibrated such that the average labor-market tightness θ in our simulations matches its empirical counterpart. The latter is measured as the average ratio of the unemployment to vacancy rates. To obtain the vacancy rate data series, we follow Petrosky-Nadeau and Zhang (2020) by combining the series from Barnichon (2010), which covers the period from January 1972 to November 2000, with the series from the Job Openings and Labor Turnover

[4]To match moments based on quarterly data, we aggregate the simulated monthly data on a quarterly basis.

Survey (JOLTS) of the BLS, which covers the period from December 2000 to December 2019. The latter series is constructed by dividing the number of job openings by the civilian labor force.

Workers' bargaining power ξ is calibrated to match the real wage elasticity to labor productivity as in Hagedorn and Manovskii (2008). We compute real wage from BLS data as the product of labor productivity and the labor income share. We then use the HP-filtered logarithm of the resulting series to estimate the real wage elasticity.

We use the Den Haan *et al.* (2000) matching function

$$\mathcal{M}(v, 1-n) = \frac{v(1-n)}{(v^{\chi} + (1-n)^{\chi})^{1/\chi}}, \tag{40}$$

to guarantee that the job finding and vacancy filling probabilities stay in the interval $[0, 1]$. The parameter χ is internally calibrated to match the empirical average job finding rate. We directly set ρ to match the ratio of the average job finding rates between Blacks and whites which is about 0.70 based on data from Karahan *et al.* (2021). We set job separation probabilities δ^A and δ^B to jointly match the ratio of the average job separation rates for Blacks and whites and the average separation rate for the overall labor force in the data. The ratio of the average Black to white male job separation rates is about 1.92 and is taken from Cajner *et al.* (2017). The series for the aggregate job finding and separation rates are constructed as in Shimer (2005) using CPS data on short-term unemployment from the BLS.

Goods markets parameters. The utility households obtain by consuming the CM good is linear, while the utility from the DM good takes the form

$$\mathrm{u}(x) = A\frac{x^{1-a}}{1-a}, \tag{41}$$

with parameters $a \in (0, 1)$ and $A > 0$. Buyers meet firms in the DM at the rate

$$\alpha(n) = \zeta\frac{n}{1+n}, \tag{42}$$

where n is the measure of active firms and ζ determines matching efficiency. For firms' cost function, we simply assume $\mathrm{c}(x) = x$.

We set the monthly discount factor β to 0.997, consistent with an average annual real interest rate of 3.37%. The remaining parameters related to goods markets are calibrated following Lagos and Wright (2005) and Berentsen *et al.* (2011). DM utility parameters A and a are calibrated to match the average and interest-rate elasticity of money demand (i.e., the inverse of money velocity). We use the the sum of M1 and the Money Market Deposit Accounts held at commercial

Table 3: SMM calibrated parameters

Parameter	Description	Value	Moment	Frequency	Data	Model
κ	Vacancy cost	1.462	Average θ	Monthly	0.567	0.577
b	Flow value of unemployment	0.952	Unemployment volatility	Quarterly	0.113	0.113
χ	Parameter of the LM matching fun.	1.427	Average JFP	Monthly	0.399	0.399
ξ	Worker bargaining weight	0.041	Elast. of avg. wage to labor prod.	Quarterly	0.526	0.526
ρ_y	Persistence parameter of y_t process	0.967	Autocorr. of labor prod.	Quarterly	0.778	0.756
σ_y	Volatility parameter of y_t process	0.011	SD of labor prod.	Quarterly	0.006	0.006
A	Level parameter of DM utility	1.495	Average money demand	Quarterly	24.9%	24.9%
a	Curvature parameter of DM utility	0.228	Elast. of money demand to ι	Quarterly	-0.806	-0.806
ζ	Parameter of the DM matching fun.	0.130	Elast. of u to ι	Monthly	0.241	0.241
φ	Buyer bargaining weight	0.414	Average price markup	Monthly	0.381	0.381

banks in the United States as our measure of aggregate money supply.[5][6]

We choose the monthly Moody's composite yield on Aaa-rated long-term U.S. corporate bonds as the empirical counterpart of ι, the opportunity cost of holding money in the model.[7] It is easy to see that this series is non-stationary. To remedy that, we decompose the interest-rate series into two components: a trend, $\bar{\iota}_t$, and a cycle, $\hat{\iota}_t$, such that $\iota_t = \bar{\iota}_t + \hat{\iota}_t$ where $\hat{\iota}_t$ follows the AR(1) process

$$\hat{\iota}_t = \rho_{\hat{\iota}}\hat{\iota}_{t-1} + \sigma_{\hat{\iota}}\varepsilon_{\hat{\iota},t}, \tag{43}$$

with $\varepsilon_{\hat{\iota}} \sim \mathcal{N}(0,1)$. The cyclical component is extracted using the HP filter with parameter $\lambda = 126,000$. Estimation of the AR(1) yields $\rho_{\hat{\iota}} = 0.939$ and $\sigma_{\hat{\iota}} = 0.0002$. $\bar{\iota}_t$ is assumed to follow a very persistent discrete Markov process with 5 states. Appendix A presents the state values and the estimated transition probabilities.

The DM matching efficiency ζ is calibrated to match the average monthly interest-rate elasticity of unemployment in the US data following Ait Lahcen *et al.* (2022). Finally, buyers' bargaining power is calibrated as in Aruoba *et al.* (2011) to match the average US mark-up of 36% as reported by De Loecker *et al.* (2020).

The model is solved using a global solution method. This is crucial to preserve the nonlinear dynamics inherent to the search and matching framework (Petrosky-Nadeau and Zhang, 2017). We apply the SMM procedure to the model's solution to jointly calibrate the internal parameters as detailed in Appendix A. Table 3 summarizes the results of the calibration.

Model vs. data. Table 4 compares descriptive statistics on unemployment based on data and model simulations. Overall, the model does a good job in replicating key data moments. In terms of levels, the model overstates Black unemployment by about 1.84pp, but the ordering of

[5] For an extended discussion of this measure see Lucas and Nicolini (2015)

[6] We extend the M1 series back to January 1948 with the pre-1959 M1 series produced by Rasche (1987), available from the Federal Reserve Bank of St. Louis at the following link: https://research.stlouisfed.org/aggreg/.

[7] This corresponds to the nominal interest rate earned on a safe but illiquid bond held from CM to CM. See Krishnamurthy and Vissing-Jorgensen (2012) for a discussion of the liquidity of treasuries vs. corporate bonds.

Table 4: Labor market statistics: Model v. Data

	Population U rate	White U rate	Black U rate
US data			
Average	6.23%	5.48%	11.80%
Standard deviation	1.61%	1.47%	3.09%
Standard deviation, HP-cycle	0.77%	0.73%	1.20%
Standard deviation, log HP-cycle	**11.32%**	11.97%	9.63%
Model simulations			
Average	6.47%	5.51%	13.64%
Standard deviation	1.77%	1.58%	3.20%
Standard deviation, HP-cycle	1.00%	0.90%	1.74%
Standard deviation, log HP-cycle	**11.32%**	11.74%	10.01%

Notes: Statistics are computed using quarterly averages of monthly data. US data covers the period from January 1972 to December 2019. Cyclical series are computed as (log-)deviations from an HP trend with $\lambda = 1600$*. Model-based average statistics are computed by averaging over 1,000 simulations with a length of 576 months each. Statistics targeted in the calibration are in bold.*

unemployment across different groups is the same as in the data. More interestingly, the model is able to replicate to a large extent the difference in unemployment volatility between Blacks and whites, both in absolute terms and in (log-)deviations from trend.

Table 5 compares the semi-elasticity implied by the model simulations to its empirical counterpart. We estimate the semi-elasticty from both model and data as the slope coefficient on a level-log OLS regression of the racial unemployment gap on the labor market tightness. As the table shows, the model gets both the direction and the magnitude of the semi-elasticity right. A 1% decrease in labor market tightness above trend is associated with an increase above trend of the racial unemployment gap of 1.6pp in the data compared to 1.8pp in the model simulations. These results provide further evidence that the model is able to capture the main mechanisms at work in the data.

5.2 Nonlinear response of the racial unemployment gap.

Steady-state comparative statics. We first investigate the different responses to labor market conditions between Blacks and whites by focusing on steady-state comparative statics. In particular, we look at the unemployment gap $u^B - u^A$, given by

$$u^B - u^A = \frac{\delta^B}{\delta^B + (1-\rho)f(\theta)} - \frac{\delta^A}{\delta^A + f(\theta)} \tag{44}$$

Table 5: Regression of racial unemployment gap on θ: Model v. Data

	LM tightness θ, log-HP cycle	
	Data	Simulations
	(1)	(2)
Constant	0.000	0.000***
	(0.000)	(0.000)
Racial U gap, HP cycle	-0.016***	-0.018***
	(0.001)	(0.000)
Observations	192	192'000
R^2	0.413	0.396

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Statistics are computed using quarterly averages of monthly data. US data covers the period from January 1972 to December 2019. Model-based regression is estimated using data from 1,000 simulations with a length of 576 months each. The racial unemployment gap is measured as the difference between the HP-filtered cyclical components of the Black and white unemployment rates. All series are detrended using the HP filter with $\lambda = 1600$.

The responsiveness of this gap with respect to a percentage decrease in labor-market tightness is

$$-\frac{\partial(u^B - u^A)}{\partial\theta}\theta = \varepsilon_{f,\theta}[u^B(1-u^B) - u^A(1-u^A)], \quad \text{where } \varepsilon_{f,\theta} = \frac{\theta f'(\theta)}{f(\theta)}. \tag{45}$$

The effect is positive when $u^B \leq 0.5$, since $u^A < u^B$. Thus, worse labor-market conditions, i.e. lower tightness θ, lead to an increase in the unemployment gap.

To investigate the state dependency of this effect, consider how it changes for a percentage decrease in market tightness:

$$\begin{aligned}\frac{\partial^2[u^B - u^A]}{\partial\theta^2}\theta^2 = \varepsilon^2_{f,\theta}\left[u^B(1-u^B)(1-2u^B) - u^A(1-u^A)(1-2u^A)\right] \\ - \frac{\partial\varepsilon_{f,\theta}}{\partial\theta}\theta\left[u^B(1-u^B) - u^A(1-u^A)\right].\end{aligned} \tag{46}$$

There are two effects. The first one originates from changes in the term $u^B(1-u^B) - u^A(1-u^A)$. With $u^A < u^B$, this first effect is positive when $u^B \leq 1/2 - \sqrt{3}/6 \approx 0.21$.[8] The second effect comes from nonlinearities in the matching function. Particularly, changes in the elasticity of f w.r.t. θ, which interact with the responsiveness of the unemployment gap. For empirically

[8] This follows because the cubic equation $x(1-x)(1-2x)$ is strictly increasing on the domain $(0, 1/2 - \sqrt{3}/6)$.

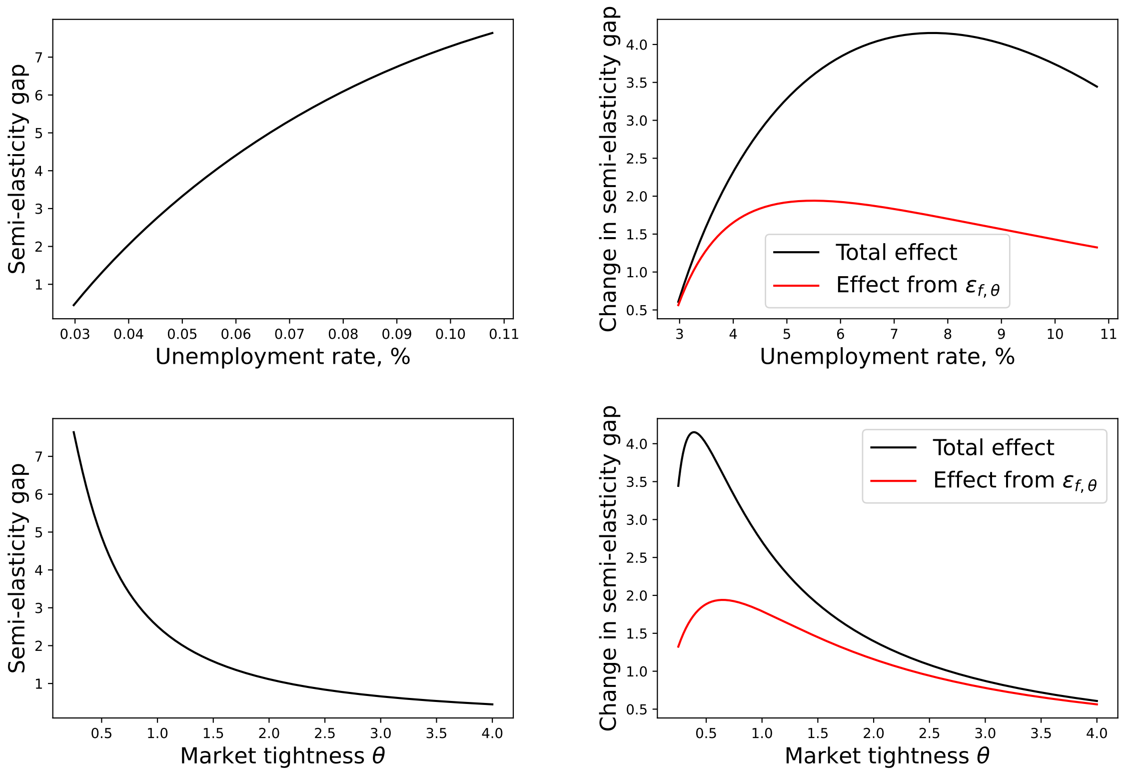

Figure 1: The local responsiveness of the unemployment gap to changes in market tightness.

Notes: The semi-elasticity gap in the left-hand panels is calculated as $-\frac{\partial(u^B-u^A)}{\partial\theta}\theta$, the change in the right-hand panels as $\frac{\partial^2(u^B-u^A)}{\partial\theta^2}\theta^2$, and the effect of $\varepsilon_{f,\theta}$ in the right-hand panels as $-\frac{\partial\varepsilon_{f,\theta}}{\partial\theta}\theta\left[u^B(1-u^B)-u^A(1-u^A)\right]$. The effects are plotted against both the overall steady-state unemployment rate (upper panels) and market tightness (lower panels), and are calculated based on the steady-state equations $u^A=\frac{\delta^A}{\delta^A+f(\theta)}$ and $u^B=\frac{\delta^B}{\delta^B+(1-\rho)f(\theta)}$, and the calibrated parameters in Table 3.

plausible unemployment rates, the overall second-order effect of a change in market tightness on the unemployment gap is positive for matching functions with $\varepsilon_{f,\theta}$ (weakly) decreasing in θ. This includes the Den Haan *et al.* (2000) matching function that we use. Thus, changes in market tightness have a particularly strong effect on the unemployment gap in states with low tightness, or equivalently, in states with high aggregate unemployment.

For our calibrated model, we plot in Figure 1 the responsiveness of the unemployment gap with respect to market tightness for different steady-state levels of unemployment (upper panels) and market tightness (lower panels). As we see from the left-hand panels, the responsiveness is larger in states with high unemployment or equivalently, low market tightness. The right-hand panels plot the second-order effect $\frac{\partial^2[u^B-u^A]}{\partial\theta^2}\theta^2$ and the part of it which comes from changes in the elasticity of the matching function, i.e., the term $-\frac{\partial\varepsilon_{f,\theta}}{\partial\theta}\theta\left[u^B(1-u^B)-u^A(1-u^A)\right]$. We see that changes in the matching elasticity matter, but mostly for low unemployment rates or equivalently, high levels of market tightness.

Table 6: Decomposition of changes in the local responsiveness of the unemployment gap.

	$\theta = 3.74$		$\theta = 0.50$		$\theta = 0.28$
	$u = 3.00\%$		$u = 6.50\%$		$u = 10.00\%$
	$u^A = 2.52\%$		$u^A = 5.53\%$		$u^A = 8.60\%$
	$u^B = 6.62\%$		$u^B = 13.82\%$		$u^B = 20.51\%$
$u^B(1-u^B) - u^A(1-u^A)$	0.0372	0.0669	0.0669	0.0844	0.0844
$\varepsilon_{f,\theta}$	0.1320	—	0.7296	—	0.8628
$\tilde{\varepsilon}_{u^B,\theta} - \tilde{\varepsilon}_{u^A,\theta}$	0.0049 pp	0.0088 pp	0.0488 pp	0.0616 pp	0.0729 pp

Notes: The local responsiveness $\tilde{\varepsilon}_{u^B,\theta} - \tilde{\varepsilon}_{u^A,\theta}$ is calculated as $-\frac{\partial(u^B-u^A)}{\partial\theta}\theta = \varepsilon_{f,\theta}[u^B(1-u^B)-u^A(1-u^A)]$ based on the steady-state equations for unemployment, i.e., $u^A = \frac{\delta^A}{\delta^A+f(\theta)}$ and $u^B = \frac{\delta^B}{\delta^B+(1-\rho)f(\theta)}$, and the calibrated parameters in Table 3. It captures the percentage-point increase in the unemployment gap $u^B - u^A$ for a one-percentage decrease in labor-market tightness θ.

A numerical example for how a change in market tightness affects the responsiveness of the unemployment gap is provided in Table 6. We change steady-state tightness in two steps such that overall unemployment increases first from 3% to 6.5%, and then from 6.5% to 10%. When moving towards a steady state with higher unemployment, we first look at what happens to the local responsiveness of the unemployment gap if the matching elasticity remains fixed. Then, we account for the change in the matching elasticity to calculate the overall effect. On the one hand, moving from a steady state with 3% unemployment to one with 6.5% unemployment, the local effect of a percentage change in market tightness on the unemployment gap increases from 0.0049 pp to 0.0488 pp. Approximately 90% of this change is driven by the changing matching elasticity—keeping the matching elasticity fixed, the local responsiveness increases to only 0.0088 pp. On the other hand, moving from a steady state with 6.5% unemployment to one with 10% unemployment, the local responsiveness increases from 0.0488 pp to 0.0729 pp, with only 47% percent of this change being driven by the matching elasticity.

Generalized impulse response functions. In light of the discussion above, we further investigate how the unemployment rates of the two groups react differently to shocks hitting the economy. To capture the strong nonlinearities of the model, we compute the generalized, nonlinear, impulse response function

$$GIRF_Y(k, \varepsilon_t, \Omega_t) = \mathbb{E}[Y_{t+k}|\varepsilon_t, \Omega_t = \omega_t] - \mathbb{E}[Y_{t+k}|\Omega_t = \omega_t], \tag{47}$$

where $\Omega_t = \omega_t$ is the state of the economy at the beginning of period t and ε_t is an innovation to the exogenous variable at time t. This formulation was proposed, among others, by Gallant *et al.* (1993) and Koop *et al.* (1996). Equation (47) measures the change caused by a shock ε_t in the expectation of Y_{t+k} conditional on the state $\Omega_t = \omega_t$. In a nonlinear model, the shape of the

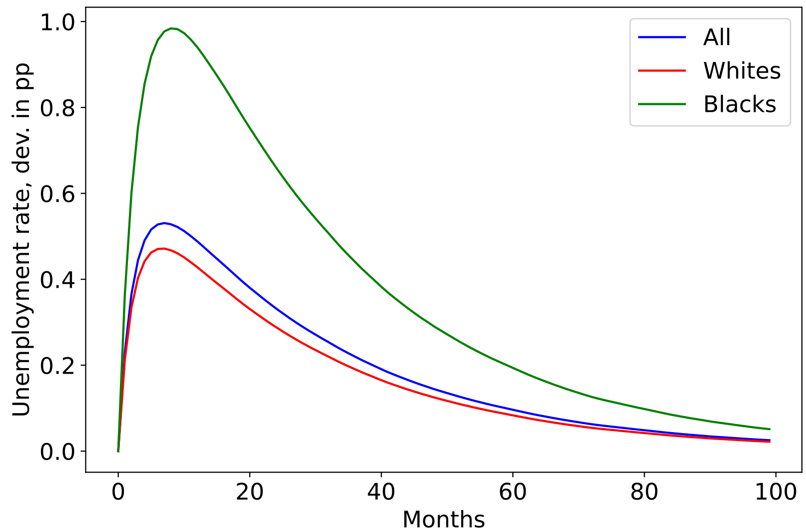

Figure 2: Reaction of unemployment to a negative productivity shock.
Notes: Unconditional GIRFs for u following a 1 standard deviation negative productivity shock. The unconditional GIRFs are computed by averaging over 1,000 conditional GIRFs. Each conditional GIRF is computed by averaging over 10,000 simulations of 100 months each starting from the same initial state. The 1,000 initial states are drawn randomly from the ergodic distribution of the calibrated model.

GIRF will in general be a function of the state of the economy at the moment the shock hits. To take account of that, we draw randomly 1,000 initial states from the ergodic distribution of the calibrated model's state variables. From each initial state we run 10,000 simulations with the shock and 10,000 simulations without the shock, each simulation lasting 100 months. The conditional GIRF at each initial state is the difference between the conditional expectations over these two simulation sets. By averaging across the initial states, we obtain the average unconditional GIRF given by

$$\mathbb{E}[GIRF_Y(k, \varepsilon_t, \Omega_t)] = \mathbb{E}[Y_{t+k}|\varepsilon_t] - \mathbb{E}[Y_{t+k}], \tag{48}$$

where the expectation is computed over the ergodic distribution of the state Ω_t. Figure 2 depicts the average unconditional GIRFs for aggregate, Blacks' and whites' unemployment rates following a negative productivity shock. At the peak, aggregate unemployment increases by 0.53pp while whites' unemployment increases by 0.47pp. Blacks' unemployment increases by 0.98pp, double the increase for whites. In addition, the effect of the shock is quite persistent, in particular for Blacks.

Figure 3 depicts the state-dependent reaction of unemployment to productivity shocks. We compute the GIRFs conditional on the level of unemployment being above (high u) or below (low u) its unconditional mean. For both groups, the reaction of unemployment to a one standard deviation negative productivity shock is much stronger when unemployment is already high. This

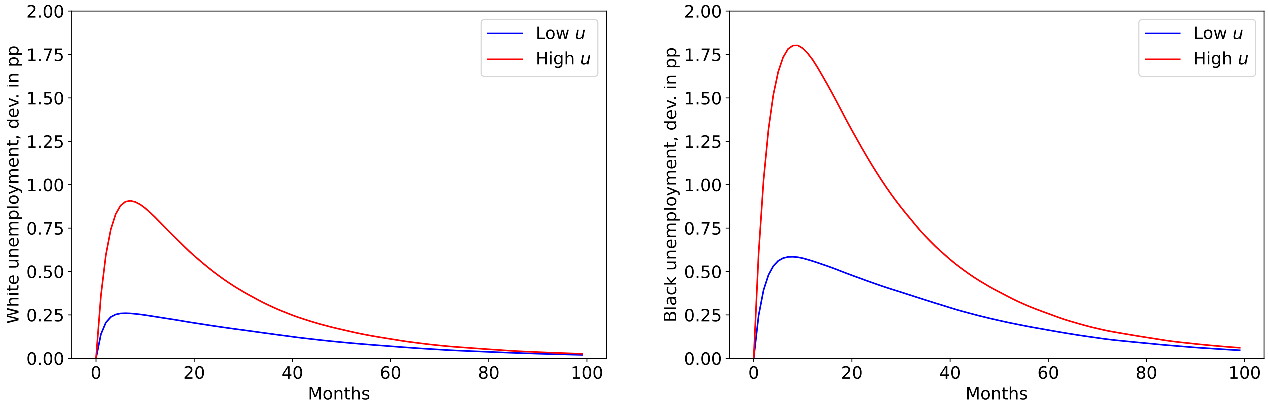

Figure 3: Reaction of u^A and u^B to a negative productivity shock under high and low aggregate unemployment.

Notes: Conditional GIRFs for u^A and u^B following a 1 standard deviation negative productivity shock. The conditional GIRFs are computed by averaging over 1,000 GIRFs conditional on aggregate unemployment above or below its unconditional expectation. Each conditional GIRF is computed by averaging over 10,000 simulations of 100 months each starting from the same initial state. The 1,000 initial states are drawn randomly from the ergodic distribution of the calibrated model.

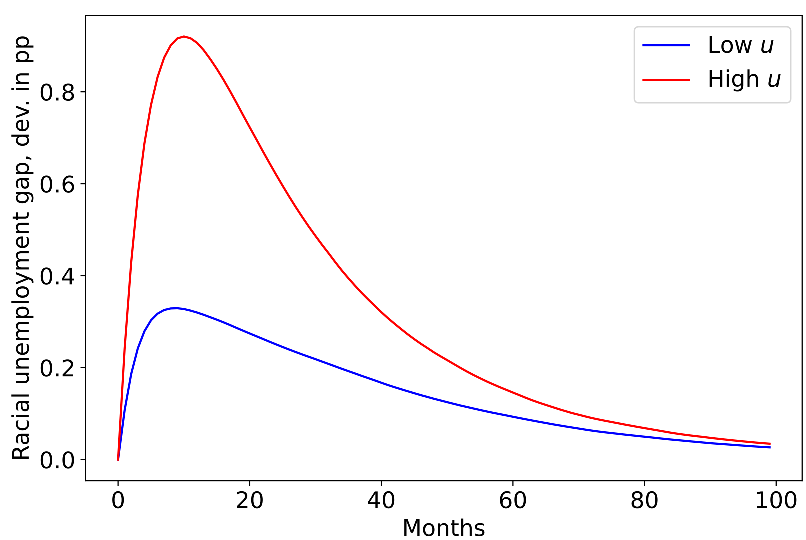

Figure 4: Reaction of the racial unemployment gap to a negative productivity shock under high and low aggregate unemployment.

Notes: Conditional GIRFs for $u^B - u^A$ following a 1 standard deviation negative productivity shock. The GIRFs are computed by averaging over 1,000 GIRFs conditional on unemployment being above or below its unconditional mean. Each conditional GIRF is computed by averaging over 10,000 simulations of 100 months each starting from the same initial state. The 1,000 initial states are drawn randomly from the ergodic distribution of the calibrated model.

is highlighted by the recent literature focusing on nonlinearities in the standard labor search model (e.g. Ljungqvist and Sargent, 2017; Petrosky-Nadeau *et al.*, 2018; Petrosky-Nadeau and Zhang,

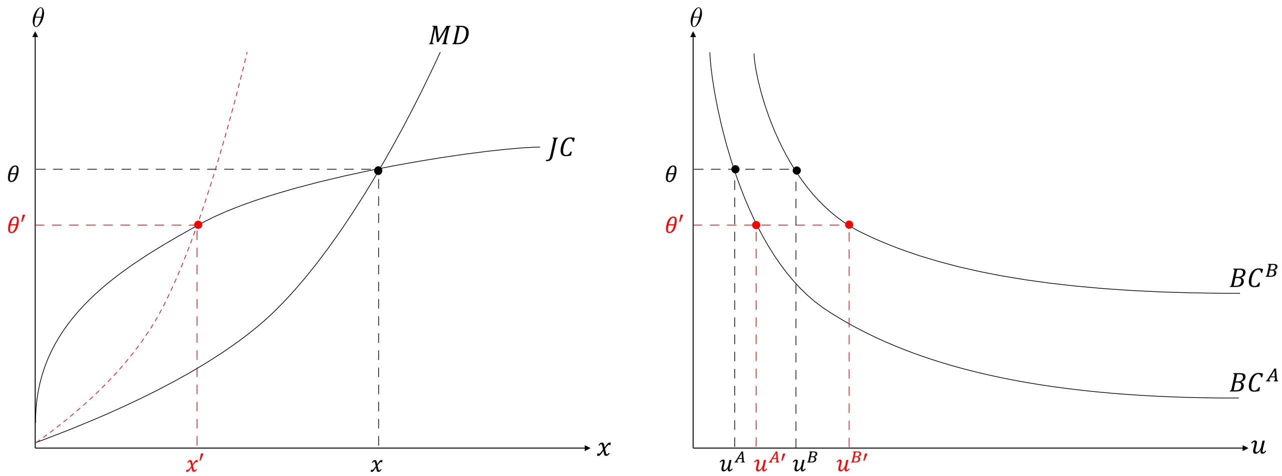

Figure 5: Effect of an increase in ι on unemployment.

2020; Bernstein *et al.*, 2021). Figure 3 shows that this result translates to a setting where two separate groups face the same labor-market tightness.

Comparing the two panels of Figure 3 reveals that the state-dependence of the reaction of unemployment to shocks is stronger for Blacks compared to whites. This can be seen by comparing the difference between the blue and red lines for Blacks and whites. In absolute value, the difference in the increase in unemployment between the two states is much larger for Blacks compared to whites. This is in line with our result above that the semi-elasticity of the unemployment gap is decreasing in θ (i.e., increasing in unemployment).

In conclusion, not only Blacks' unemployment increases more relative to whites' unemployment following a negative productivity shock but it's reaction to further shocks becomes stronger as unemployment increases.

5.3 Inflation tax and the racial unemployment gap

Figure 5 highlights the mechanism through which changes in the inflation and nominal interest rates affect unemployment of Blacks and whites in our model. The left panel depicts money demand (MD curve) and job creation (JC curve) in the (x, θ)-space. The JC curve depicts labor-market tightness as a function of DM consumption (and real balances). Higher DM trade means more profits for firms and hence higher entry. The MD curve maps labor-market tightness into DM consumption. Higher entry of firms means more trade opportunities in the DM which leads to a higher demand for money. The intersection of the two curves determines the equilibrium θ and x. Through the Beveridge curves for groups A and B, θ determines their respective unemployment rates as depicted in the right panel of Figure 5.

An increase in inflation leads to an increase in the cost of holding money through the Fisher equation $\iota_t = \frac{1+\pi_t}{\beta} - 1$—the nominal rate moves one-for-one with inflation. This is depicted in Figure 5 as a change in the MD curve which implies a lower DM consumption for any given θ. The resulting new equilibrium θ' is lower, leading to an increase in unemployment for both groups. However, this transmission from θ to u^A and u^B in the steady state depends on the slope of the two Beveridge curves. In particular, the group with the highest semi-elasticity is going to suffer from a higher increase in its unemployment as explained in the introduction. This is clearly seen in the left panel of Figure 5, where moving from θ to θ' leads to a higher increase in u^B compared to u^A. It is also clear from the curvature of the two Beveridge curves that this effect is state-dependent and increasing in aggregate unemployment (decreasing in θ) as discussed in the previous section.

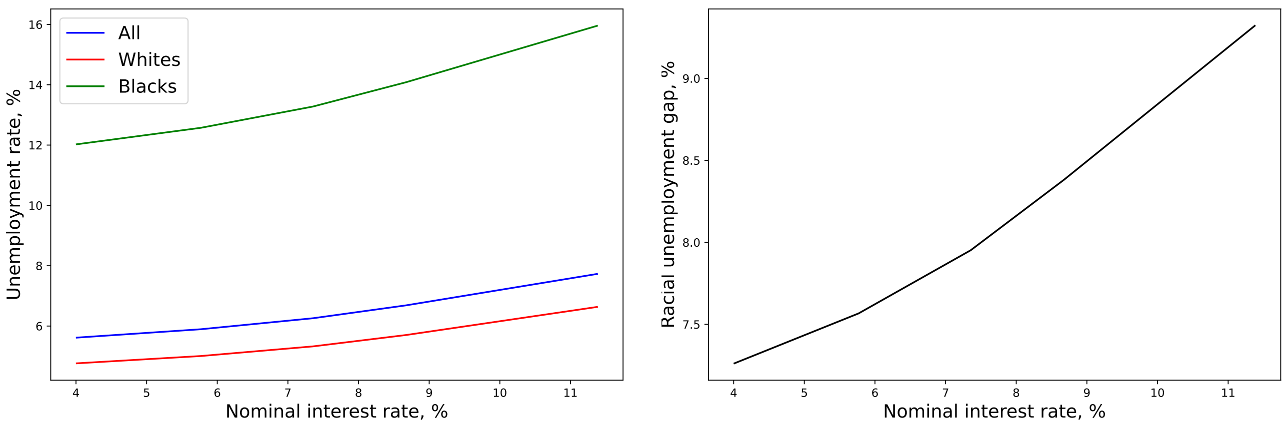

Figure 6: Model simulations: nominal interest rates vs. unemployment.

To quantify the previous discussion, we simulate the effect of an increase in inflation, and thus the trend nominal interest rate, on various measures of unemployment using the calibrated model. In particular, for each level of $\bar{\iota}$, we average over 1,000 simulations each the same length as the data (576 months). Figure 6 and Table 7 present the results of these simulations. As seen in the left panel of Figure 6, both white and Black unemployment are increasing in $\bar{\iota}$. However, the latter increases more which translates into a higher racial unemployment gap as seen in the right panel of Figure 6. Table 7 reports more details. In particular, increasing trend inflation from the Friedman rule (FR) to 10% increases whites' average unemployment rate from 4.48% to 8.22%.[9] It increases Blacks' average unemployment from 11.38% to 18.84%. In absolute terms, the unemployment cost of a 10% inflation rate for Blacks is about 7.46pp, double the cost for whites of 3.73pp. However, in relative terms, whites' average unemployment rate increases by 83.48% relative to its level at the FR whereas Blacks' average unemployment rate increases by 65.55%.

[9]Our definition of the Friedman rule here, i.e., $\bar{\iota} = 0$, is slightly different from the usual definition in the New Monetarist literature, i.e., $\iota_t = 0$, since we allow for temporary deviations through shocks to $\hat{\iota}_t$.

Table 7: Unemployment cost of trend inflation for Blacks and whites

Inflation rate	**Interest rate**	**Average unemployment**		**Difference with FR**	
		Whites	**Blacks**	**Whites**	**Blacks**
-3.26%	0.00% (FR)	4.48%	11.38%	-	-
0.00%	3.37%	4.67%	11.81%	0.19pp	0.43pp
2.50%	5.95%	5.04%	12.64%	0.55pp	1.26pp
5.00%	8.54%	5.64%	13.95%	1.15pp	2.56pp
10.00%	13.70%	8.22%	18.84%	3.73pp	7.46pp

Notes: Average quarterly unemployment based on simulated data. The statistics are computed for each level of $\bar{\iota}$ by averaging over 1,000 simulations each of the same length as the data (576 months).

Table 8: Unemployment volatility as a function of trend inflation for Blacks and whites

Inflation rate	**Interest rate**	**Average unemployment volatility**	
		Whites	**Blacks**
-3.26%	0.00% (FR)	0.36%	0.79%
0.00%	3.37%	0.42%	0.92%
2.50%	5.95%	0.58%	1.21%
5.00%	8.54%	0.91%	1.78%
10.00%	13.70%	2.72%	4.33%

Notes: Unemployment volatility is measured as the standard deviation of the HP-filtered cyclical component of unemployment over a quarterly frequency. The statistics are computed for each level of $\bar{\iota}$ by averaging over 1,000 simulations each of the same length as the data (576 months).

Table 8 illustrates the volatility inducing effect of a higher trend inflation. Increasing trend inflation from the Friedman rule to 10% increases white unemployment volatility from 0.36% to 2.72%, while it increases Black unemployment volatility from 0.79% to 4.33%.

What do we learn from this in terms of policy? The Federal Reserve's current inflation target is 2%. Table 8 tells us that a substantial increase in trend inflation above target from 2.5% to 5% would increase unemployment volatility for Blacks by almost twice as much as for whites, i.e., from 1.21% to 1.78% for Blacks compared to a move from 0.58% to 0.91% for whites. It would also increase the average unemployment gap between Blacks and whites by about 0.71pp.

Generalized impulse response functions. Next, we turn to the role of trend inflation in shaping the reaction of the racial unemployment gap to exogenous shocks hitting the economy. Figure 7 depicts conditional GIRFs for three different levels of trend inflation. A higher trend inflation rate, through a higher opportunity cost of holding money, reduces the fundamental surplus fraction which increases the reaction of unemployment to shocks as discussed in Ait Lahcen *et al.* (2022). This effect is strongest for Blacks' unemployment rate where the reaction to the shock under high trend inflation is on average 0.72pp higher than the reaction under low trend inflation. For whites, this difference amounts to only about 0.36pp. As a consequence, the reaction of the racial unemployment gap to a negative productivity shock is increasing in trend inflation as seen in Figure 8.

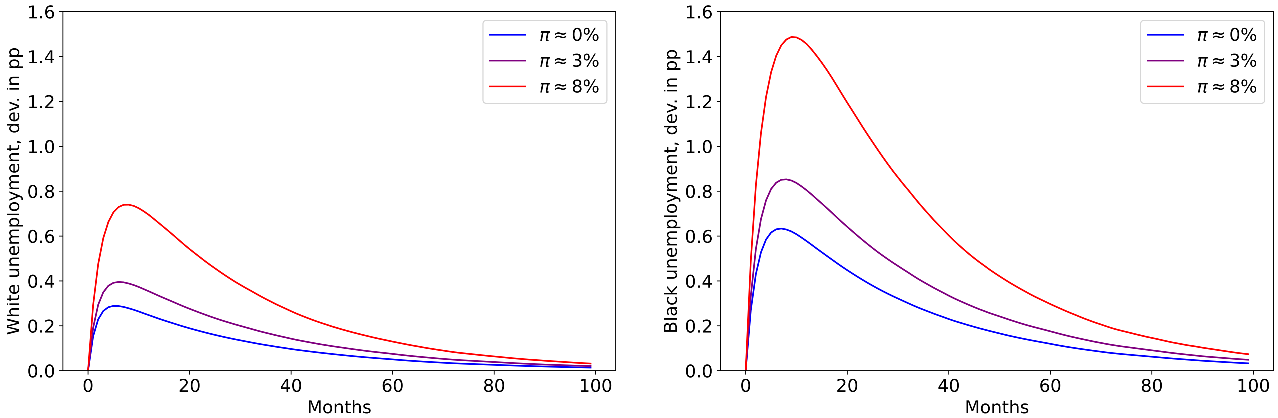

Figure 7: Reaction of u^A and u^B to a negative productivity shock for various trend inflation levels.
Notes: Average conditional GIRFs for u^A and u^B following a 1 standard deviation negative productivity shock. The conditional GIRFs are computed by averaging over 1,000 GIRFs conditional on three levels of trend inflation. Each conditional GIRF is computed by averaging over 10,000 simulations of 100 months each starting from the same initial state. The 1,000 initial states are drawn randomly from the ergodic distribution of the calibrated model.

Welfare cost of inflation. Another way of measuring the differential effect of the inflation tax is to compare its effects on the welfare of the two groups. Given the path of exogenous variables $\{\bar{\iota}_t, \hat{\iota}_t, y_t\}_{t=0}^{\infty}$, we define average welfare for Blacks as

$$\mathcal{W}^B(\{\bar{\iota}_t, \hat{\iota}_t, y_t\}_{t=0}^{\infty}) = \mathbb{E}\sum_{t=0}^{\infty} \beta^t(\lambda\alpha(n_t)[\mathrm{u}(x_t) - \mathrm{g}(x_t)] + (\lambda - \mu_t^B)w_t^B + \mu_t^B l)/\lambda \quad (49)$$

and for whites as

$$\mathcal{W}^A(\{\bar{\iota}_t, \hat{\iota}_t, y_t\}_{t=0}^{\infty}) = \mathbb{E}\sum_{t=0}^{\infty} \beta^t((1-\lambda)\alpha(n_t)[\mathrm{u}(x_t) - \mathrm{g}(x_t)] + (1 - \lambda - \mu_t^A)w_t^A + \mu_t^A l)/(1-\lambda) \quad (50)$$

where l is the value of leisure that corresponds to the flow value of unemployment b, from which we subtract unemployment benefits, measured using a 0.4 wage replacement rate as in Mitman and Rabinovich (2015). Since productivity is endogenous in our model, we set unemployment benefits to 0.4 of the mean of the ergodic distribution of the economy-wide average wage. This leaves us with $l = 0.554$.

We compute welfare for different levels of trend inflation by simulating the model economy for each level subject to productivity and cyclical interest-rate shocks. This approach takes into account the nonlinearities of the model and the way they interact with aggregate uncertainty

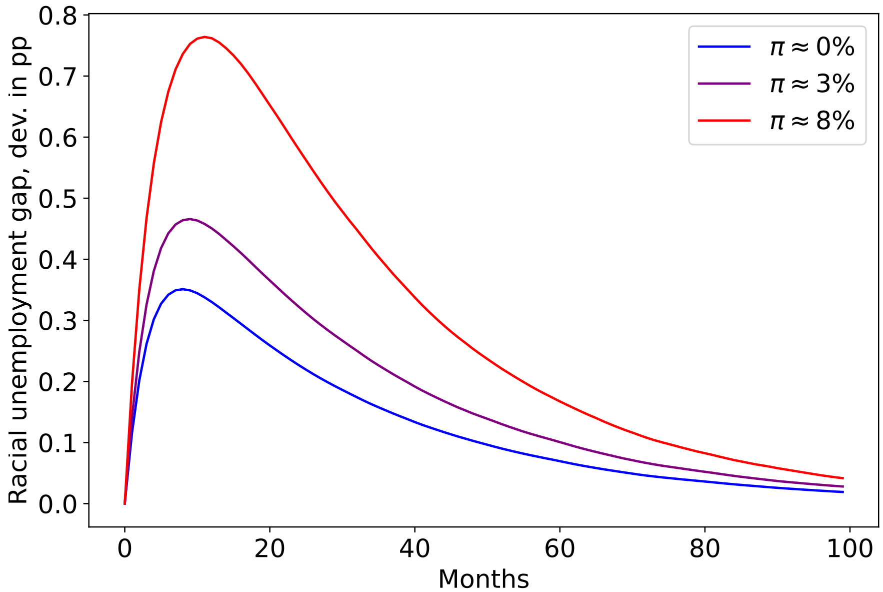

Figure 8: Reaction of the racial unemployment gap to a negative productivity shock for various trend inflation levels.

Notes: Average conditional GIRFs for $u^B - u^A$ following a 1 standard deviation negative productivity shock. The GIRFs are computed by averaging over 1,000 GIRFs conditional on three levels of trend inflation. Each conditional GIRF is computed by averaging over 10,000 simulations of 100 months each starting from the same initial state. The 1,000 initial states are drawn randomly from the ergodic distribution of the calibrated model.

Table 9: Welfare cost of inflation for Blacks and whites for different levels of $\bar{\imath}$

Trend inflation	**Trend interest rate** $\bar{\imath}$	**Average welfare cost,** $(1-\Delta(\bar{\imath}))\%$	
		Whites	**Blacks**
-3.26%	0.00%	-	-
0.00%	3.37%	0.66%	0.68%
2.50%	5.95%	1.74%	1.81%
5.00%	8.53%	3.05%	3.26%
10.00%	13.70%	6.25%	7.13%

Notes: Data are based on monthly model simulations. For each $\bar{\imath}$, we run 1,000 simulations each 1,000 months long subject to productivity and cyclical interest-rate shocks. We then burn the first 424 observations to keep each simulation at the same length as our data. $\Delta(\bar{\imath})$ is the share of total consumption at the Friedman rule that equalizes average welfare levels at $\bar{\imath}$ and the Friedman rule.

when measuring welfare. In particular, we compute welfare under the path $\{\bar{\imath}_t = 0, \hat{\imath}_t, y_t\}_{t=0}^{\infty}$, the definition we used above for the Friedman rule. Denote with a star the resulting allocation. We then define the welfare cost of inflation as the share of total consumption agents are willing to give up to live in a zero trend nominal interest-rate environment instead of $\bar{\imath} > 0$. Let $(1 - \Delta)$ be this share where Δ solves

$$\mathcal{W}^B(\{\bar{\iota}_t, \hat{\iota}_t, y_t\}_{t=0}^{\infty}) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t(\lambda\alpha(n_t^*)[\mathrm{u}(\Delta x_t^*) - \mathrm{g}(\Delta x_t^*)] + \Delta((\lambda - \mu_t^{B*})w_t^{B*} + \mu_t^{B*}l)/\lambda \tag{51}$$

for Blacks and

$$\mathcal{W}^A(\{\bar{\iota}_t, \hat{\iota}_t, y_t\}_{t=0}^{\infty}) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t((1-\lambda)\alpha(n_t^*)[\mathrm{u}(\Delta x_t^*) - \mathrm{g}(\Delta x_t^*)] + \Delta((1-\lambda - \mu_t^{A*})w_t^{A*} + \mu_t^{A*}l)/(1-\lambda) \tag{52}$$

for whites. Table 9 reports the average welfare cost measured in total consumption for each group at different levels of $\bar{\iota}$ relative to the Friedman rule. Increasing inflation from the Friedman rule to 10% lowers welfare for whites by 6.25% and for Blacks by 7.13%. The almost 1pp difference is tightly linked to the disparate effect of trend inflation on unemployment discussed above. Measured this way, this difference in welfare cost takes into account the differential effect on both the level of unemployment as well as its volatility.

6 Conclusion

The purpose of this paper is to expand on a basic insight: that a group with a higher steady-state level of unemployment will have a higher variability in unemployment as a mechanical result of the standard labor flows model. We explore this basic insight in an attempt to understand the dynamics of the racial unemployment gap. We present a calibrated model which nests the standard DMP model, but also allows us to discuss the impact of long-run interest rates and inflation. Theoretically, we decompose the movements in unemployment into different sources, elucidating the mechanisms underlying labor market movements. Quantitatively, we reasonably match a range of non-targeted moments, and provide a number of simulation results.

In response to productivity shocks, we show that the model predicts a much stronger rise in unemployment for Black workers following a negative productivity shock, so that the racial unemployment gap is strongly counter-cyclical. Moreover, this response is greater when unemployment is already high, perhaps rationalizing previous findings regarding the role of a "high pressure" economy in mitigating the racial unemployment gap. However, our policy experiments show that higher trend inflation has a more negative impact on Black workers than white workers, indicating that a higher inflation target might be self-defeating in ameliorating the racial unemployment gap.

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Appendix A Computations and calibration

Numerical solution. We solve the model numerically using a fitted value function iteration algorithm.[10] The space of the endogenous state variables u_{t-1} and γ_{t-1} is discretized using grids of 10 equidistant points. We bilinearly interpolate the expectation part of the two Bellman operators between the grid points of the endogenous state variables in order to improve accuracy. We approximate the continuous state stochastic processes for y_t and $\hat{\iota}_t$ with a 30-state Markov chain using the Rouwenhorst (1995) procedure.[11] For the productivity shock process in particular, Petrosky-Nadeau and Zhang (2017) show that this procedure provides a better approximation when solving the DMP model nonlinearly. We model the trend component $\bar{\iota}_t$ as a very persistent Markov chain with 5 states. To compute the state values, we divide the distribution of $\bar{\iota}_t$ into five bins separated by the distribution's quintiles and take the average value of each bin. This yields the values: $\{4.01\%, 5.77\%, 7.36\%, 8.67\%, 11.38\%\}$. The following transition probabilities are estimated by maximum likelihood as in Chatterjee and Corbae (2007):

$$\begin{pmatrix} 0.991 & 0.009 & 0 & 0 & 0 \\ 0.009 & 0.982 & 0.009 & 0 & 0 \\ 0 & 0.009 & 0.982 & 0.009 & 0 \\ 0 & 0 & 0.009 & 0.982 & 0.009 \\ 0 & 0 & 0 & 0.009 & 0.991 \end{pmatrix}.$$

Calibration procedure. The set of model parameters $\{\kappa, b, \chi, \xi, \rho_y, \sigma_y, A, a, \zeta, \phi\}$ is jointly calibrated following a Simulated Method of Moments (SMM) procedure.[12] Define Θ as the vector containing the above set of parameters, μ as the vector of the targeted empirical moments and $\mu_s(\Theta)$ as the vector of their model-based counterparts. The simulated moments $\mu_s(\Theta)$ are obtained by averaging over $S = 1,000$ model simulations each of length $T = 1,000$ using a random draw s of productivity and interest-rate shocks.[13] The burn-in period is set to 424 months such that the length of the remaining simulated series matches the length of the empirical data series (576 months). Finally, we solve for the vector of parameters $\hat{\Theta}$ that minimizes the distance $G(\Theta) = \mu - \frac{1}{S}\sum_{s=1}^{S} \mu_s(\Theta)$ such that

$$\hat{\Theta} = \arg\min_{\Theta} G(\Theta)^T W^{-1} G(\Theta) \tag{53}$$

[10] Our Python code makes extensive use of the Numba library for just-in-time compilation and parallelization (Lam *et al.*, 2015).

[11] We use the Rouwenhorst routine from the QuantEcon Python library (Sargent and Stachurski, 2014).

[12] See Ruge-Murcia (2012) and references therein.

[13] To match empirical moments based on quarterly data, we aggregate our monthly simulations quarterly and compute the corresponding model-based moments.

where W is a semi-definite weighting matrix.[14] We set W to an identity matrix since our model is exactly identified.

[14] We use the percent difference to avoid unintended weighting.