University of Zurich

Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 462

Providing Innovation Incentives for the Green Transition

Armin Schmutzler

December 2024

Providing Innovation Incentives for the Green Transition

Armin Schmutzler*

December 2024

Abstract

By affecting prices and thereby market shares of green and brown firms, *product innovations* and *process innovations* influence industry emissions even when they do not directly affect the emission intensity of the innovating firm. Using a differentiated two-stage duopoly, this paper therefore analyzes the effects of environmental policy on such innovations, and it asks how these effects differ from each other and from those of *environmental innovations* that directly reduce the emission intensity. The paper investigates the determinants of R&D investments, showing in particular that incentives for certain types of potentially beneficial innovations may be negative. Moreover, it analyzes how suitable policies can foster green innovation.

JEL Codes: Q55, L13
Keywords: Innovation, Environmental Policy, Imperfect Competition

*Department of Economics, University of Zurich and CEPR. Email: armin.schmutzler@econ.uzh.ch.
I am grateful to Lucas Bretschger, Ken Gillingham, Matti Liski, Wolfram Schlenker, Geert van Moer and Philipp Wichardt and to seminar audiences at Collegium Helveticum, ETH Zurich, Helsinki, Leipzig (AuROe), Mannheim (MCEE 2023), Rostock, Rome (EARIE 2023), University of Zurich, and the PSE-ETH Workshop on the Economics of the Transition to a Sustainable Society for helpful discussions.

1 Introduction

Successfully confronting ecological challenges such as climate change will require the replacement of brown, heavily polluting products with green, less polluting substitutes. Major innovations will be necessary to induce such a transition. This paper therefore analyzes policies for supporting green innovations. The automobile industry is useful to illustrate the issues. As the transportation sector is responsible for around a quarter of global CO2 emissions, tremendous efforts have been made to replace traditional vehicles with internal combustion engines (ICEVs) with electric vehicles (EVs). Automobile producers have recently spent almost £70 billion per year on R&D related to electric vehicles.[1] Many policy instruments have been used to support these developments. R&D subsidies directly target innovation behavior. Public investments into complementary infrastructure increase the value of the green product to consumers, and adoption subsidies reduce the purchasing price. Although the latter two instruments do not directly target R&D, they influence the market equilibrium and thus the profits that firms can obtain from innovation; thus, they are likely to affect innovation outcomes. Motivated by such examples, this paper asks: How suitable are different instruments for inducing innovations that foster the transition from brown to green products?

The literature on innovation in environmental economics focuses on reductions in specific emissions or marginal abatement costs.[2] The example of the automobile industry strongly suggests that this focus may be too narrow, as any innovation that increases the market share of green products can help to reduce total emissions. For example, a green firm can engage in process innovations that cut production costs, without any direct effect on emission intensity. Moreover, it can invest in the quality of such products, making them more valuable for consumers.[3] One may conjecture that such activities are likely to increase its market share, resulting in lower total emissions, as the green product is less polluting than the brown product.[4]

Therefore, this paper not only deals with environmental innovations that directly target specific emissions, but also with pure process and product innovations. The analysis has several crucial ingredients. First, both brown and green firms can carry out R&D investments. Second, competition in the product market is imperfect, which is an important element in cases such as the automobile industry. Third, consumers have heterogeneous tastes for the products of green and brown firms, which may reflect explicit environmental preferences as well as preferences concerning other product characteristics.

The paper first asks under which circumstances innovations reduce aggregate emissions, showing that the answer is more involved than one might expect and that it depends on the type of the innovation in a subtle way: Some innovations that might appear to reduce emissions turn out not to do so. Moreover, I find that firms may be reluctant to engage in certain types of environmentally beneficial investment. Because they intensify competition, firms would abstain from such investments even if they were costless. Finally, I identify circumstances that foster innovations, which is useful for analyzing the effects of various environmental policies and their desirability from a welfare perspective.

[1]https://www.bdo.co.uk/en-gb/news/2021/top-20-global-carmakers-spend-another-71-7bn-on-r-and-d-as-electric-vehicle-rollout-gathers-pace

[2]See, for instance, the literature cited in footnotes 13 and 14 below.

[3]For instance, automobile producers can invest to improve acceleration or increase the range of EVs.

[4]See Forsythe, Gillingham, Michalek, and Whitefoot (2023) for related evidence on adoption behavior.

In more detail, I analyze a two-stage duopoly, with R&D investments preceding price competition.[5] One of the products (the *green product*) has better environmental properties than the other (the brown product), with lower specific emissions in production and/or consumption. In addition, products may differ in other aspects. In the baseline model treated in Sections 2-4, consumers choose whether to buy a unit of the brown product or a unit of the green product.[6] I assume that consumers can be ordered on a continuum according to their willingness to pay (WTP) for the green product from very low (*conventional consumers*) to very high (*green consumers*). Despite this terminology, green consumers do not necessarily value the green product highly *because* of its environmental features—the preferences may reflect other features as well.

The benchmark duopoly model, adapted from Schmutzler (2024), allows for *vertical differentiation*, where even the most conventional consumers have a higher WTP for the green product, as well as for *horizontal differentiation*, where the most conventional consumers prefer the brown product to the green product. This nests the textbook models of Hotelling (1929) and Shaked and Sutton (1982) as special cases. However, the framework is more general.[7]

In the first (investment) stage of the game, the firms can engage in various types of innovation activities. *Process innovations* reduce production costs. *Product innovations* increase the WTP. *Environmental innovations* lead to lower specific emissions. In this framework, I address the following questions:

1. What are the environmental effects of innovations, and how do they depend on the type of innovation (process, product or environmental) and the identity of the innovator (brown or green firm)? Through which channels do these effects arise?

2. What determines firms' incentives to carry out process innovations, product innovations and environmental innovations, respectively?

3. How should policy instruments be chosen to induce desirable R&D investments? Is direct R&D support necessary or do market instruments such as emissions taxes suffice? Which instruments induce "green" investments without compromising too much on other goals?

Environmental Effects: Focusing first on innovations that do not affect specific emissions, a natural conjecture is that, by improving its relative position, innovations of the green firm increase its market share and thus reduce total emissions.[8] Indeed this is exactly what happens for process innovations and for "non-targeted" product innovations that increase the WTP for the green product for all consumers by the same amount.

[5]The paper thus belongs to the literature on environmental policy with imperfect competition that developed in response to the early papers of Buchanan and Stubblebine (1962) and Buchanan (1969). Examples include Barnett (1980) for monopoly, Katsoulacos and Xepapadeas (1995) for Cournot oligopoly and Lange and Requate (1999) for differentiated price competition. Requate (2006) surveys the literature.

[6]The benchmark model assumes that products are sufficiently valuable that consumers will never choose the outside option.

[7]Contrary to the standard Hotelling model, even under horizontal differentiation, it allows for the plausible case that the valuations for green and brown products are positively correlated. Conversely, contrary to the Shaked-Sutton textbook model, even under vertical differentiation, the framework caters for the case that valuations for the green and brown product are negatively correlated.

[8]The relation between WTP and market shares discussed in this paragraph is as in Schmutzler (2024) but there I neither discuss emissions nor pro-environmental preferences and R&D investments.

The same logic applies to product innovations of the green firm that are targeted toward conventional consumers, meaning that they increase their WTP more than for green consumers. Surprisingly, however, even in the benchmark model, where rebound effects from increasing total output are ruled out by assumption, a product innovation of the green firm that targets green consumers may increase total emissions.[9] Intuitively, the firm takes advantage of the green consumers after the increase in WTP by increasing its price, which reduces its market share despite the direct positive effect of higher WTP. With horizontal differentiation, the price effect is not strong enough to outweigh the immediate effect, so that the green firm increases its market share, and total emissions decline. With vertical differentiation, however, the price effect dominates so that, paradoxically, green product innovations increase the market share of the brown firm and thereby aggregate emissions.

When consumers have intrinsic pro-environmental preferences, additional effects arise. First, under the plausible assumption that the reduction in green emissions triggers a higher increase in the WTP for green consumers than for conventional consumers, the green firm may increase prices after an environmental innovation (that reduces specific emissions). As a result, an environmental innovation of the green firm can reduce its market share and thus increase aggregate emissions under vertical differentiation despite the direct emission-reducing effect of the innovation. Second, environmental innovations of the brown firm are a double-edged sword for related but different reasons. Whereas such innovations directly reduce aggregate emissions, they increase the appeal of the brown product to consumers that would otherwise have bought the green product. This leads to a market share reallocation toward the brown product, which tends to increase aggregate emissions with horizontal differentiation, possibly outweighing the beneficial effect of lower emission intensity. This result is similar to a rebound effect, resulting from the reallocation of market shares rather than changes in total output.

Determinants of Innovation: The incentives of a firm for each type of R&D investment depend on the effect of the investment on the firm's own profit in the resulting price equilibrium. Incentives for process innovations and for non-targeted product innovations are generally positive, as such innovations have positive effects on the margin and the market share of the innovating firm. For targeted product innovations and environmental innovations, it is crucial to consider their effects on the intensity of competition between the two firms. If the green firm targets green consumers by introducing improvements that they value more than conventional consumers, this increases the taste heterogeneity between consumers, thus softening competition, which adds to the positive effect that the product becomes more attractive for consumers. As a result, profits unambiguously increase. In contrast, if the green firm engages in innovations that target conventional consumers, it intensifies competition by reducing consumer heterogeneity. Whereas the direct beneficial effect of higher WTP still dominates for vertical differentiation, this is not true for horizontal differentiation. Therefore, surprisingly, innovation incentives are negative. By similar arguments, no matter whether product differentiation is horizontal or vertical, the brown firm does not want to engage in innovations targeted toward green consumers: The latter type of investment would increase the similarity with the green products and thereby reduce differentiation.

Therefore, not all types of innovations are beneficial for a firm, even ignoring investment costs. Thus, the investment game may have a boundary equilibrium with zero investments of one firm. For process innovations and non-targeted product innovations, however, an

[9]Recall that, in this model, total output is fixed.

interior equilibrium exists in which both firms invest under quite general conditions. In this equilibrium, the green firm invests more than the brown firm if and only if it is stronger in the sense that it would have a higher market share in the absence of innovation. Moreover, if the willingness to pay for the green firm's product uniformly increases, the difference in investments between the green and the brown firm will increase. This change results in a reduction in total emissions.

To separate the direct effect of any change in the economic environment on emissions from the investment effect, it is useful to ask how the investment difference between the green and the brown firm can be maximized among all parameter constellations that would yield the same market share and thus the same total emissions in the absence of investments. The right approach will depend on whether the brown firm or the green firm is stronger, and, in the latter case, on the size of its lead.

Policy Analysis: Environmental policy affects total emissions through the second stage (keeping the investment levels fixed) and through the first stage (taking the effects on investment levels into account). For instance, the second-stage effects of emissions taxes are entirely analogous to those of process innovations of a green firm, as such taxes give it a cost advantage, which it turns into a higher market share, thereby reducing total emissions. The first-stage effect of taxes on the investment of the green firm is also positive, reinforcing the direct emission-reducing effect of the tax.[10] The second-stage effects of adoption subsidies for the green product are identical to those of non-targeted product innovations and thus similar to those of emissions taxes. By contrast, R&D subsidies for the green firm exclusively operate through the first stage: By reducing its investment costs, they increase the investment of this firm, thereby inducing lower investments of the brown firm. Reflecting the above discussions, subsidies for process innovations and (non-targeted) product innovations of the green firms thus reduce overall emissions, whereas the effects are more subtle for targeted product innovations.[11]

Turning to welfare statements, under imperfect competition, it is generally difficult to achieve the first-best with a single instrument. In the current context, the problem is exacerbated because of potential distortions in investments and prices. To illustrate the implications of a firm's pricing decisions on total welfare, note that the firm ignores three types of effects. A price increase (a) is beneficial for the competitor and (b) bad for consumers. Given imperfect competition, the latter effect usually dominates the former. If the green firm increases prices, this (c) is also bad for the environment as it shifts outputs to the brown firm. All told, without policy interventions prices of the green firm are therefore too high.[12]

Innovations can have direct welfare effects (that do not reflect price changes) and indirect, price-induced effects. If second-period prices have been taken care of by adequate instruments, only the direct effects matter. Then there is no reason to support process innovations, as they do not *directly* affect the competitor, consumers and the environment. However, perfect internalization of externalities with price instruments is unlikely to be feasible. As argued above, equilibrium prices are biased upwards for the green firm. Process

[10]Recall, however, that, in a more general setting, product innovations that are targeted to green consumers might increase aggregate emissions (see Section 5).

[11]For instance, R&D subsidies for innovations targeted toward green consumers may reduce the market share of the green firm under vertical integration and thus increase aggregate emissions.

[12]For the brown firm, the case is less clear: While lower prices would reduce the dead-weight loss from imperfect competition, the resulting output reallocation would typically also lead to higher emissions.

innovations lower the green firm's price more strongly than the brown firm's and thus work toward a preferable allocation. Environmental innovations can be worth supporting even if second-period prices are unbiased: They directly reduce environmental damages, a beneficial effect that the investing firms do not take into account even when pricing externalities have been internalized with adequate instruments.

Generalization: The benchmark model is sufficiently flexible to comprise several interesting cases of duopolistic interaction. However, it faces the obvious limitation that the demand structure is quite specific, resulting in the feature that total equilibrium output is independent of environmental policy and investment activities. Section 5 therefore treats a more general model to deal with these shortcomings. Several of the qualitative features of the benchmark model carry over to this case, and simple conditions address the occurrence of rebound effects. Contrary to the benchmark model, these effects can now not only result from market share reallocations but also from increases in total output. The downside of the more general model is that, though understanding the determinants of innovation incentives is still possible, explicitly calculating the innovation equilibrium is not.

Relation to the Literature: I am not aware of any paper analyzing whether and how environmental policy can encourage the transition from brown to green products in differentiated oligopoly markets by influencing different types of R&D investment. This paper fills the gap by providing conditions under which policy supports green innovations and an output reallocation away from the brown firm and thereby fosters lower aggregate emissions. It also points to possible caveats: Some ostensibly green policies may lead to higher emissions.

The paper contributes to a long-standing debate concerning environmental policy and innovation. At a general level, theory suggests that market-based policy instruments such as emissions taxes and tradeable permits can help to provide incentives for environmental innovations.[13] There is some evidence supporting this view.[14] To repeat, the extant literature focuses mainly on innovations that reduce specific emissions or abatement costs. By contrast, the current paper deals with process innovations and non-environmental product innovations as well, which is crucial because of their prevalence and their effects on the market share of green products.

Another distinguishing aspect of this paper is its focus on consumer heterogeneity. This appears critical to understand developments such as the transition from ICEVs to EVs. Consumers may differ in the extent of their pro-environmental preferences as well as their hedonic valuation for other aspects of the products. While previous work has investigated environmental policy when some consumers have pro-environmental preferences,[15] the analysis of the effects of policy on innovation and emissions in such a setting has not

[13] Early theoretical contributions include Downing and White (1986), Malueg (1989), Milliman and Prince (1989), Biglaiser and Horowitz (1994), Parry (1995); they were developed further by Kennedy and Laplante (1999) and Requate and Unold (2003)

[14] Studies like Newell, Jaffe, and Stavins (2010), Hassler, Krusell, and Olovsson (2021), Acemoglu, Aghion, Bursztyn, and Hemous (2012), Noailly and Smeets (2015), and Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen (2016) find price effects of innovation, thus providing indirect evidence that market-based instruments could affect innovation (via their effect on prices. Johnstone, Hascic, and Popp (2008), Rogge, Schneider, and Hoffmann (2011) and Calel and Dechezleprêtre (2016) deal with the innovation effects of specific policies. Extensive surveys are Popp (2010), Popp (2019) and Popp, Pless, Haščič, Johnstone et al. (2020).

[15] See, for instance, Arora and Gangopadhyay (1995), Cremer and Thisse (1999), Moraga-Gonzalez and Padron-Fumero (2002), Bansal and Gangopadhyay (2003) Lombardini-Riipinen (2005), Bansal (2008).

received much attention.[16]

One aspect of the welfare analysis in this paper is that price instruments typically do not suffice to obtain the first-best allocation. It shares this feature with previous work on environmental innovation. However, these papers usually rely on the existence of knowledge spillovers.[17] My paper shows that, even in the absence of spillovers, providing adequate innovation incentives under imperfect competition is a complex problem, and the right approach differs for process, product and environmental innovations.[18]

Focusing on the automobile industry, Holland, Mansur, and Yates (2021) model the transition from conventional cars to electric vehicles. Contrary to my paper, they do not ask how environmental policy affects innovation and thereby emissions, and imperfect competition does not play a role. Forsythe et al. (2023) use stated preference approaches to identify substantial willingness to pay for improvements that fall into the category of product innovations (e.g., extended range and improvements in acceleration) and they also find evidence that cost reductions would foster the adoption of EVs.

Finally, I need to clarify the relation to the companion paper (Schmutzler (2024)) which introduces the duopoly model that the second-stage of the model relies on and derives equilibrium prices, outputs and profits. It also addresses some of the comparative statics that are discussed in the current paper in more detail. It puts more emphasis on the technical assumptions that are required for the equilibrium to hold and on the relation between the model and existing textbook models in industrial organization. By contrast, it does not deal with the relation between policy, innovation and the environment, which is the focus of the current paper, and it does not consider welfare implications.

The paper is organized as follows. Section 2 describes the assumptions of the model and interprets the set-up. Section 3 analyzes the second-stage (price) equilibrium and the resulting economic and environmental outcomes, taking investments and policy decisions as given. Section 4 turns to innovation behavior and discusses the policy effects on innovation and emissions. Section 5 shows how the analysis carries over to more general demand functions. Section 6 concludes.

[16] Sengupta (2012) and Langinier and Chaudhuri (2020) consider emission-reducing innovations with green consumers. Both papers focus on monopolists. Schmitt (2024) deals with a duopoly, but focuses exclusively on environmental innovations.

[17] Because of the public goods character of knowledge production, it is generally not possible to provide adequate innovation incentives by relying *exclusively* on emissions taxes, so that direct innovation policies may be called for. See, for instance, Carraro and Siniscalco (1994), Carraro and Soubeyran (1996), Katsoulacos and Xepapadeas (1996).

[18] More broadly, the paper is related to the IO literature on innovation. In line with the results of this paper, authors like Bagwell and Staiger (1994) and Leahy and Neary (1997) have argued that, absent spillovers, process innovations of different firms tend to be strategic substitutes, while different types of investment of one firm are complements in Athey and Schmutzler (1995). Related to all these papers, Schmutzler (2013) provides a detailed discussion of complementarities between demand-enhancing and mark-up increasing measures in the context of increasing competition. However, none of these contributions deals with effects on pollution or with environmental policy.

2 The Model

Consider a duopoly where firms choose R&D investments in Stage 1, before engaging in differentiated price competition in Stage 2. Section 2.1 describes the assumptions for the pricing stage. Section 2.2 introduces the investment stage. Finally, Section 2.3 discusses the framework. Firms differ in the intensity with which they pollute the environment and potentially also in other product characteristics. Consumers have heterogeneous preferences, resulting in different valuations for the green (less polluting) and the brown (more polluting) product. For a large part of the analysis, it does not matter whether those consumers who value the green product more than others do so because of pro-environmental preferences or for other reasons. The model of Sections 2.1 and 2.2 allows for both possibilities, whereas Section 2.3 focuses on the case that some consumers actually value the green product *because* of its environmental characteristics.

2.1 Second Stage: Pricing Decisions

The second stage model is adapted from the slightly more formal treatment in Schmutzler (2024) to which it adds emissions.[19] Two firms $i \in \{B, G\}$ produce a brown product B and a green product G, respectively, with constant marginal production cost c_i. Denote the output of firm i as x_i and its emissions as e_i. The products differ in specific emissions $\eta_i := \frac{e_i}{x_i}$, with $\eta_B > \eta_G$. Thus, total emissions are

$$E = \eta_B x_B + \eta_G x_G. \tag{1}$$

The firms simultaneously set prices p_i. Consumers decide which of the two products to buy.[20] There is a unit mass of consumers who are uniformly distributed on the interval $[0, 1]$. Consumer $k \in [0, 1]$ values product i at v_k^i. The following notation is important:

$$\begin{aligned} \Delta_C &:= c_G - c_B \\ \Delta_p &:= p_G - p_B \\ \Delta_k &:= v_k^G - v_k^B \\ \Psi &:= \Delta_1 - \Delta_0 \end{aligned}$$

Note that increases of Δ_1 and Δ_0 mean that the valuation of the green product relative to the brown product increases, whereas increases in Ψ stand for greater preference heterogeneity. Facing prices p_i, consumer k buys from firm G if $v_k^G - p_G > v_k^B - p_B$ (or $\Delta_k > \Delta_p$) and from firm B if $v_k^G - p_G < v_k^B - p_B$ (or $\Delta_k < \Delta_p$). This results in demand functions $x_B(p_B, p_G)$ and $x_G(p_G, p_B)$.

The first assumption restricts the possible valuation vectors.

Assumption 1. *All valuation vectors (v_k^B, v_k^G) lie on a straight line.*

Using Assumption 1 and uniformity, the distribution of consumer valuations is determined by the valuations of the extreme consumer types, (v_0^B, v_0^G) and (v_1^B, v_1^G). Moreover,

$$\Delta_k = \Delta_0 + k (\Delta_1 - \Delta_0). \tag{2}$$

The following assumption imposes restrictions on valuations.

[19]Specifically, Assumptions 1 to 3 are stated more formally in the companion paper.

[20]I assume that the valuations for both products are sufficiently high that no consumer wants to choose the outside option; see Schmutzler (2024) for a formal condition.

Assumption 2. *(i)* $v_1^G > v_0^G$*;*

(ii) $\Delta_1 > \Delta_0$ *or, equivalently,* $v_1^G - v_1^B > v_0^G - v_0^B$*;*

(iii) $\Delta_1 = v_1^G - v_1^B > 0$.

Part (i) states that $k = 1$ has higher WTP for the green product than $k = 0$, so that Assumption 1 implies more generally that v_k^G is increasing in k. Part (ii) implies the single-crossing condition that $v_k^G - v_k^B$, the difference in WTP for the green and brown product, is increasing in k. (iii) requires that at least the consumer with the strongest valuation for the green product prefers it to the brown product if prices are equal.

The assumptions made so far leave us with substantial flexibility regarding the nature of competitive interaction.

(i) Horizontal Differentiation ($\Delta_0 < 0$) **(ii) Vertical Differentiation** ($\Delta_0 > 0$)

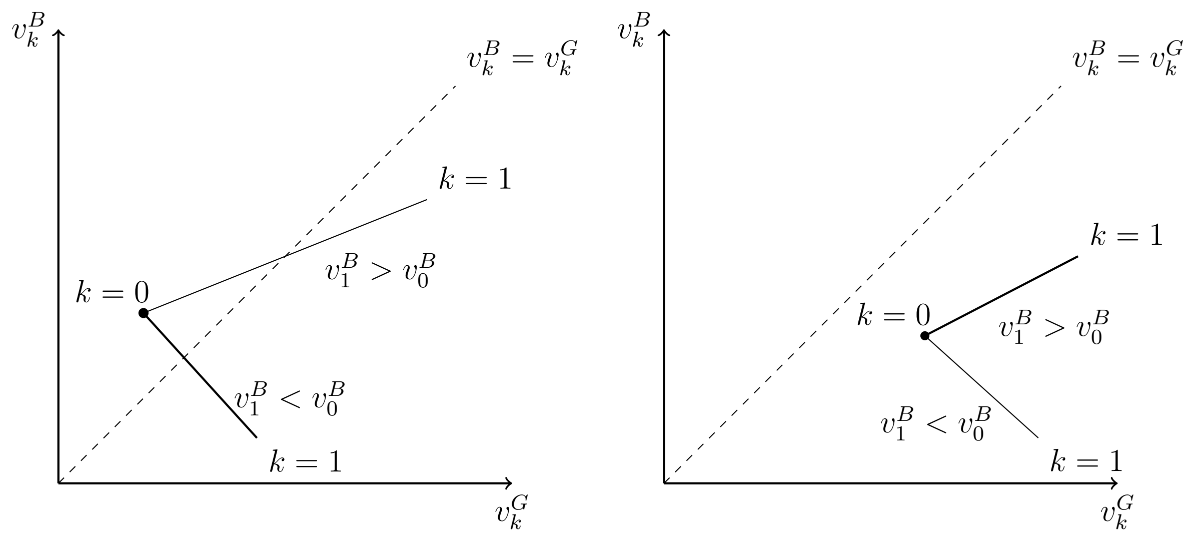

Figure 1: Illustration of Assumption 1.

Note: In panel (i), the downward-sloping line corresponds to the support in a Hotelling model with negative WTP correlation. In panel (ii), the upward-sloping line corresponds to the support in a Shaked-Sutton model with positive WTP correlation.

Figure 1 illustrates the possibilities. With horizontal differentiation ($\Delta_0 < 0$; Figure 1(i)), at least some consumers prefer the brown product to the green product at equal prices, whereas with vertical differentiation all consumers prefer the green product.[21] Horizontal differentiation includes the special case of a standard textbook Hotelling model with linear transportation costs and uniformly distributed consumers, where the support of the valuation distribution is downward sloping in (v_k^G, v_k^B)-space (see Figure 1(i)), with extreme points symmetric to the diagonal.[22] With vertical differentiation ($\Delta_0 > 0$; Figure 1(ii)), all consumers prefer the green product at equal prices. Moreover, the assumptions

[21] Assumption 2(iii) rules out the possibility that all consumers prefer the brown product at equal prices.

[22] Suppose consumers at 0 and 1 have gross valuations v for the products of firms B and G, respectively. With transportation costs $t > 0$, the valuation vectors for the consumers at 0 and 1 are $(v^B, v^G) = (v, v-t)$ and $(v^B, v^G) = (v - t, v)$, respectively. Thus, the support is downward-sloping and symmetric at the diagonal.

of IO textbook models such as e.g. Shaked and Sutton (1982) imply that the support of the consumer distribution is an upward-sloping line, reflecting the notion that consumers who have higher WTP for the green product also have higher WTP for the brown product (v_k^B is increasing in k).[23]

The final, informally stated, assumption guarantees the existence of a full coverage equilibrium where all consumers are served and both firms sell a positive output.[24]

Assumption 3. *(i) Cost and demand asymmetries between the green firm and the brown firm are sufficiently small.*

(ii) The WTP for each product is sufficiently high.

2.2 First Stage: Innovation Decisions

I now introduce the assumptions on the first stage. Starting from some initial constellation of specific emissions ($\underline{\eta}_B$,$\underline{\eta}_G$), marginal costs ($\underline{c}_B$ and $\underline{c}_G$) and WTP parameters ($\underline{v}_0^i$ and $\underline{v}_1^i$ and hence $\underline{\Delta}_0$ and $\underline{\Delta}_1$), each firm can carry out R&D investments, that is, engage in an innovation activity of size $y_i \in [0, y_i^{max}]$ at cost $K(y_i)$, where $K(.)$ an increasing function and $y_i^{max} > 0$. The following different interpretations of the innovation are possible.

Definition 1. ***Innovation Types***

*(i) A **process innovation** of firm i corresponds to a reduction of c_i by y_i.*

*(ii) For firm i, a **non-targeted product innovation** corresponds to a simultaneous increase of v_k^i for all $k \in [0,1]$ by y_i.*[25]

*(iii) For firm i, a **product innovation targeting green consumers** corresponds to an increase of v_1^i by y_i (for fixed v_0^i).*

*(iv) For firm i, a **product innovation targeting conventional consumers** corresponds to an increase of v_0^i by y_i (for fixed v_1^i).*

*(v) An **environmental innovation** of firm i corresponds to a reduction of η_i by y_i.*

While process innovations aim at cost reductions, product innovations increase WTP. For non-targeted product innovations, all consumers benefit in the same way. By contrast, Assumption 1 implies that, for innovations targeting green consumers, the increase in WTP is increasing in k; for innovations targeting conventional consumers, it is decreasing.

Avoiding boundary solutions in the first or second stage requires that the innovations are not too large. This can be stated in a slightly casual way as follows:

Assumption 4. *$y_i^{max} > 0$ is sufficiently small.*

[23] As discussed in detail in Schmutzler (2024), the remaining cases in Figure 1 (horizontal differentiation with positive correlation and vertical differentiation with negative correlation) are rarely treated in the literature but make perfect economic sense.

[24] For a technical statement, see Assumption 3 in Schmutzler (2024), restated in Appendix A.1.

[25] Equivalently, it corresponds to a simultaneous increase of v_0^i and v_1^i by y_i.

In more detail, I first require that y_i^{max} is small enough that Assumption 2 and 3 hold before and after the investment. Second, in the special case of process innovations, I further assume that $y_i^{max} \leq c_i$ to avoid negative costs. Third, for environmental innovations, I require that $\underline{\eta}_B - y_i^{max} > \underline{\eta}_G$, so that the brown firm can never become less polluting than the green firm.

Usually, I treat innovations as resulting in ceteris paribus improvements in one of the three dimensions, cost, product quality (WTP), and environmental quality (specific emissions). However, I also discuss the possibility that product innovations (in particular, those targeting green consumers) simultaneously reduce emissions and increase the WTP for a firm's product. I refer to innovations that increase WTP without affecting specific emissions as *pure product innovations* and to innovations that reduce specific emissions without affecting WTP as *pure environmental innovations*.

2.3 Interpretation of the Framework

After having described the framework, I now discuss two points concerning its interpretation. First, I show that a specific model where consumers value good environmental properties of a product to different degrees fits the general set-up. Second, I show how the difference terms Δ_0, Δ_1 and Δ_C can be interpreted as reflecting policy.

2.3.1 Example: Intrinsic Pro-Environmental Preferences

As argued above, a higher WTP for the green product does not necessarily reflect explicit pro-environmental preferences. However, such preferences can be incorporated as a special case of the general setting, assuming that each product has two dimensions, reflecting consumption quality and environmental quality, respectively. The quality of firm i in the consumption dimension is parameterized by $Q_i \in \mathbb{R}^+$ and in the environmental dimension by $R_i \in \mathbb{R}^+$, where R_i is negatively related to specific emissions η_i. Consumers $k \in [0,1]$, appreciate quality in both dimensions, but differ in the relative willingness to pay (WTP) for each component.[26] They are uniformly distributed on the interval $[0,1]$. The WTPs of the most extreme consumers are captured by parameters $\overline{r}$, $\underline{r}$, $\overline{q}$ and $\underline{q}$, such that

$$\overline{r} > \underline{r} > 0;\ \overline{q} > 0;\ \underline{q} > 0. \tag{3}$$

The WTPs are assumed to be given by:

$$v_k^B = \left(\underline{r} + k\left(\overline{r} - \underline{r}\right)\right) R_B + \left(\underline{q} + k\left(\overline{q} - \underline{q}\right)\right) Q_B;$$
$$v_k^G = \left(\underline{r} + k\left(\overline{r} - \underline{r}\right)\right) R_G + \left(\underline{q} + k\left(\overline{q} - \underline{q}\right)\right) Q_G.$$

Using (3), note that consumers are ordered so that the WTP of those with high k reacts more positively to better environmental properties ($\overline{r} > \underline{r}$), while for consumption properties, both possibilities are possible, depending on whether $\overline{q} > \underline{q}$ (positively correlated valuations) or $\overline{q} < \underline{q}$ (negatively correlated valuations). Appendix A.2 provides conditions guaranteeing that assumptions (A2) and (A3) hold in this specific model, so that the analysis in Section 3 that relies on these assumptions can be applied.[27]

[26] This approach abstracts from the problem that consumers may not be able to easily assess environmental quality, for instance, because of attention constraints (see Schmitt (2024)).

[27] In this specific setting, horizontal differentiation ($\Delta_0 < 0$) requires $\Delta_Q := Q_G - Q_B < 0$, whereas vertical differentiation ($\Delta_0 > 0$) requires $\Delta_Q > 0$.

2.3.2 Interpretation of the Difference Terms Δ_0, Δ_1 and Δ_C

In Section 3, it will turn out that Δ_0, Δ_1 and Δ_C are crucial determinants of most economic outcomes of the price game described in Section 2.1: Margins, outputs and profits are fully determined by the differences in WTP and costs captured in Δ_0, Δ_1 and Δ_C. Changes in these expressions can be interpreted in several ways. First, they can reflect exogenous changes in preferences or technology. Second, they can result from innovation activities of the firms in Stage 1. Third, as I now explain in more detail, they can capture government policies.

Quite obviously, an *emissions tax* t translates into a reduction in Δ_C, as it imposes a higher tax burden per unit of the brown product than for the green product.[28] An *adoption subsidy* S for the green product corresponds to a uniform increase in the WTP v_k^G for each consumer or, equivalently, to a simultaneous increase of Δ_0 and Δ_1 by the same amount.[29] *Infrastructure investments* by the state that improve the consumption properties of all consumers for the green product can be captured in the same way.

To dig deeper, it is useful to refer to the specific setting of Section 2.3.1, in which the higher WTP for the green product of consumers with high k reflects explicit preferences for the relatively good environmental characteristics of this product and stronger sensitivity to improvements in these environmental properties. Then a policy that improves the consumption characteristics of the green product, but not of the brown product, can be captured as an increase in Δ_1 (with Δ_0 fixed).[30] Conversely, a policy that improves the environmental characteristics of the brown product, but not of the green product, can be captured as an increase in Δ_0 (with Δ_1 fixed).

3 Analysis of the Price Game

Section 3.1 briefly summarizes the analysis of the price equilibrium from Schmutzler (2024). Section 3.2 deals with the determinants of equilibrium emissions. Section 3.3 identifies sources of welfare losses in the pricing stage.

3.1 Equilibrium Characterization

Under suitable conditions, the pricing game has an equilibrium with full coverage, where consumers with k below a cut-off k^* buy product B and those with $k > k^*$ buy product G. To find this equilibrium, first note that, for arbitrary prices (p_B, p_G), consumer k is indifferent between the products if

$$v_0^B + k\left(v_1^B - v_0^B\right) - p_B = v_0^G + k\left(v_1^G - v_0^G\right) - p_G. \tag{4}$$

[28] The costs per unit output including the tax are $c_G + t\eta_G$ for the green product and $c_B + t\eta_B$ for the brown product, where $\eta_G < \eta_B$.

[29] Consumer k prefers the green product to the brown product if $v_k^G + S - v_k^B > p_k$ rather than $v_k^G - v_k^B > p_k$ as in the case without subsidies.

[30] To see this, assume that that the policy increases R_G, leaving everything else fixed, and that $\underline{r} = 0$, so that consumer $k = 0$ does not care for environmental quality.

Using the uniformity of the consumer distribution and assuming full coverage with an interior cut-off, (4) immediately gives demand functions as

$$x_B(p_B, p_G) = \frac{\Delta_p - \Delta_0}{\Delta_1 - \Delta_0} \tag{5}$$

$$x_G(p_G, p_B) = \frac{\Delta_1 - \Delta_p}{\Delta_1 - \Delta_0} \tag{6}$$

The first-order conditions lead to the candidate equilibrium

$$p_B^* = \frac{c_G + 2c_B + \Delta_1 - 2\Delta_0}{3} \tag{7}$$

$$p_G^* = \frac{c_B + 2c_G - \Delta_0 + 2\Delta_1}{3}. \tag{8}$$

Schmutzler (2024) confirms that these prices form an equilibrium under Assumptions 1-3. Unsurprisingly, both equilibrium prices are increasing in both costs. Moreover, both prices are increasing in Δ_1, but decreasing in Δ_0. Intuitively, an increase in Δ_1 (the WTP of the greenest consumer) increases the heterogeneity in the WTP for the two products, thereby softening competition, whereas an increase in Δ_0 reduces heterogeneity, thereby intensifying competition.

3.2 Determinants of Equilibrium Emissions

Inserting equilibrium prices into demand functions gives equilibrium outputs:

$$x_B^* = x_B^*(\Delta_0, \Delta_1, \Delta_C) = \frac{\Delta_1 - 2\Delta_0 + \Delta_C}{3\,(\Delta_1 - \Delta_0)} \tag{9}$$

$$x_G^* = x_G^*(\Delta_0, \Delta_1, \Delta_C) = \frac{2\Delta_1 - \Delta_0 - \Delta_C}{3\,(\Delta_1 - \Delta_0)} \tag{10}$$

Inserting equilibrium outputs x_B^* and x_G^* in (1), immediately leads to the following characterization of equilibrium emissions in the second stage.

Proposition 1. *Total equilibrium emissions in the second stage are given as*

$$E^* = \frac{(\eta_B + 2\eta_G)\,\Delta_1 - (2\eta_B + \eta_G)\,\Delta_0 + (\eta_B - \eta_G)\Delta_C}{3\,(\Delta_1 - \Delta_0)} \tag{11}$$

I now show how changes in parameters affect market shares and emissions, distinguishing between the cases without and with environmental preferences.

3.2.1 No Intrinsic Environmental Preferences

As a benchmark, it is useful to first abstract from intrinsic pro-environmental preferences, treating Δ_0 and Δ_1 as exogeneous.

(NO EP) Δ_0 *and* Δ_1 *are both independent of* η_B *and* η_G*.*

Any change of Δ_0, Δ_1 or Δ_C reduces total emissions if and only if it increases the market share of the green firm. A natural conjecture would be that this happens whenever the green firm becomes stronger, meaning that Δ_0 or Δ_1 increases or Δ_C falls. The following result substantiates this result while clarifying its limitations.

Corollary 1. ***Determinants of Total Emissions***

(i) Total emissions are increasing in η_G and η_B.

(ii) A reduction in Δ_C increases the market share of firm G and reduces total emissions.

(iii) If (NO EP) holds, a uniform increase in Δ_0 and Δ_1 increases the market share of firm G and reduces total emissions.

(iv) Suppose (NO EP) holds and $\Delta_C = 0$.

(a) Under horizontal differentiation, increases in Δ_0 and Δ_1 both increase the market share of firm G and reduce total emissions.

(b) Under vertical differentiation, an increase in Δ_0 increases the market share of firm G and reduces total emissions, whereas an increase in Δ_1 reduces it.

Result (i) is a mechanical implication of the definition of specific emissions, as long as they do not affect WTPs (which is ruled out by (NO EP)). The remaining results are also straightforward to prove, so I focus on the intuition.[31] Result (ii) confirms the conjecture that, if the green firm gets stronger in the sense of having lower costs, then its market share and emissions increase: As the costs of the green firm fall in relative terms, it reduces the price. The brown firm follows suit, but by a smaller amount. Therefore, its reaction will not fully compensate the direct market share effect of the green firm's price reduction. Result (iii) similarly shows how uniform WTP improvements of the green firm reduce total emissions.[32] Result (iv) shows that the effects of changes in Δ_0 and Δ_1 on emissions depend on whether differentiation is horizontal or vertical. The intuition relies on the interplay of direct demand effects and price-mediated effects. For instance, for fixed prices, an increase in Δ_1 shifts demand to firm 1. However, this direct effect is dampened by the price reactions. As the increased differentiation in consumer tastes softens competition, both prices increase, but more so for firm G that exploits the increased willingness to pay for the green product. With horizontal differentiation ($\Delta_0 < 0$), the direct demand effect dominates, resulting in an output shift to the green firm. More surprisingly, with vertical differentiation the price-induced effect dominates the direct effect so that total emissions fall. Thus, an improvement in the relative WTP of the greenest consumers for the green product need not reduce G's emissions – with vertical differentiation, G takes advantage of the increasing WTP with higher prices that reduce its market share and thus increase total emissions. An increase in Δ_0 (reflecting an increase in the WTP of the conventional consumers for the green firm) similarly affects market shares and thus total emissions directly and via price adjustments. However, as discussed above, the demand shift intensifies competition, inducing stronger price reductions for the brown firm than for the green firm. Nonetheless, the direct positive demand effect now dominates for both horizontal and vertical differentiation, and the green firm's market share increases.

Figure 2 illustrates the effects of WTP changes for a specific parameterization with symmetric marginal costs $\Delta_C = 0$. It describes combinations of Δ_0 and Δ_1 for which market shares and hence total emissions are constant. The Δ_1-axis divides the region with

[31]The results are implied by Corollary 2 in Schmutzler (2024), where a formal proof is given. The following discussion in this paragraph closely mirrors the discussion in the companion paper.

[32]Contrary to the previous case, the firm's market share increase goes hand in hand with higher prices of both firms.

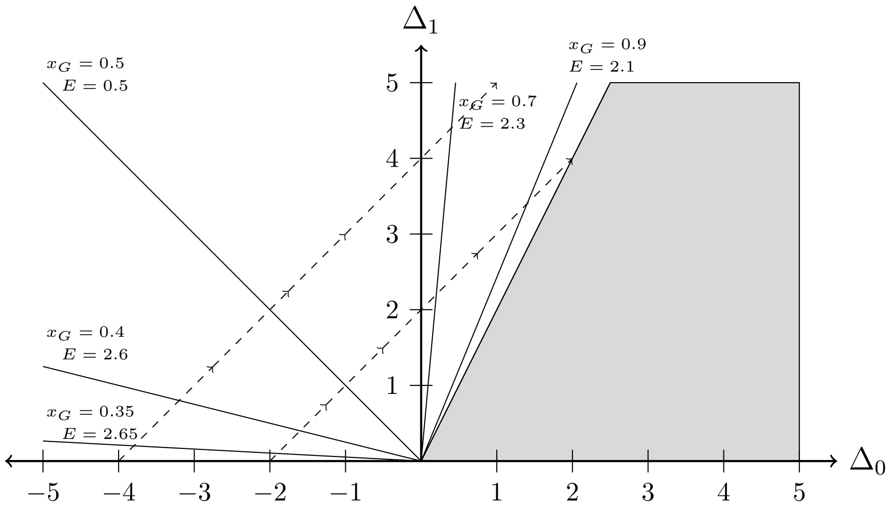

Figure 2: Determinants of Total Emissions

Note: The lines represent combinations of Δ_0 and Δ_1 for which market shares and total emissions are constant. They correspond to $\Delta_C = 0$ and specific emissions $\eta_B = 3$ and $\eta_G = 2$

horizontal differentiation (to the left) and the region with vertical differentiation.[33] The bisector of the second quadrant corresponds to symmetric horizontal differentiation where the brown and the green firm are equally strong and thus share the market between them ($x_G = \frac{1}{2}$). Moving from the left corner of the figure to the right by uniformly increasing Δ_0 and Δ_1, following the directions of the arrows on the dashed lines, the green firm becomes stronger and its market share strictly increases, inducing a fall in total emissions. While an increase in Δ_0 (for fixed Δ_1) also increases the market share of the green firm and reduces total emissions, the figure shows that an increase in Δ_1 (for fixed Δ_0) only has this effect for horizontal differentiation, but not for vertical differentiation.

3.2.2 Intrinsic Environmental Preferences

Corollary 1 is a useful benchmark, but it abstracts from environmental preferences, as there is no link between specific emissions and WTP, which is entirely exogenous. The following alternative assumption accounts for this link:

(EP) Δ_0, Δ_1 *and* $\Delta_1 - \Delta_0$ *are weakly increasing in* η_B *and weakly decreasing in* η_G.

Condition (EP) reflects the WTP for good environmental properties of a product. Using Assumption 1, it implies that Δ_k is weakly increasing in η_B and weakly decreasing in η_G for all $k \in (0, 1)$.[34] To capture effects of specific emissions on WTP (resulting from environmental preferences), part (i) of Corollary 1 needs to be adjusted by considering the interplay of the direct effect of a change in specific emissions and the indirect effect that reflects changes in WTP. In one case, the overall effect is clear-cut:

[33]The darkly shaded area does not satisfy the parameter restrictions implied by Assumption 2 and 9, the formal version of Assumption 3.

[34]The analysis does not require that Δ_0 and Δ_1 are *fully* determined by specific emissions. The WTP for one product can also be affected by improvements in the consumption characteristics, without any concomitant change in environmental properties.

Corollary 2. *Suppose that (EP) holds and that $\Delta_C = 0$. Then for horizontal differentiation, total emissions are increasing in specific emissions η_G.*

In this case, the direct effect of an increase in η_G on total emissions captured in Corollary 1 is reinforced by the resulting effect on market shares: For higher η_G, the lower WTP for this product reduces its market share, which increases aggregate emissions further, by adding to the direct effect that the emissions of the green product are more polluting. In other cases, the direct and the indirect effect are countervailing. For instance, with vertical differentiation, the reduction in WTP for the green product following an increase in its specific emissions can trigger a price reduction and thereby an increase in its market share, which could dominate the direct effect in principle. Moreover, an increase in η_B could also reduce aggregate emissions. With higher emissions of the brown product, consumers may switch to the green product, mitigating the adverse effect that the brown product has become more polluting.

3.2.3 Taking Stock: How (not) to Reduce Emissions

The previous section described the determinants of total emissions in the second stage of the game. To repeat, the results can either be interpreted as showing how policy could directly influence emissions when investment levels are fixed or, conversely, as describing how innovation would affect emissions, with policy kept fixed. Obviously, equation (1) implies that any *ceteris paribus* reduction in the specific emissions of either firm or any *ceteris paribus* increase in the market share of the green firm will reduce aggregate emissions. It is therefore tempting to conclude that environmental innovations of either firm or pure product and process innovations of the green firm will reduce total emissions. While the above analysis partly confirms this logic, various caveats exist.

Caveat 1: Using Green Innovations to Exploit Consumers. As argued above, a green firm that has engaged in innovations targeted toward green consumers faces an incentive to increase its prices so much that the net effect on its market share may become negative. Specifically, with symmetric consumers, this happens with vertical differentiation. Emissions unambiguously increase for pure product innovations targeted at green consumers in that case. While the effect is also present for pure environmental innovations, emissions may still fall because the green firm's specific emissions decline.

Caveat 2: Environmental Innovations by Brown Firms – A Mixed Blessing. As argued above, a ceteris paribus reduction in the specific emissions of the brown firm clearly reduces aggregate emissions. However, the innovation may affect WTP if some consumers appreciate the improved environmental properties or if other aspects of the product also improve. Then the change in specific emissions is no longer ceteris paribus. It may therefore result in an increase in the brown firm's market share that may dominate the direct effects of lower specific emissions.[35]

Caveat 3: Rebound Effects. The analysis assumes that total output in the market is fixed. Once this assumption is dropped, additional rebound effects need to be taken into account. For instance, Section 5 considers a more general model where a product innovation not only increases the green firm's market share, but may also attract new consumers or induce some of its consumers to buy a larger quantity of the green product.

[35] Under vertical differentiation, however, the caveat does not arise (with symmetric costs) if the improvement is mainly appreciated by green consumers. Then it can be captured as a reduction in Δ_1. According to Corollary 1(iii), this would lead to an increase in the green firm's market share.

Then, it is simple to show that even if the innovation increases the green firm's market share, increased total demand may compensate this beneficial effect, and total emissions could increase.

3.3 Welfare Considerations

I now provide some thoughts on the optimal policy in the pricing stage, assuming for the moment that investment levels and thus quality and production costs are fixed. I use the notation $\mathbf{p} := (p_B, p_G)$ and $\mathbf{\Delta}_v := (\Delta_0, \Delta_1)$. I denote equilibrium outputs as $x^i(\mathbf{p}; \mathbf{\Delta}_v)$, profits as $\pi^i(\mathbf{p}; \mathbf{\Delta}_v)$ and consumer surplus as $\sigma(\mathbf{p}; \mathbf{\Delta}_v)$. Moreover, environmental damages are captured by an increasing function $D(E)$. Further, let

$$\delta(\mathbf{p}; \mathbf{\Delta}_v) := D(\eta_B x_B(\mathbf{p}; \mathbf{\Delta}_v) + \eta_G x_G(\mathbf{p}; \mathbf{\Delta}_v))$$

refer to the damages arising in the equilibrium corresponding to the price vector $\mathbf{p}$ and the WTP differences captured by $\mathbf{\Delta}_v$. Thus, second-period welfare is

$$W^2(\mathbf{p}; \mathbf{\Delta}_v) = \pi_B(\mathbf{p}; \mathbf{\Delta}_v) + \pi_G(\mathbf{p}; \Delta_v) + \sigma(\mathbf{p}; \mathbf{\Delta}_v) - \delta(\mathbf{p}; \mathbf{\Delta}_\mathbf{v}).$$

The following simple statements are easy to derive:

Corollary 3. *(i) π^i is increasing in p_j and concave and hence single-peaked in p_i.*
(ii) Consumer surplus $\sigma(\mathbf{p}; \Delta_v)$ is decreasing in both prices.
(iii) Damages $\delta(\mathbf{p}; \Delta_v)$ are decreasing in p_B and increasing in p_G.

(i) and (ii) are standard. As to (iii), an increase in the price of firm i reduces its own demand x_i and increases competitor demand x^j. As total demand is fixed, these effects obviously have the same size, so that (from $\eta_B > \eta_G$) a price increase of firm G increases total emissions and damages, whereas a price increase of firm B decreases them. For convenience, I add the following assumption.

Assumption 4: *$W^2(\mathbf{p}; \Delta_v)$ is concave in prices.*

Given Assumption 4, an interior maximum for total welfare satisfies

$$\frac{\partial W_2}{\partial p_i} = \frac{\partial \pi^i}{\partial p_i} + \left(\frac{\partial \pi^j}{\partial p_i} + \frac{\partial \sigma}{\partial p_i} - \frac{\partial \delta}{\partial p_i} \right) = 0 \tag{12}$$

The first term $\frac{\partial \pi^i}{\partial p_i}$ is zero in the second-period equilibrium without policy interference. The terms in bracket capture three different externalities. A price increase of firm i is beneficial for the competitor and bad for consumers. Given imperfect competition, the latter effect dominates the former, so that $\frac{\partial \pi^j}{\partial p_i} + \frac{\partial \sigma}{\partial p_i} < 0$. As $\frac{\partial \delta}{\partial p_G} > 0$ by Corollary 3, equilibrium prices of firm G are thus unambiguously too high in equilibrium. For Firm B, this is less clear. Following well-known considerations in environmental policy under imperfect competition, there is a trade-off between correcting the product market distortions and the environmental distortions, which can be optimally resolved with suitable taxes or subsidies.[36]

[36] See, for instance, chapter 6 in Phaneuf and Requate (2016) for a useful discussion.

4 Innovation Decisions

I now turn to the firms' R&D investment decisions. Recall from Section 2.2 that, at the outset of the game, each firm is characterized by its costs $\underline{c}_i$, the valuations ($\underline{v}_0^i$ and $\underline{v}_1^i$) and specific emissions $\underline{\eta}_i$. Denote the resulting initial cost and WTP differences as $\underline{\Delta}_C$, $\underline{\Delta}_0$ and $\underline{\Delta}_1$. As the analysis of the second-stage game has shown that equilibrium outputs, margins and hence profits depend exclusively on Δ_0, Δ_1 and Δ_C, innovations can be captured as changes in these difference terms. For example, when interpreting y_i as a process innovation, the cost difference resulting from the first-stage innovations can be written as $\Delta_C(y_G, y_B)$.[37] This function is decreasing in y_G and increasing in y_B.[38] Innovations of firm B move Δ_C, Δ_0 and Δ_1 in the opposite direction as innovations of G. Thus, for each type of innovation, the relevant difference terms Δ_C, Δ_0 or Δ_1 can be written as functions of y_i and y_j to obtain gross profits as a function $\Pi_i(y_i, y_j)$ of investments.[39] Finally, parameters will be chosen so that the net profit function $\Pi^i(y_i, y_j) - K(y_i)$ is concave.[40]

4.1 The Determinants of Innovation Incentives

The candidate for an interior investment equilibrium is given by the system of FOCs

$$\frac{\partial \Pi_B}{\partial y_B} = K'(y_B) \tag{13}$$

$$\frac{\partial \Pi_G}{\partial y_G} = K'(y_G) \tag{14}$$

Investment in this candidate equilibrium is thus determined by equating marginal investment costs with the marginal benefits of investment $\frac{\partial \Pi_i}{\partial y_i}$ for the respective firm, commonly referred to as *innovation incentives.* However, the assumptions made so far do not guarantee that an interior equilibrium exists: The following result shows that innovation incentives need not be positive. Thus, firms may want to forego opportunities to increase the WTP for their products even if they were available at zero cost.

Lemma 1. ***The Sign of Innovation Incentives***

(i) Incentives for process innovations and non-targeted product innovations are positive.

(ii) Incentives for targeted product innovations are positive, except for

(a) innovations of the brown firm targeted at green consumers;

(b) innovations of the green firm targeted at conventional consumers under horizontal differentiation.

[37] This notation suppresses the dependence of the cost difference on initial costs.

[38] Similarly, a non-targeted product innovation of the green firm corresponds to a uniform increase in Δ_0 and Δ_1, and a targeted innovation of G corresponds to increases of either Δ_0 or Δ_1.

[39] For instance, with process innovations, $\Pi_i(y_i, y_j) := \pi_i^*\big(\Delta_0, \Delta_1, \Delta_C(y_i, y_j)\big)$. As $\Delta_C(y_B, y_G) = \underline{\Delta}_C - y_G + y_B$, this gives $\pi_G^* = \frac{\left(2\underline{\Delta}_1 - \underline{\Delta}_0 - \underline{\Delta}_C + y_G - y_B\right)^2}{9\left(\underline{\Delta}_1 - \underline{\Delta}_0\right)}$.

[40] This will not only include the requirement that the investment cost function is sufficiently convex; in addition, the WTP and parameters must satisfy suitable conditions; see below.

The result follows from equilibrium profits, which are[41]

$$\pi_B^* = \pi_B^*(\Delta_0, \Delta_1, \Delta_C) = \frac{(\Delta_1 + \Delta_C - 2\Delta_0)^2}{9\,(\Delta_1 - \Delta_0)} \tag{15}$$

$$\pi_G^* = \pi_G^*(\Delta_0, \Delta_1, \Delta_C) = \frac{(2\Delta_1 - \Delta_0 - \Delta_C)^2}{9\,(\Delta_1 - \Delta_0)} \tag{16}$$

The positive incentives in part (i) of Lemma 1 are in line with the intuition that being stronger in any dimension benefits a firm. The negative incentives in (ii) reflect the intuition that positive effects of such innovations that come from higher demand at fixed prices are outweighed by adverse effects from higher competition. In light of this discussion, an equilibrium with positive investments need not exist for all types of innovation.

4.2 Process Innovations and Non-Targeted Product Innovations

I now explicitly calculate the equilibrium investments in two important cases where interior equilibria exist, and I discuss the determinants of investments. Specifically, I consider process innovations and non-targeted product innovations. To avoid notational clutter, I assume that costs are *initially* symmetric ($\underline{\Delta}_C = 0$), and for the explicit calculations of equilibrium investments, I focus on quadratic investment costs:

$$K(y_i) = \kappa y_i^2 \text{ with a positive constant } \kappa \tag{17}$$

The following assumption will guarantee existence of an interior investment equilibrium.

Assumption 5. *One of the following conditions holds:*

(i) $\underline{\Delta}_1 > -\underline{\Delta}_0$ *and* $\kappa > \frac{1}{3(2\underline{\Delta}_1 - \underline{\Delta}_0)}$ *or*

(ii) $\underline{\Delta}_1 < -\underline{\Delta}_0$ *and* $\kappa > \frac{1}{3(2\underline{\Delta}_0 - \underline{\Delta}_1)}$

4.2.1 Equilibrium Investments and Emissions

Using (15) and (16), gross profits in the two-stage game can be written as functions of first-period investments:

$$\Pi_B(y_B, y_G) = \frac{(\underline{\Delta}_1 + (y_B - y_G) - 2\underline{\Delta}_0)^2}{9(\underline{\Delta}_1 - \underline{\Delta}_0)} - \kappa y_B^2$$

$$\Pi_G(y_G, y_B) = \frac{(2\underline{\Delta}_1 - (y_B - y_G) - \underline{\Delta}_0)^2}{9(\underline{\Delta}_1 - \underline{\Delta}_0)} - \kappa y_G^2$$

This leads to the following characterization of the SPE:

Proposition 2. *Suppose that Assumption 5 holds and* $\underline{\Delta}_C = 0$. *Then the investment game has a unique equilibrium with positive investments given by*

$$y_B^* = \frac{3\kappa\underline{\Delta}_1 - 6\kappa\underline{\Delta}_0 - 1}{3\kappa\,(9\kappa(\underline{\Delta}_1 - \underline{\Delta}_0) - 2)}$$

$$y_G^* = \frac{6\kappa\underline{\Delta}_1 - 3\kappa\underline{\Delta}_0 - 1}{3\kappa\,(9\kappa(\underline{\Delta}_1 - \underline{\Delta}_0) - 2)}$$

The proposition implies the following comparative statics results.

[41] The result follows directly from the expressions for equilibrium prices ((7) and (8)) and outputs ((15) and (16)).

Corollary 4. *Suppose that Assumption 5 holds and* $\underline{\Delta}_C = 0$.

(i) *(a) The investment of firm B is decreasing in* $\underline{\Delta}_0$ *for* $\kappa > \frac{1}{9\underline{\Delta}_1}$*;*

(b) For vertical differentiation, the investment of firm B is increasing in $\underline{\Delta}_1$*;*

(c) For horizontal differentiation, the investment of firm B is decreasing in $\underline{\Delta}_1$ *for* $\kappa > \frac{1}{9|\underline{\Delta}_0|}$.

(ii) *(a) The investment of firm G is increasing in* $\underline{\Delta}_0$ *for* $\kappa > \frac{1}{9\underline{\Delta}_1}$*;*

(b) For vertical differentiation, the investment of firm G is decreasing in $\underline{\Delta}_1$*;*

(c) For horizontal differentiation, the investment of firm G is increasing in $\underline{\Delta}_1$ *for* $\kappa > \frac{1}{9|\underline{\Delta}_0|}$.

(iii) $\Delta_y^* := y_G^* - y_B^*$ *is*

(a) increasing in $\underline{\Delta}_0$ *for* $\kappa > \frac{1}{9\underline{\Delta}_0}$*;*

(b) decreasing in $\underline{\Delta}_1$ *for vertical differentiation, increasing in* $\underline{\Delta}_1$ *for horizontal differentiation for* $\kappa > \frac{1}{9\underline{\Delta}_0}$*;*

(c) increasing after a uniform increase in $\underline{\Delta}_0$ *and* $\underline{\Delta}_1$.

To interpret the results, it is important to note that, according to Assumption 5, κ needs to be sufficiently large. This is obviously consistent with the various downward restrictions on κ in Corollary 4.

Results (i) and (ii) focus on the individual equilibrium investments, whereas (iii) addresses the difference in the investments of the two firms. Intuitively, if the green firm invests more than the brown firm, its market share will be higher than it would be in the absence of investments, thereby working toward lower emissions. Moreover, it is straightforward to show that total emissions are a decreasing function of the investment difference Δ_y^*. Thus, the investment difference captures the aspects of the investment decisions that are relevant from an environmental perspective.

Figure 3 illustrates the result for costs that are initially symmetric. The bisector of the second (left) quadrant corresponds to parameters for which firms are symmetrically horizontally differentiated at the outset: In this symmetric situation, equilibrium investments of both firms are the same. Thus, symmetry still obtains after the investment decisions have been made. To the right of the bisector, when the green firm is stronger, it invests more than the competitor, so that its lead is reinforced by investment. The converse statement holds for the opposite case that the brown firm is stronger. Uniform WTP increases for firm G, as captured by moves to the upper right along the dashed lines, lead to increases in Δ_y^*.[42]

To see the economic intuition for Corollary 4, first note that, very generally, incentives to innovate depend on the effects on equilibrium outputs and margins that the innovation would have. The results in Corollary 4 reflect two main ideas. First, for both firms, own

[42]Between the dashed (horizontal and vertical) lines, the lines corresponding to constant investment differences are downward-sloping. When the green firm is strong ($\Delta_0 > -\frac{1}{9}$), however, the lines corresponding to constant investment differences are increasing, with a higher Δ_y^* where competition is intense ($\underline{\Delta}_0$ is large and $\underline{\Delta}_1$ is small). The converse relation holds when the brown firm is relatively weak.

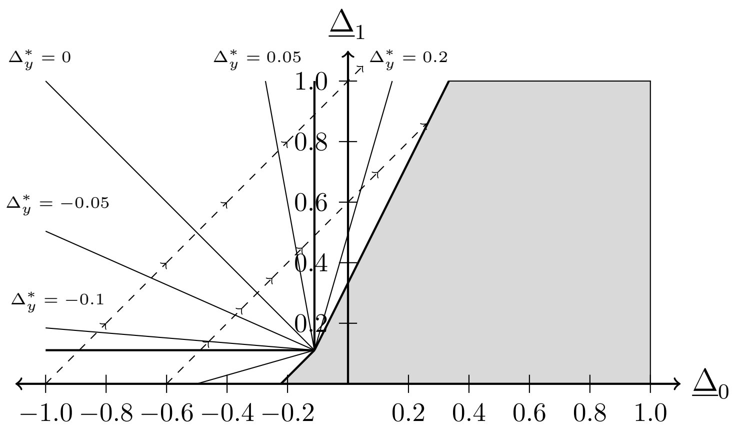

Figure 3: Investment Equilibrium for Non-targeted Product Innovation

Note: The lines describe combinations of $\underline{\Delta}_0$ and $\underline{\Delta}_1$ for which the investment difference Δ_y is constant. Assumptions: $\kappa = 1$, $\underline{\Delta}_C = 0$, $\eta_B = 3$ and $\eta_G = 2$. The parameter restrictions guarantee that an interior investment equilibrium exists.

process innovations and non-targeted product innovations have positive effects in the output and the margin dimension. Second, output and margin expansions are complements: Output expansions are more valuable when margins are high and, conversely, margin expansions are more valuable when outputs are high. This principle immediately explains the results on the effects of uniform increases in $\underline{\Delta}_0$ and $\underline{\Delta}_1$ in (iii): As such increases increase the initial outputs and margins of firm G and decrease those of firm B, they increase the incentives to increase margins and outputs by innovation. These effects are mutually reinforcing, leading to higher equilibrium investments of the green firms and lower investments of the brown firm and thus also to a higher Δ_y^*. In the remaining cases, countervailing effects come into play. For instance, an increase in $\underline{\Delta}_1$ increases the equilibrium margin of firm B, but reduces its output. The higher margin increases the value of expanding output, the lower output reduces the value of increasing margins. As a result, there are countervailing effects on innovation incentives, and it is not clear without direct calculation which of them dominates.

One can easily calculate aggregate emissions in the SPE of the investment game by inserting the equilibrium investments from Proposition 2 in equation (11).

Corollary 5. *Aggregate emissions in the investment game with* $\underline{\Delta}_C = 0$ *are given as*

$$E^* = \frac{(\eta_B + \eta_G) + 3\kappa\left((2\underline{\Delta}_0 - \underline{\Delta}_1)\eta_B + (\underline{\Delta}_0 - 2\underline{\Delta}_1)\eta_G\right)}{2 - 9\kappa\left(\underline{\Delta}_1 - \underline{\Delta}_0\right)} \tag{18}$$

E^* is decreasing in $\underline{\Delta}_0$, the WTP difference between the green and brown product for conventional consumers. Not only does the parameter increase directly reduce total emissions for given investment levels as a result of market share adjustments—by Corollary 4, it also increases Δ_y^*, which reinforces the direct effect(ii). A uniform increase in $\underline{\Delta}_0$ and $\underline{\Delta}_1$ reduces E^* for analogous reasons. Finally, E^* is decreasing in $\underline{\Delta}_1$ for horizontal differentiation, but increasing for vertical differentiation. Though this distinction makes the argument more subtle, the intuition is still that the investment effect of the parameter change (captured by Corollary 4) reinforces the direct effect from Corollary 1: When the increase in $\underline{\Delta}_1$ reduces total emissions for fixed investments (for horizontal differentiation), it also increases green investments at the expense of brown investments, conversely when the direct effect on emissions is positive (for vertical differentiation).

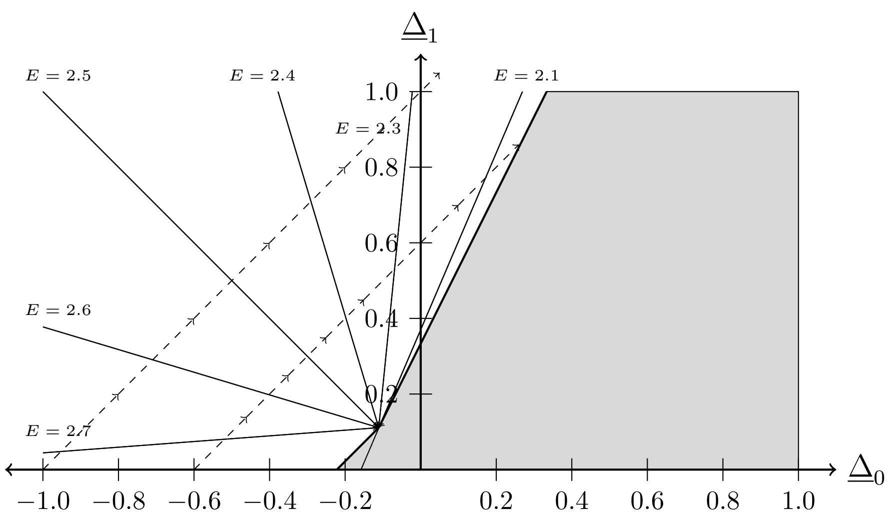

Figure 4: Total Emissions with Endogenous Non-targeted Product Innovations

Note: The lines describe combinations of $\underline{\Delta}_0$ and $\underline{\Delta}_1$ for which total emissions are constant, given equilibrium investments. Assumptions: $\kappa = 1$, $\underline{\Delta}_C = 0$, $\eta_B = 3$ and $\eta_G = 2$. The parameter restrictions guarantee that an internal investment equilibrium exists.

Figure 4 illustrates the result for $\underline{\Delta}_C = 0$. As in Figure 2, the straight lines give combinations of WTP differences for which total emissions are constant.[43] The emission levels are now given by (18) and thus take into account the potential differences between the investments of the two firms. This makes the effects of WTP differences on total emissions more pronounced. For instance, with exogeneous investments and the parameterization used for the figures, the relatively low total emission level of 2.3 can only be achieved under vertical differentiation (see Figure 2), whereas it can arise even with horizontal differentiation with endogeneous investments: If the green firm has a higher market share than the competitor, it will invest more. Thus, it will expand its lead, leading to a reduction in total emissions below what would be possible in the absence of investments.

To sum up the analysis so far, it reveals the direct effects of WTP changes on emissions (for fixed investments) as well as their effects on investments. Combining both effects shows how WTP changes affect total emissions when their effects on investment are taken into account. A central observation is that the direct negative effect of uniform WTP increases of firm 1 is strengthened when the reaction of investments is taken into account, because the WTP change increases Δ_y^* in favor of the green firm.

I now address a complementary question that helps to isolate the direct effects of parameter changes on emissions from those induced by changes in investment behavior. Suppose that a policy targets a specific emissions level without taking investment effects into account. Which of the available policies reaching the target would be most beneficial in terms of increasing Δ_y^* in favor of the green firm? Technically, this amounts to maximizing Δ_y^* along an iso-emission line such as those given in Figure 2. The following

[43]The axes now refer to the first-stage WTP differences rather than those at the beginning of the second stage.

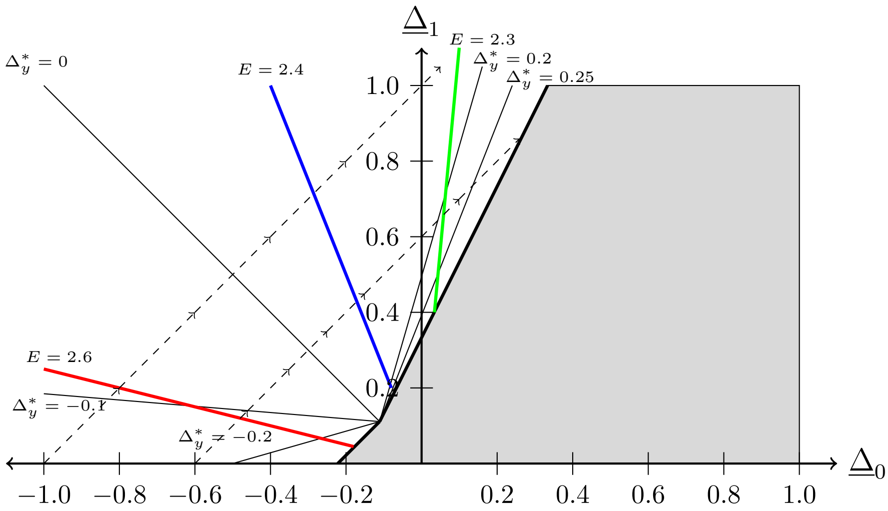

Figure 5: Maximizing Δ_y^*

Note: The thick lines correspond to the iso-emission lines for $E = 2.6$ (red) and $E = 2.3$ (green), respectively. The thin lines correspond to iso-investment curves for $\Delta_y^* = -0.2$, -0.1, 0, 0.2 and 0.25, respectively (in clockwise order). Assumptions: $\kappa = 1$, $\underline{\Delta}_C = 0$, $\eta_B = 3$ and $\eta_G = 2$.

result essentially answers the question by identifying the direction in which moves along the indifference curve increase Δ_y^*.

Corollary 6. *Increasing Δ_0 along the static iso-emissions line increases Δ_y^* if and only if $\frac{1}{2} < x_G < \frac{2}{3}$ or, equivalently,*

$$\frac{\eta_B + 2\eta_G}{3} < E < \frac{\eta_B + \eta_G}{2} \tag{19}$$

Condition (19) requires emissions to be lower than for equal market shares, but not too strongly. For instance, in Figure 5, the condition holds if $E = 2.4$. For this (blue) iso-emissions line, Δ_y^* is therefore highest in the lower right corner where competition is most intense. By contrast, for $E = 2.3$ (green line), where (19) is violated, the green firm is initially relatively strong and emission are correspondingly low. Along this line, Δ_y^* is highest where Δ_0 and Δ_1 are initially small, so that competition is relatively favorable for the brown firm. Finally, for $E = 2.6$ (red line), the green firm is initially relatively weak and emission are correspondingly high. Among these constellations, Δ_y^* is highest (least negative) where Δ_0 is low and Δ_1 is initially high. Thus, in this case, softer competition tends to shift investment in the direction of firm G. All told, to induce investment by the green but not by the brown firm, policy should choose points along iso-emission lines as follows: (i) For low emissions, the brown firm should be strengthened (low Δ_0 and Δ_1); (ii) For high emissions, competition should be as soft as possible (low Δ_0 and high Δ_1); (iii) For intermediate emission levels, competition should be as intense as possible (high Δ_0 and low Δ_1).

4.3 Policy

As explained in Section 2.3, the parameters in (18) can be interpreted as capturing policy effects. Thus, for instance, the discussion following Corollary 5 shows how adoption subsidies (captured by uniform increases in $\underline{\Delta}_0$ and $\underline{\Delta}_1$) lead to lower subsidies: They not only directly reduce emissions by shifting output to the green firm; in addition, the improvement in the green firm's relative position increases its investment incentives, thus reinforcing the direct effect.

Even though the framework abstracts from knowledge spillovers and thus from a source of positive externalities, the scope for market failure is substantial. Achieving optimal incentives is demanding, not least because of imperfect competition. Additional complications arise because there are intrinsic asymmetries between the two firms which would necessitate differential treatment. The optimal policy will clearly depend on whether the regulator can only control investments or whether she can optimally choose prices as well.

Starting with the latter case, there would be no reason to support process innovations of the green firm. Even though these innovations would shift output toward the green firm and thereby reduce environmental damages, this could also be achieved by adequate choice of prices (e.g., via environmental taxes). The issues are more complex for product innovations. Abstracting from price effects, such innovations typically have an adverse (business-stealing) effect on the competitor and a positive effect on consumers. Thus, there is no reason to expect the equilibrium investments to maximize welfare, even when prices are chosen optimally in the second stage. From a purely environmental perspective, however, there is no reason to intervene, except for those innovations that also reduce specific emissions.

The scope for beneficial interventions affecting investment decisions becomes larger if the regulator cannot directly affect prices, which adjust according to the second-stage Nash equilibrium without regulation. This seems like a plausible possibility in view of the recent political experience in the U.S. where legislators opposed a bill that relied heavily on carbon taxes, whereas they supported the "Inflation Reduction Act" that relied heavily on subsidies.[44] In this case, there even is a rationale for supporting process innovations of the green firm. The discussion in Section 3.3 suggests that prices of the green firm are too high in the second period relative to those of the brown firm. Process innovations address this problem and can therefore help to alleviate the externality problem. Next, consider a product innovation that improves the consumption features of the green product, without any effect on specific emissions. Corollary 1 showed that such an innovation typically increases the green firm's market share and thus reduces emissions.[45] Thus, supporting such an innovation may help to correct for the adverse environmental effect of excessive second-stage prices. Similarly, in the presence of intrinsically green consumers, the arguments for supporting environmental innovations usually get reinforced by their possible positive market share effect on the green firm.

[44]See https://www.whitehouse.gov/cleanenergy/inflation-reduction-act-guidebook/

[45]Recall the exception of an innovation targeted toward green consumers under vertical differentiation.

5 Generalization

I now replace the benchmark model of Section 2 with a more general framework. This allows conclusions about the robustness of the insights on how innovations affect emissions and how innovation incentives are determined.

5.1 Set-Up

The central assumptions are as follows:

Assumption 6. *Firm $i \in \{B, G\}$ has a demand function $x^i(p_i, p_j; \theta_i, \theta_j)$ for $j \neq i$, which is decreasing in p_i and θ_j, increasing in p_j and θ_i and continuously differentiable (at least $\mathcal{C}^1$) in all arguments wherever both equilibrium prices and outputs are positive.*

For convenience, I state the second assumption on profit rather than demand functions.

Assumption 7. *For $i = 1, 2$, the profit function π^i is twice continuously differentiable and concave in own prices wherever both prices and the resulting outputs are positive; the function satisfies strategic complementarities, $\pi^i_{ij} := \frac{\partial^2 \pi^i}{\partial p_i \partial p_j} > 0$; moreover $|\pi^i_{ii}| > \pi^i_{ij}$.*

Interpreting θ_i as WTP parameters, the demand and profit functions resulting from the benchmark model satisfy Assumptions 6 and 7.[46] These assumptions suffice for existence of second-stage equilibria. It is convenient to impose uniqueness directly:

Assumption 8. *For any $\mathbf{Y} := (c_B, c_G, \theta_B, \theta_G)$, there exists a unique product market equilibrium $(p^*_B(\mathbf{Y}), p^*_G(\mathbf{Y}))$, where p^*_B and p^*_G are differentiable functions.*

Recall that, in the benchmark model, process innovations and product innovations increase the innovator's output and reduce the output of the competitor. The following terminology captures these features.

Definition 2. ***Regular innovations***

*(i) An innovation is **regular** if it increases the innovator's equilibrium output and reduces the competitor's output.*

*(ii) A regular innovation is **strictly regular** if it does not reduce total output.*

*(iii) A strictly regular innovation is **purely business-stealing** if it does not affect total output.*

[46] Depending on which type of innovation θ_i is supposed to capture, it can correspond to v^i_0, v^i_1 or as a parameter that uniformly shifts v^i_0 and v^i_1 in the benchmark model.

5.2 Effects of Innovation on Emissions

I first ask how regular innovations affect emissions, without otherwise specifying the type of innovation. I then give conditions for process and product innovations, respectively, to be regular.

5.2.1 The Effects of Regular Innovations

I now state a very simple general result on the effects of regular innovations on total emissions that follows directly from equation (1). To this end, let $\phi_i \in \{\theta_i, -c_i\}$ be a parameter capturing the strength of firm i in the demand or cost dimension so that the effect of the innovation corresponds to the effect of a change in ϕ_i.

Proposition 3. *Denote the outputs corresponding to* (p_B^*, p_G^*) *as* (x_B^*, x_G^*).
(i) A regular innovation of firm G reduces total emissions if and only if

$$\frac{\frac{\partial x_G^*}{\partial \phi_G}}{\left|\frac{\partial x_B^*}{\partial \phi_G}\right|} < \frac{\eta_B}{\eta_G}. \tag{20}$$

(ii) A strictly regular innovation of firm B increases overall emissions.

Compared to the benchmark model where total output is fixed, so that innovations are purely business stealing, an innovation of firm G can now increase total emissions because it increases total output. Condition (20) rules out such rebound effects by making sure that the possible adverse effect of an increase in total output does not dominate the beneficial effect that the green firm (with its relatively low specific emissions) has a higher market share. Result (ii) holds because, for a strictly regular innovation, the adverse effects of the increase in market share of the brown firm are exacerbated by the possible increase in total output.

5.2.2 Regularity of Process Innovations

To understand the scope of Proposition 3, I now provide conditions under which innovations are regular, starting with process innovations.

Proposition 4. *Suppose the costs of firm i fall. Then:*
(i) Both prices fall.
(ii) Suppose that $\frac{\partial x^i}{\partial p_j} \leq \left|\frac{\partial x^i}{\partial p_i}\right|$ *holds locally. (a) Then the effect on firm i's own output is positive. (b) The effect on the output of firm $j \neq i$ is negative if*

$$\frac{\partial p_i^*/\partial c_i}{\partial p_j^*/\partial c_i} > \frac{|\partial x^j/\partial p_j|}{\partial x^j/\partial p_i}.$$

As in the benchmark model, (i) holds because the incentive to reduce prices resulting from a cost reduction extends to the competitor by strategic complementarities. (ii), which guarantees regularity, takes into account that the parameter change potentially affects outputs directly or via the effects on both equilibrium prices (see Section A.5 for more detail). (ii)(a) shows that the direct positive effect of firm i's cost reduction and the effect of lower own prices on its own output dominate the adverse effect of lower competitor prices. Contrary to the benchmark model where total output is fixed, an increase in the output of firm i does not guarantee a reduction in the output of firm j. The condition in (ii)(b) fills the gap by guaranteeing that the reduction in x_j induced by the reduction in p_i dominates the increase in x_j induced by the reduction in p_j.

5.2.3 Regularity of Product Innovations

I now turn to product innovations. The benchmark model highlighted the critical role of price effects of product innovation for total output and emissions. This motivates the following terminology.

Definition 3. *(i) A product innovation of player i* ***increases (reduces) demand sensitivity*** *if $\frac{\partial^2 x^i}{\partial p_i \partial \theta_i} < 0$ ($\frac{\partial^2 x^i}{\partial p_i \partial \theta_i} > 0$).*
(ii) A product innovation ***softens (intensifies) competition*** *if $\pi^i_{i\theta_i} > (<)\ 0$.*

The relation between the two concepts is reflected by the following equation:

$$\pi^i_{i\theta_i} = \frac{\partial x^i}{\partial \theta_i} + (p_i - c_i)\frac{\partial^2 x^i}{\partial p_i \partial \theta_i}. \tag{21}$$

As Assumption 6 implies $\partial x^i / \partial \theta_i > 0$, product innovations that reduce demand sensitivity soften competition, while those that increase demand may intensify competition. This distinction is important for the price effects of product innovations:

Proposition 5. *Consider a product innovation of firm i.*
(i) If the innovation softens competition, it increases both prices; if it intensifies competition, it reduces both prices.
(ii) (a) The innovation increases firm i's output if

$$\frac{\partial x^i}{\partial \theta_i} + \frac{\partial x^i}{\partial p_j}\frac{\partial p^*_j}{\partial \theta_i} > -\frac{\partial x^i}{\partial p_i}\frac{\partial p^*_i}{\partial \theta_i}.$$

(b) It reduces competitor j's output if

$$\frac{\partial x^j}{\partial \theta_i} + \frac{\partial x^j}{\partial p_j}\frac{\partial p^*_j}{\partial \theta_i} < -\frac{\partial x^j}{\partial p_i}\frac{\partial p^*_i}{\partial \theta_i}.$$

Result (i) follows readily from the facts that competition-softening (intensifying) innovations shift reaction curves in (out) and that, because prices are strategic complementarities, these price effects reinforce each other. The results in (ii), which jointly guarantee that the innovation of firm i is regular, result from the interplay of direct demand effects with the indirect effects induced by price changes. I focus on the case that product innovations soften competition; the case of competition-intensifying innovations is analogous. As reflected in the term $\frac{\partial x^i}{\partial p_j}\frac{\partial p^*_j}{\partial \theta_i}$, the price increase of firm j following the competition-softening innovation of firm i reinforces the positive direct effect $\frac{\partial x^i}{\partial \theta_i}$; whereas firm i's own price effect weakens these two positive effects (as captured by $\frac{\partial x^i}{\partial p_i}\frac{\partial p^*_i}{\partial \theta_i}$). The condition in (ii)(a) guarantees that the positive effects dominate.[47]

[47]In the benchmark model, the direct effects of a product innovation in the strong dimension will dominate, so that an increase in a firm's own quality increases its output.

Trivially, in the extreme case of the benchmark model that quality does not affect total output (quality improvements are purely business-stealing), the output effects on the competitor are exactly opposite to those on the investing firm. Hence, the condition for a positive effect on the innovator's output is then identical to the condition for a negative effect on the competitor. More generally, a negative effect on firm j's output requires that the joint effect of the induced price changes does not dominate the direct negative effect ($\frac{\partial x^j}{\partial \theta_i}$). If innovation softens competiton, then the effect of firm j's own higher equilibrium price, $\frac{\partial x^j}{\partial p_j}\frac{\partial p_j^*}{\partial \theta_i}$, strengthens the negative direct effect, whereas the effect of firm i's higher equilibrium price, $\frac{\partial x^j}{\partial p_i}\frac{dp_i^*}{d\theta_i}$ weakens the positive direct effect. The condition in (ii)(b) guarantees that the negative effects dominate.

The above results show that the major part of the analysis of the second-stage game carries over from the benchmark to the general model. A calculation of the investment equilibrium is impossible without specifying functional forms. However, it is possible to make some general statements on investment incentives.

Proposition 6. ***Innovation Incentives***

(i) The incentives of firm i to engage in a process innovation are positive if and only if

$$x^i\left(p_i^*, x_j^*\right) > (p_i - c_i)\frac{\partial x^i}{\partial p_j}\frac{dp_j^*}{dc_i} \tag{22}$$

(ii) The incentives of firm i to engage in a product innovation are positive if

(a) the innovation softens competition or

(b) the innovation intensifies competition and

$$\frac{\frac{\partial x^i}{d\theta_i}}{\frac{\partial x^i}{\partial p_j}} > \left|\frac{dp_j^*}{d\theta_i}\right|. \tag{23}$$

To understand the result, note more generally that the marginal profit effect of changing any parameter ϕ of the pricing game is given by

$$\frac{d\pi_i^*}{d\phi} = \frac{\partial \pi_i^*}{d\phi} + \frac{\partial \pi_i^*}{\partial p_j}\frac{dp_j^*}{d\phi} \text{ for } i = 1, 2; j \neq i. \tag{24}$$

The first term in (24) is the direct effect on profits of the firm under consideration; the second term captures the effect that is mediated by a price change of the competitor.[48] The conditions in Proposition 6 make sure that the indirect effects either reinforces the direct effect (for a competition-softening product innovation) or that it at least does not dominate the direct effect (as guaranteed by (22) and (23)).

In summary, this section analyzed in a more general way than the benchmark model how the intensity of product market competition affects the relation between innovation and emissions. Moreover, it provides a deeper understanding of the conditions required for innovation incentives to be positive.

[48] Reflecting the logic of the envelope theorem, $\frac{\partial \pi_i^*}{\partial p_i}\frac{dp_i^*}{d\phi} = 0$ as $\frac{\partial \pi_i^*}{\partial p_i} = 0$. Hence, there is no need to account for effects intermediated by a change in firm i's price.

6 Conclusions

This paper has investigated the incentives for engaging in innovations that foster the green transition. Contrary to a large body of work on environmental innovation, the analysis emphasizes that suitable product and process innovations can reduce total emissions by market share reallocation. The analysis relies on a differentiated duopoly with a brown firm and a green firm. Both firms can engage in process innovations, product innovations or environmental innovations. A large part of the analysis has been carried out in a simple but flexible discrete-choice model. In this model, I first derived plausible conditions under which innovations of the green firm lower aggregate emissions, while innovations of the brown firm increase total emissions. For process innovations and non-targeted product innovations, this essentially results from output relocation toward the green firm. For targeted product innovations and for environmental innovations, various caveats need to be considered. Product innovations targeted at green consumers increase total emissions under vertical differentiation, as the green firm opts to exploit the green consumers. For environmental innovations, the reduction in the green firm's specific emissions reinforces this market share effect when consumers have environmental preferences, whereas environmental innovations of the brown firm only lead to lower aggregate emissions if the beneficial effects from the firm's lower specific emissions dominate adverse effects from its increased market share.

The results suggest that firms may have no incentives for product innovations targeting consumers who prefer the competitor, as such innovations would intensify competition. By contrast, incentives for process innovations and for non-targeted product innovations are positive. Increases in the strength of the green firm (uniform increases in WTP relative to the competitor) will increase the difference between its investments and those of the brown firm, thus reinforcing the positive environmental firms of a strong green firm.

The analysis also contains some results regarding the rationale for directly subsidizing innovations. When suitable price policies are available, there is no need for supporting process innovations of green firms; however, this changes when price instruments are not available. By contrast, there is a case for supporting environmental innovations of the green firm even when price instruments are available.

The benchmark model has several obvious limitations, most notably the fixed total output. The analysis of Section 5 has already shown that, to a large extent, these limitations can be overcome with a more abstract model with more general demand functions, at the cost that the results become less sharp. A possible complementary approach might be to maintain the discrete-choice setting of Section 2, but attempt to work with more general preference distributions. Of course, other generalizations are conceivable, such as allowing for more than two firms as well as for the case that firms sell both brown and green products.

A Appendix

This appendix contains proofs and calculations required to corroborate the results mentioned in the main text.

A.1 Formal Statement of Assumption 3

The precise requirement of Assumption 3 is that:[49]

Assumption 9. *(i)* $\Delta_1 - 2\Delta_0 + \Delta_C > 0$

(ii) $2\Delta_1 - \Delta_0 - \Delta_C > 0$

(iii) *(a)* $v_0^G > \frac{c_B + 2c_G - \Delta_0 + 2\Delta_1}{3}$ *or (b)* $\min\{v_0^B, v_1^B\} > \frac{\Delta_1 - 2\Delta_0 + 2c_B + c_G}{3}$

A.2 Example: Heterogeneous Environmental Preferences

I now show under which conditions the example of Section 2.3.1 satisfies Assumptions 2 and 9. Define $\Delta_R := R_G - R_B$. Assumptions 2, 9(i) and 9(ii) require that the following conditions hold.

$$\begin{aligned}
\text{Assumption 2(i)'}:\ & (\overline{r} - \underline{r})\, R_G > \left(\underline{q} - \overline{q}\right) Q_G \\
\text{Assumption 2(ii)'}:\ & \overline{r}\Delta_R + \overline{q}\Delta_Q > \underline{r}\Delta_R + \underline{q}\Delta_Q \\
\text{Assumption 2(iii)'}:\ & \overline{r}\Delta_R + \overline{q}\Delta_Q > 0 \\
\text{Assumption 9(i)'}:\ & \overline{r}\Delta_R + \overline{q}\Delta_Q > 2\left(\underline{r}\Delta_R + \underline{q}\Delta_Q\right) - \Delta_C \\
\text{Assumption 9(ii)'}:\ & \overline{r}\Delta_R + \overline{q}\Delta_Q > \frac{\left(\underline{r}\Delta_R + \underline{q}\Delta_Q\right) + \Delta_C}{2}
\end{aligned}$$

Using the fact that the last four inequalities are all downward restrictions on $\overline{r}\Delta_R + \overline{q}\Delta_Q$, it is straightforward to show that Assumption 2(i), 9(ii)' and 9(ii)' can be jointly satisfied. By suitable choices of the parameters in the example of Section 2.3.1 one can finally make sure that Assumption 9(iii)(a) or (b) holds as well. Essentially, v_0^B, v_0^G and v_1^B have to be high enough. For any given parameterization satisfying the remaining conditions, this can be achieved by simultaneously increasing these three parameters as well as v_1^G by the same sufficiently high amount, as this leaves the WTP differences unaffected.

A.3 Proof of Corollary 2

Let $\Delta_0 = \Delta_0(\eta_B, \eta_G)$ and $\Delta_1 = \Delta_1(\eta_B, \eta_G)$. Using $\Delta_C = 0$, (11) gives

$$E = \frac{(\eta_B + 2\eta_G)\,\Delta_1(\eta_B, \eta_G) - (2\eta_B + \eta_G)\,\Delta_0(\eta_B, \eta_G)}{3\left(\Delta_1(\eta_B, \eta_G) - \Delta_0(\eta_B, \eta_G)\right)}$$

Taking the derivative and rearranging gives

$$\frac{\partial E}{\partial \eta_G} = \frac{2\Delta_1 - \Delta_0}{3\Delta_1 - 3\Delta_0} + \frac{\left(\Delta_0 \frac{\partial \Delta_1}{\partial \eta_G} - \Delta_1 \frac{\partial \Delta_0}{\partial \eta_G}\right)(\eta_B - \eta_G)}{3\left(\Delta_0 - \Delta_1\right)^2}$$

Using Assumption 2(ii), Assumption 9(ii) with $\Delta_C = 0$, (EP) and the fact that $\Delta_0 < 0 < \Delta_1$ for horizontal differentiation gives the result.

[49] See Schmutzler (2024) for a detailed discussion.

A.4 Proofs for the Investment Game

A.4.1 Proof of Proposition 2

The proof is the same for process innovations and non-targeted product innovations.

With $\Psi := \Delta_1 - \Delta_0$, one can rewrite (gross) profits as

$$\pi_B = \frac{1}{9}\frac{(\Psi - \Delta_0 + (y_B - y_G))^2}{\Psi}$$
$$\pi_G = \frac{1}{9}\frac{(2\Psi + \Delta_0 - (y_B - y_G))^2}{\Psi}$$

Thus, for firm B, the FOC is

$$\frac{\partial}{\partial y_B}\left((\underline{\Psi} + (y_B - y_G) - \underline{\Delta}_0)^2 - 9\underline{\Psi}\kappa\,(y_B)^2\right) =$$
$$2\underline{\Psi} - 2\Delta_0 + 2y_B - 2y_G - 18\Psi y_B = 0$$

Thus, for firm G, the FOC is

$$\frac{\partial}{\partial y_G}\left((2\underline{\Psi} - (y_B - y_G) + \underline{\Delta}_0)^2 - 9\underline{\Psi}\kappa\,(y_G)^2\right) =$$
$$4\underline{\Psi} + 2\Delta_0 - 2y_B + 2y_G - 18\Psi y_G = 0$$

The SOC for both firms is

$$\kappa > \frac{1}{9\underline{\Psi}}. \tag{25}$$

The solution of the system of FOCs is

$$y_B = \frac{1}{3}\frac{3\kappa\underline{\Psi} - 3\kappa\underline{\Delta}_0 - 1}{\kappa\,(9\kappa\underline{\Psi} - 2)}$$
$$y_G = \frac{1}{3}\frac{6\kappa\underline{\Psi} + 3\kappa\underline{\Delta}_0 - 1}{\kappa\,(9\kappa\underline{\Psi} - 2)}$$

Inserting $\underline{\Psi} := \underline{\Delta}_1 - \underline{\Delta}_0$ and rearranging gives the expressions in the proposition.
These expressions are both positive if

$$\kappa > \frac{2}{9\underline{\Psi}} = \frac{2}{9\left(\underline{\Delta}_1 - \underline{\Delta}_0\right)} \tag{26}$$
$$\kappa > \frac{1}{3(\underline{\Psi} - \underline{\Delta}_0)} = \frac{1}{3(\underline{\Delta}_1 - 2\underline{\Delta}_0)} \tag{27}$$
$$\kappa > \frac{1}{6\underline{\Psi} + 3\underline{\Delta}_0} = \frac{1}{3\,(2\underline{\Delta}_1 - \underline{\Delta}_0)} \tag{28}$$

The first of these conditions also makes sure that the SOC (25) holds, so that the solution of the FOCs is an equilibrium with positive investments of both firms if conditions (26) – (28) all hold. If $\underline{\Delta}_1 > -\underline{\Delta}_0$, then Assumption 5 corresponds to (28), and this condition implies (26) and (27). If $\underline{\Delta}_1 < -\underline{\Delta}_0$, then Assumption 5 corresponds to (27), and this condition implies (26) and (28). This proves the result.

A.4.2 Proof of Corollary 4

(i) The results follow from

$$\frac{\partial y_B^*}{\partial \underline{\Delta}_0} := \frac{\partial}{\partial \underline{\Delta}_0} \frac{1}{3} \frac{6\kappa\underline{\Delta}_0 - 3\kappa\underline{\Delta}_1 + 1}{\kappa\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)} = \frac{1 - 9\kappa\underline{\Delta}_1}{\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)^2}$$
$$\frac{\partial y_B^*}{\partial \underline{\Delta}_1} := \frac{\partial}{\partial \underline{\Delta}_1} \frac{1}{3} \frac{6\kappa\underline{\Delta}_0 - 3\kappa\underline{\Delta}_1 + 1}{\kappa\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)} = \frac{9\kappa\underline{\Delta}_0 + 1}{\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)^2}$$

(ii) The results follow from

$$\frac{\partial y_G^*}{\partial \underline{\Delta}_0} := \frac{\partial}{\partial \underline{\Delta}_0} \frac{1}{3} \frac{3\kappa\underline{\Delta}_0 - 6\kappa\underline{\Delta}_1 + 1}{\kappa\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)} = \frac{9\kappa\underline{\Delta}_1 - 1}{\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)^2}$$
$$\frac{\partial y_G^*}{\partial \underline{\Delta}_1} := \frac{\partial}{\partial \underline{\Delta}_1} \frac{1}{3} \frac{3\kappa\underline{\Delta}_0 - 6\kappa\underline{\Delta}_1 + 1}{\kappa\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)} = -\frac{9\kappa\underline{\Delta}_0 + 1}{\left(9\kappa\underline{\Delta}_0 - 9\kappa\underline{\Delta}_1 + 2\right)^2}$$

(iii) The equilibrium investment difference is given as

$$y_G^* - y_B^* = \frac{\underline{\Delta}_0 + \underline{\Delta}_1}{9\kappa(\underline{\Delta}_1 - \underline{\Delta}_0) - 2} \tag{29}$$

Taking derivatives gives the result.

A.4.3 Proof of Corollary 6

Using (10) and $\Delta_C = 0$, iso-output lines are of the form $x_G = \frac{2\underline{\Delta}_1 - \underline{\Delta}_0}{3(\underline{\Delta}_1 - \underline{\Delta}_0)}$ or, equivalently,

$$\Delta_1 = \frac{\Delta_0(1 - 3x_G)}{2 - 3x_G}. \tag{30}$$

Inserting this expression into the investment difference (29) gives

$$\Delta_y = 3\Delta_0 \frac{2x_G - 1}{4 - 6x_G + 9k\Delta_0}. \tag{31}$$

Taking the derivative with respect to Δ_0 immediately gives the result for market shares. The result for emissions follows directly from (1).

A.5 General Model

Output effects: Propositions 4 and 5, which guarantee regularity, rely on the following simple relations that hold for an arbitrary parameter ϕ_i, for which demand functions are differentiable and for $i, j \in \{1, 2\}$ and $j \neq i$,

$$\frac{dx_i^*}{d\phi_i} = \frac{\partial x_i^*}{d\phi_i} + \frac{\partial x_i^*}{\partial p_i}\frac{\partial p_i^*}{\partial \phi_i} + \frac{\partial x_i^*}{\partial p_j}\frac{\partial p_j^*}{\partial \phi_i}$$

$$\frac{dx_j^*}{d\phi_i} = \frac{\partial x_j^*}{d\phi_i} + \frac{\partial x_j^*}{\partial p_i}\frac{\partial p_i^*}{d\phi_i} + \frac{\partial x_j^*}{\partial p_k}\frac{\partial p_k^*}{d\phi_i} \tag{33}$$

In general, therefore, each parameter change can thus have a direct effect (the first term on the r.h.s) and two indirect, price-mediated effects. For $\phi_i = -c_i$, however, the term $\frac{\partial x_j^*}{d\phi_i}$ disappears.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): "The environment and directed technical change," *American Economic Review*, 102, 131–166.

Aghion, P., A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen (2016): "Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry," *Journal of Political Economy*, 124, 1–51.

Arora, S. and S. Gangopadhyay (1995): "Toward a theoretical model of voluntary overcompliance," *Journal of Economic Behavior & Organization*, 28, 289–309.

Athey, S. and A. Schmutzler (1995): "Product and process flexibility in an innovative environment," *The RAND Journal of Economics*, 557–574.

Bagwell, K. and R. W. Staiger (1994): "The sensitivity of strategic and corrective R&D policy in oligopolistic industries," *Journal of International Economics*, 36, 133–150.

Bansal, S. (2008): "Choice and design of regulatory instruments in the presence of green consumers," *Resource and Energy economics*, 30, 345–368.

Bansal, S. and S. Gangopadhyay (2003): "Tax/subsidy policies in the presence of environmentally aware consumers," *Journal of Environmental Economics and Management*, 45, 333–355.

Barnett, A. H. (1980): "The Pigouvian tax rule under monopoly," *The American Economic Review*, 70, 1037–1041.

Biglaiser, G. and J. K. Horowitz (1994): "Pollution regulation and incentives for pollution-control research," *Journal of Economics & Management Strategy*, 3, 663–684.

Buchanan, J. M. (1969): "External diseconomies, corrective taxes, and market structure," *The American Economic Review*, 59, 174–177.

Buchanan, J. M. and W. Stubblebine (1962): "Externality. Classic Papers in Natural Resource Economics," .

Calel, R. and A. Dechezleprêtre (2016): "Environmental policy and directed technological change: evidence from the European carbon market," *Review of Economics and Statistics*, 98, 173–191.

Carraro, C. and D. Siniscalco (1994): "Environmental policy reconsidered: The role of technological innovation," *European Economic Review*, 38, 545–554.

Carraro, C. and A. Soubeyran (1996): "Environmental taxation, market share, and profits in oligopoly," *Environmental Policy and Market Structure*, 23–44.

Cremer, H. and J.-F. Thisse (1999): "On the taxation of polluting products in a differentiated industry," *European Economic Review*, 43, 575–594.

Downing, P. B. and L. J. White (1986): "Innovation in pollution control," *Journal of Environmental Economics and Management*, 13, 18–29.

Forsythe, C. R., K. T. Gillingham, J. J. Michalek, and K. S. Whitefoot (2023): “Technology Advancement is Driving Electric Vehicle Adoption,” *Proceedings of the National Academy of Science*, 120, e2219396120.

Hassler, J., P. Krusell, and C. Olovsson (2021): “Directed technical change as a response to natural resource scarcity,” *Journal of Political Economy*, 129, 3039–3072.

Holland, S. P., E. T. Mansur, and A. J. Yates (2021): “The Electric Vehicle Transition and the Economics of Banning Gasoline Vehicles,” *American Economic Journal: Economic Policy*, 13, 316–344.

Hotelling, H. (1929): “Stability in Competition,” *Economic Journal*, 39, 41–57.

Johnstone, N., I. Hascic, and D. Popp (2008): “Renewable energy policies and technological innovation: evidence based on patent counts,” Tech. rep., National Bureau of Economic Research.

Katsoulacos, Y. and A. Xepapadeas (1995): “Environmental policy under oligopoly with endogenous market structure,” *The Scandinavian Journal of Economics*, 411–420.

——— (1996): *Environmental innovation, spillovers and optimal policy rules*, Springer.

Kennedy, P. and B. Laplante (1999): “Environmental Policy and Time Consistency: Emission Taxes and Emissions Trading,” in *Environmental Regulation and Market Power: Competition, Time Consistency and International Trade*, ed. by E. Petrakis, E. Sartzetakis, and A. Xepapadeas, Edward Elgar, 116–144.

Lange, A. and T. Requate (1999): “Emission taxes for price-setting firms: differentiated commodities and monopolistic competition,” in *Environmental Regulation and Market Power*, Edward Elgar Publishing, 1–26.

Langinier, C. and A. R. Chaudhuri (2020): “Green technology and patents in the presence of green consumers,” *Journal of the Association of Environmental and Resource Economists*, 7, 73–101.

Leahy, D. and J. P. Neary (1997): “Public policy towards R&D in oligopolistic industries,” *American Economic Review*, 642–662.

Lombardini-Riipinen, C. (2005): “Optimal tax policy under environmental quality competition,” *Environmental and Resource Economics*, 32, 317.

Malueg, D. A. (1989): “Emission credit trading and the incentive to adopt new pollution abatement technology,” *Journal of Environmental Economics and Management*, 16, 52–57.

Milliman, S. R. and R. Prince (1989): “Firm incentives to promote technological change in pollution control,” *Journal of Environmental Economics and Management*, 17, 247–265.

Moraga-Gonzalez, J. L. and N. Padron-Fumero (2002): “Environmental policy in a green market,” *Environmental and Resource Economics*, 22, 419–447.

Newell, R. G., A. B. Jaffe, and R. N. Stavins (2010): "The induced innovation hypothesis and energy-saving technological change," in *Technological Change and the Environment*, Routledge, 103–132.

Noailly, J. and R. Smeets (2015): "Directing technical change from fossil-fuel to renewable energy innovation: An application using firm-level patent data," *Journal of Environmental Economics and Management*, 72, 15–37.

Parry, I. W. (1995): "Optimal pollution taxes and endogenous technological progress," *Resource and Energy Economics*, 17, 69–85.

Phaneuf, D. J. and T. Requate (2016): *A course in environmental economics: theory, policy, and practice*, Cambridge University Press.

Popp, D. (2010): "Innovation and climate policy," *Annual Review of Resource Economics*, 2, 275–298.

——— (2019): "Environmental policy and innovation: a decade of research," .

Popp, D., J. Pless, I. Haščič, N. Johnstone, et al. (2020): "Innovation and entrepreneurship in the energy sector," Tech. rep., National Bureau of Economic Research.

Requate, T. (2006): "Environmental policy under imperfect competition," *The International Yearbook of Environmental and Resource Economics*, 2007, 120–207.

Requate, T. and W. Unold (2003): "Environmental policy incentives to adopt advanced abatement technology:: Will the true ranking please stand up?" *European Economic Review*, 47, 125–146.

Rogge, K. S., M. Schneider, and V. H. Hoffmann (2011): "The innovation impact of the EU Emission Trading System â Findings of company case studies in the German power sector," *Ecological Economics*, 70, 513–523.

Schmitt, S. Y. (2024): "Investments in Environmental Quality under Limited Attention," Working paper, University of Bamberg.

Schmutzler, A. (2013): "Competition and investment: A unified approach," *International Journal of Industrial Organization*, 31, 477–487.

——— (2024): "Hotelling meets Shaked and Sutton: A Unified Linear Model of Product Differentiation," Working paper, University of Zurich.

Sengupta, A. (2012): "Investment in cleaner technology and signaling distortions in a market with green consumers," *Journal of Environmental Economics and Management*, 64, 468–480.

Shaked, A. and J. Sutton (1982): "Relaxing price competition through product differentiation," *The Review of Economic Studies*, 49, 3–13.