B Supplementary Material for "Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Boom"

B.1 Additional Proofs for the Baseline Model

B.1.1 Proofs of Propositions A.1 and A.2

To prove these results, we start by defining the function $I(s) \equiv (\varepsilon-1)(1-s)^{1-\psi}+s^{1-\psi}-\varepsilon\frac{\eta_B}{\eta}$ and characterize its zeros in the following two Lemmas.

Lemma B.1 *Assume that $\varepsilon \geq 2^{1-\psi}$. Over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}},1\right)$, the function $I(s)$ has:*

1. *no zero if $\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}$;*
2. *one zero with $\frac{1}{\varepsilon}<\frac{\eta_B}{\eta}<\frac{1}{2^{1-\psi}}$ and this zero satisfies $I'(s^*)<0$;*
3. *no zero if $\frac{\eta_B}{\eta}>\frac{1}{2^{1-\psi}}$ and i) $\varepsilon\geq 2$ or ii) $\frac{\eta_B}{\eta}>\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$;*
4. *two zeros if $\frac{1}{2^{1-\psi}}<\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and $\varepsilon<2$, the first zero satisfies $I'\left(s_1^*\right)>0$ and the second zero satisfies $I'\left(s_2^*\right)<0$.*

Proof. Differentiating $I(s)$, we obtain

$$I'(s)=\left(s^{-\psi}-(\varepsilon-1)(1-s)^{-\psi}\right)(1-\psi), \tag{B-1}$$

$$I''(s)=-\psi\left(s^{-\psi-1}+(\varepsilon-1)(1-s)^{-\psi-1}\right)(1-\psi)<0.$$

Therefore the function I is concave in s and always decreasing in s for s large enough (since $I'(1)=-\infty$).

Further, at the boundaries of the interval, one gets:

$$I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)=(\varepsilon-1)\left[\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}-\frac{\eta_B}{\eta}\right],$$

and we get that $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)>0$ if and only if $\frac{\eta_B}{\eta}<\frac{1}{2^{1-\psi}}$. In addition $I(1)=1-\varepsilon\frac{\eta_B}{\eta}$, and we obtain that $I(1)>0$ if and only if $\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}$. Since $\varepsilon>2^{1-\psi}$, we get that $I(1)>0\Rightarrow$

$I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 0$. As I is concave, it has no zeros for $\varepsilon < 1/2^{1-\psi}$. This establishes part 1 of Lemma B.1.

Assume now that $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$, then $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 0$ but $I(1) < 0$, since I is concave, then I has only 1 zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ and this zero features $I'(s^*) < 0$. This establishes part 2 of Lemma B.1.

Consider now the case where $\frac{\eta_B}{\eta} > \frac{1}{2^{1-\psi}}$, so that $I(1) < 0$ and $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$. Then either I has 2 zeros (one for I increasing and one for I decreasing) or I has no zero. First, note that I is decreasing on $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ if $I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$. In that case, we have

$$\begin{aligned} I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) &< 0 \iff \left(\frac{\eta_B}{\eta}\right)^{\frac{-\psi}{1-\psi}} - (\varepsilon-1)\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)^{-\psi} < 0 \\ &\iff \frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}. \end{aligned}$$

If $\varepsilon \geq 2$, then $\frac{1}{2^{1-\psi}} \geq \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$ so that $\frac{\eta_B}{\eta} > \frac{1}{2^{1-\psi}} \Rightarrow \frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Then, I has no zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$. This establishes part 3i) of Lemma B.1.

We now consider the case where $\varepsilon < 2$, and $\frac{\eta_B}{\eta} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$, then I has a maximum, which is reached at $s = \tilde{s}$, where $\tilde{s}$ solves $I'(\tilde{s}) = 0$. Using (B-1), we get $\tilde{s} = \left[1+(\varepsilon-1)^{\frac{1}{\psi}}\right]^{-1}$ and

$$I(\tilde{s}) = \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} - \varepsilon\frac{\eta_B}{\eta}.$$

Therefore,

$$I(\tilde{s}) > 0 \iff \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}.$$

We note that when $\varepsilon < 2$, $\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$, so that $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ immediately implies $\frac{\eta_B}{\eta} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Therefore, if $I(\tilde{s}) > 0$, then I will have two zeros, the first one when I is increasing and the second one when I is decreasing. This establishes part 4) of Lemma B.1. Finally, if instead, $I(\tilde{s}) < 0$, then $I(s)$ will have no zeros, establishing part 3ii) of Lemma B.1. Note that $\frac{1}{2^{1-\psi}} \leq \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ for all ε with strict inequality unless $\varepsilon = 2$, therefore the interval $\left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}\right)$ is non-empty for $\varepsilon \neq 2$. ■

We establish a similar Lemma for the case $\varepsilon < 2^{1-\psi}$.

Lemma B.2 *Assume that* $\varepsilon < 2^{1-\psi}$*. Over the interval* $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$*, the function* $I(s)$ *has:*

1. *no zeros if* $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$*;*
2. *one zero with* $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$ *and this zero satisfies* $I'(s^*) > 0$*;*
3. *two zeros if* $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$*, the first zero satisfies* $I'(s_1^*) > 0$ *and the second zero satisfies* $I'(s_2^*) < 0$*.*
4. *no zero if* $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$*;*

Proof. The proof is similar to the previous case. With $\varepsilon < 2^{1-\psi}$, $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 0 \Rightarrow I(1) > 0$. Since I is concave, it has no zeros for $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$, which establishes part 1.

Assume now that $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$, then $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$ but $I(1) > 0$, since I is concave, then I has only 1 zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ and this zero features $I'(s^*) > 0$. This establishes part 2.

Consider now the case where $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}$. As in the previous proof, $I(1) < 0$ and $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$, so I has either 2 zeros (one for I increasing and one for I decreasing) or I no zero. I is decreasing on $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ if $I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$, which is equivalent to $\frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Otherwise, I has a maximum $\tilde{s} = \left[1+(\varepsilon-1)^{\frac{1}{\psi}}\right]^{-1}$ and we still get that $I(\tilde{s}) > 0 \iff \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$. With $\varepsilon < 2^{1-\psi}$, then we always have that $\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. We can then consider two cases: $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$. In the former case, I has 2 zeros, in the latter I has no zero (since either I decreases or its maximum is negative). This establishes parts 3) and 4). ■

We now establish Propositions A.1 and A.2. To do that, we derive the respective conditions under which each type of asymptotic equilibrium exists. Using (21), the allocation of innovation follows:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} = \frac{\kappa_g^{\varepsilon} A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}\left(\frac{1}{A_{ct}}+\frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{1}{A_{st}}\kappa_d^{\varepsilon}\left(\frac{1}{A_{st}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}. \tag{B-2}$$

Corner Asymptotic Steady State with Clean Innovation. In an asymptotic steady state where $s_{gt} \to 1$, the (B-2) grows without bonds, which in turn confirms the corner allocation for innovation. Therefore such a steady state is always possible and occurs whenever A_{g0} is sufficiently large relative to the fossil-fuel technologies.

Corner Asymptotic Steady State with Fossil-Fuel Innovation. Alternatively, consider a steady state where $s_{ft} \to 1$. Then (B-2) implies that:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} = O\left(\frac{A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}B_{ct}^{\varepsilon} + \frac{1}{A_{st}}\kappa_d^{\varepsilon}B_{st}^{\varepsilon}}\right).$$

The LHS tends toward 0 and the RHS tends toward 0 only if $B_{ct}^{\varepsilon}/A_{ct}$ grows without bound (knowing that $B_{st}^{\varepsilon}/A_{st}$ behaves similarly). This occurs if $\varepsilon\eta_B > \eta$. Therefore, we get that for $\eta_B/\eta < 1/\varepsilon$, an asymptotic steady state where all innovation occurs in the fossil-fuel technologies cannot exist. In contrast, such an asymptotic steady state occurs for $\eta_B/\eta > 1/\varepsilon$ provided that A_{g0} is sufficiently small.

Interior Asymptotic Steady State. We now analyze whether an interior asymptotic steady state is possible. There are three possible cases: A_{ct} grows faster, at the same rate or less fast than B_{ct}.

Assume first that A_{ct} grows less fast than B_{ct} (that is, $\eta\left(s_{ft}^*\right)^{1-\psi} < \eta_B$ where s_{ft}^* is the limit of s_{ft}). Then (B-2) implies that

$$\left(\frac{s_g^*}{s_f^*}\right)^{\psi} \sim \frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}\kappa_c^{\varepsilon} + A_{st}^{\varepsilon-1}\kappa_d^{\varepsilon}}. \tag{B-3}$$

The RHS can only converge asymptotically to a constant if A_{gt} and A_{ct} grow at the same rate in the long-run. This is possible only if $s_f^* = s_g^* = 1/2$, which combined with condition $\eta\left(s_{ft}^*\right)^{1-\psi} < \eta_B$, requires that $\eta_B/\eta > 2^{\psi-1}$. In addition, if $A_{g(t-1)}$ is shocked in such a way that the RHS in (B-3) increases, then s_{gt} should increase as well: so that the interior asymptotic state can only exist in a knife-edge case and it is unstable.

The case where A_{ct} and B_{ct} grow at the same rate follows the same logic since in that case (B-3) still holds up to a constant. We must then have $s_f^* = 1/2$ and $\eta\left(s_{ft}^*\right)^{1-\psi} = \eta_B$, which can only occur for $\eta_B = \eta 2^{\psi-1}$. Again this interior steady state will always be unstable.

Consider now the case where A_{ct} grows faster than B_{ct} (that is $\eta\left(s_{ft}^*\right)^{1-\psi} > \eta_B$). Then (B-2) implies that

$$\left(\frac{s_{gt}^*}{s_{ft}^*}\right)^{\psi} \sim \frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}B_{ct}^{\varepsilon} + \frac{1}{A_{st}}\kappa_d^{\varepsilon}B_{st}^{\varepsilon}},$$

which is possible only if the RHS tends toward a constant. This implies that s_{ft}^* must also satisfy $I\left(s_{ft}^*\right) = \varepsilon\eta_B/\eta$. An interior steady state will therefore exist if $I\left(s_{ft}^*\right) = 0$ has a solution in the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$. That steady state will be unstable if $I'\left(s^*_{ft}\right) < 0$ since then a shock leading to a temporarily higher s_{ft} is associated with permanently higher s_{ft}. The steady state will be stable if $I'\left(s^*_{ft}\right) > 0$.

Lemma B.1 immediately characterizes the conditions under which this case occurs for $\varepsilon \geq 2^{1-\psi}$ and we get that:

1) There is no interior asymptotic steady state if $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$;

2) There is one unstable interior asymptotic steady state if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta}$ and i) $\varepsilon \geq 2$ or ii) $\varepsilon < 2$ and $\frac{\eta_B}{\eta} \notin \left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$;

3) There are two unstable interior asymptotic steady states and one stable interior asymptotic steady state if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta}$, $\varepsilon < 2$ and $\frac{\eta_B}{\eta} \in \left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$.

Similarly, Lemma B.2 characterizes the conditions under which $I\left(s^*_{ft}\right) = 0$ has a solution in $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ for $\varepsilon < 2^{1-\psi}$ and we get that:

1) There is no interior asymptotic steady state if $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$;

2) There is one unstable interior asymptotic steady state if $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$ and a stable interior asymptotic steady state;

3) There are two unstable interior asymptotic steady states if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and a stable asymptotic steady state;

4) There is one unstable interior asymptotic steady state if $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$.

Conclusion. Bringing together the three cases establishes Propositions A.1 and A.2.

B.1.2 Complement to Proposition 5

In this Appendix, we complement Proposition 5 by showing that under the same conditions but when $A_{g0} > \overline{A_{g0}}$, the natural gas boom decreases welfare provided that $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently large, that φ_L is small and that φ_D is large as mentioned in the text.

Proof. In that case, the economy is on a green path whether the boom occurred or not. From Proposition 4, however, we get that emissions are lower without the boom for t large enough. Therefore, if the stock of carbon depends mostly on current emissions (which is the case when φ_L is sufficiently small and φ is sufficiently large enough), then S_t is lower without the boom for t large enough (though in both cases, S_t tends toward a constant). In addition, since innovation is reallocated away from clean technologies, A_{gt} is lower with the boom than without. Therefore, for t sufficiently large, we obtain that C_{Et} is also lower with the boom than without. As a result, for t large enough output is lower with the boom than without.

For T large but finite, Y_t grows approximately at the rate $\gamma^{\eta}-1$. Using (A-4), we can then write the change in welfare following (a small) boom as:

$$dU \approx \sum_{\tau=0}^{T-1} \frac{1}{(1+\rho)^{\tau}} \frac{d(Y_{\tau})^{1-\vartheta}}{1-\vartheta} + \sum_{\tau=T}^{\infty} \frac{Y_T^{1-\vartheta}}{(1+\rho)^T} \left(\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}\right)^{(\tau-T)} \left(\frac{\nu^{\lambda}\widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1} d\ln C_{E\tau}}{(1-\nu)^{\lambda} A_{P\tau}^{\lambda-1} + \nu^{\lambda}\widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1}} - \zeta dS_{\tau} \right).$$

As argued above, for T large $dS_{\tau} > 0$. Furthermore, $d\ln C_{E\tau} \approx d\ln A_{g\tau} = \sum_{u=0}^{\tau} \eta(1-\psi) s_{gu}^{-\psi} ds_{gu}$ where all $ds_{gu} < 0$, so that $d\ln C_{E\tau}$ is negative and bounded away from 0. Therefore, the second sum becomes arbitrarily large if $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently close to 1. The latter condition is met when ρ is sufficiently small and $\vartheta \leq 1$ for instance. ■

B.1.3 Proof of Proposition 6

Anticipating that the social planner allocates labor symmetrically within intermediates and that she maintains the equality $E_{it} = Q_{it}$, we can write the social planner problem as maximizing

$$U_0 = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{Y_t^{1-\vartheta}}{1-\vartheta}, \tag{B-4}$$

subject to the final good equation (2) with Lagrange parameter λ_t, the energy equation (3) with Lagrange parameter λ_{Et},

$$\lambda_{it} : E_{it} = C_{it} L_{it},$$

where for this equation and the following ones the term before the : is the associated Lagrange parameter,

$$\lambda_{Pt} : Y_{Pt} = A_{Pt} L_{Pt}$$

$$\lambda_{Lt} : L_{ct} + L_{st} + L_{gt} + L_{Pt} = L$$

$$\mu_{ct} : A_{ct} = \gamma^{\eta s_{ft}^{1-\psi}} A_{c(t-1)},\ \mu_{st} : A_{st} = \gamma^{\eta s_{ft}^{1-\psi}} A_{s(t-1)} \text{ and } \mu_{gt} : A_{gt} = \gamma^{\eta s_{gt}^{1-\psi}} A_{g(t-1)},$$

$$\chi_t : s_{ft} + s_{gt} = 1,$$

$$\omega_{Pt} : \xi_c E_{ct} + \xi_s E_{st} = P_t,$$

$$\omega_{St}: S_t = \overline{S} + \sum_{s=0}^{t+T} (\varphi_L + (1-\varphi_L)\varphi_0 (1-\varphi_d)^s) P_{t-s}.$$

The first order condition with respect to Y_t imposes that λ_t be equal to the marginal value of consumption at time t,

$$\lambda_t = \frac{Y_t^{-\vartheta}}{(1+\rho)^t}.$$

The first order condition with respect to Y_{Pt} ensures that $\frac{\partial Y_t}{\partial Y_{Pt}} = \frac{\lambda_{Pt}}{\lambda_t} \equiv p_{Pt}$, where the ratio λ_{Pt}/λ_t is the shadow price of the production input. Similarly, the first order condition with respect to E_t implies $\frac{\partial Y_t}{\partial Y_{Et}} = \frac{\lambda_{Et}}{\lambda_t} \equiv p_{Et}$. The first order condition with respect to E_{gt} implies $\lambda_{Et}\frac{\partial E_t}{\partial E_{gt}} = \lambda_{gt}$, so that $\frac{\partial E_t}{\partial E_{gt}} = \frac{\lambda_{gt}}{\lambda_t} = p_{gt}$. The first order condition with respect to Y_{ct} gives

$$\lambda_{Et}\frac{\partial E_t}{\partial E_{ct}} = \lambda_{ct} + \xi_c \omega_{Pt} \Longrightarrow \frac{\partial E_t}{\partial E_{ct}} = p_{ct} + \xi_c \tau_t,$$

with $p_{ct} \equiv \lambda_{ct}/\lambda_t$ being the shadow producer price of coal-based energy and $\tau_t = \omega_{Pt}/\lambda_t$ being the shadow price of emissions. Similarly, we have

$$\frac{\partial E_t}{\partial E_{st}} = p_{st} + \xi_s \tau_t.$$

First order conditions with respect to L_{it} for $i = c, s, g$ yield $p_{it}\partial E_{it}/\partial L_{it} = \lambda_{Lt}/\lambda_t \equiv w_t$ which is the shadow wage and similarly, $p_{Pt}\partial Y_{Pt}/\partial L_{Pt} = w_t$. Therefore, and unsurprisingly, the static optimal allocation is identical to the decentralized allocation provided that there is a carbon tax given by τ_t. Note that there is no monopoly distortion to be addressed because all sectors are equally affected and there is no roundabout production (yet the shadow wage differs from the decentralized wage by a constant).

The first order condition with respect to S_t yields

$$\omega_{St} = \lambda_t \zeta Y_t, \tag{B-5}$$

whereas the first order condition with respect to P_t implies:

$$\omega_{Pt} = \sum_{s=0}^{\infty} (\varphi_L + (1-\varphi_L)\varphi_0 (1-\varphi_d)^s) \omega_{St+s}.$$

We can rewrite this as

$$\tau_t = Y_t^{\vartheta} \sum_{s=0}^{\infty} \frac{(\varphi_L + (1-\varphi_L)\varphi_0 (1-\varphi_d)^s)}{(1+\rho)^s} \zeta Y_{t+s}^{1-\vartheta}. \tag{B-6}$$

If $\vartheta = 1$, we obtain the closed form solution of Golosov et al. (2014), namely $\tau_t = Y_t \gamma (1+\rho) \left(\frac{\varphi_L}{\rho} + \frac{(1-\varphi_L)\varphi_0}{\rho+\varphi_d} \right)$.

The first order conditions with respect to A_{it} for $i = c, s$ yield

$$\mu_{it} = \lambda_{it} \left(\frac{C_{it}}{A_{it}} \right)^2 L_{it} + \gamma^{\eta_f s_{f(t+1)}^{1-\psi}} \mu_{i(t+1)}.$$

Multiply by A_{it} and iterate forward to get

$$\mu_{it} A_{it} = \lambda_{it} \frac{C_{it}}{A_{it}} E_{it} + A_{it+1} \mu_{i(t+1)} = \sum_{s=0}^{\infty} \lambda_{it+s} \frac{C_{it+s}}{A_{it+s}} E_{it+s}.$$

The first order condition with respect to A_{gt} gives

$$\mu_{gt} = \lambda_{gt} L_{gt} + \gamma^{\eta s_{g(t+1)}^{1-\psi}} \mu_{g(t+1)},$$

which similarly leads to

$$\mu_{gt} A_{gt} = \sum_{s=0}^{\infty} \lambda_{gt+s} E_{gt+s}.$$

The first order conditions with respect to s_{ft} and s_{gt} imply

$$(1-\psi) \ln(\gamma) s_{ft}^{-\psi} (\mu_{ct} A_{ct} + \mu_{st} A_{st}) = \chi_t = (1-\psi) \ln(\gamma) s_{gt}^{-\psi} \mu_{gt} A_{gt}.$$

Therefore the innovation allocation obeys

$$\left(\frac{s_{ft}}{s_{gt}} \right)^{\psi} = \frac{\mu_{ct} A_{ct} + \mu_{st} A_{st}}{\mu_{gt} A_{gt}} = \frac{\sum_{s=0}^{\infty} \frac{1}{1+r_{t,t+s}} \left(\frac{C_{c(t+s)}}{A_{c(t+s)}} p_{c(t+s)} E_{c(t+s)} + \frac{C_{cs(t+s)}}{A_{s(t+s)}} p_{s(t+s)} E_{s(t+s)} \right)}{\sum_{s=0}^{\infty} \frac{1}{1+r_{t,t+s}} p_{g(t+s)} E_{g(t+s)}},$$

where $r_{t,t+s} = \lambda_t / \lambda_{t+s} - 1$ is the shadow interest rate between t and $t+s$. At the optimum, the allocation of innovation depends on the ratio of the social values of innovation in each sector. These social values are equal to the discounted sum of the marginal benefit of innovation in all future periods. This contrasts with the decentralized economy where the allocation of innovation is given by:

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}=\frac{\frac{C_{ct}}{A_{ct}}p_{ct}E_{ct}+\frac{C_{st}}{A_{st}}p_{st}E_{st}}{p_{gt}E_{gt}},$$

including in the presence of the carbon tax (since p_{ct} and p_{st} are pre-tax producer prices of energy). The optimal scientist allocation can be decentralized through research subsidies.

In the quantitative analysis, we add an exogenous path of emissions from the rest of the world P_t^{ROW} and direct disutility costs from carbon concentration on utility (to capture the effect of climate change on the rest of the world), see (25). The former does not affect our analysis, whereas the latter simply turns (B-5) into $\omega_{St}=\lambda_t\frac{D'(S_t)}{1-D(S_t)}Y_t-\frac{\nu'(S_t)}{(1+\rho)^t}$, so that we get $\tau_t=Y_t^{\vartheta}\sum_{s=0}^{\infty}\frac{(\varphi_L+(1-\varphi_L)\varphi_0(1-\varphi_d)^s)}{(1+\rho)^s}\left(\zeta Y_{t+s}^{1-\vartheta}-\nu'(S_{t+s})\right)$ instead of (B-6).

B.1.4 Proof of Proposition 7

We prove Proposition 7 and also establish that the shale gas boom decreases welfare provided that $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently large, that φ_L is sufficiently small and that φ_D is sufficiently large.

Proof of Part 1). With $\varepsilon\geq 2$, Proposition A.1 applies and establishes that for $\eta_B<\eta/\varepsilon$, the economy converges toward a green path, so that $s_{gt}\to 1$ and t_{switch} must be finite. We then show that from t_{switch} onward, green innovation increases over time. Using the notation f_t introduced in Appendix A.4, we get:

$$f_{t+1}(s_{gt})=\frac{\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{\gamma^{-\eta_B}}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{\gamma^{-\eta_B}}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{2\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi}.$$

Assume that $s_{gt}\geq 1/2$ and that $\eta s_{ft}^{1-\psi}>\eta_B$ then

$$\begin{aligned}&f_{t+1}(s_{gt})\\ =\;&\gamma^{(\varepsilon-1)\eta\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)}\frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}-\eta_B}}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}-\eta_B}}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi}\\ <\;&f_t(s_{gt})=1,\end{aligned}$$

therefore $s_{g(t+1)}>s_{gt}$.

Assume now that $s_{gt}\geq 1/2$ but that $\eta s_{ft}^{1-\psi}\leq\eta_B$, then:

$$\begin{aligned} f_{t+1}\left(s_{gt}\right) &= \gamma^{\varepsilon\eta_B}\frac{\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{\eta_B-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{\eta_B-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{2\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \\ &\leq \gamma^{\varepsilon\eta_B-\eta s_{ft}^{1-\psi}-\eta(\varepsilon-1)s_{gt}^{1-\psi}}\frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi}. \end{aligned}$$

We wish to establish that $\varepsilon\eta_B-\eta s_{ft}^{1-\psi}-\eta(\varepsilon-1)s_{gt}^{1-\psi}<0$. To do so, we define $h(s)\equiv\varepsilon\eta_B-\eta s^{1-\psi}-\eta(\varepsilon-1)(1-s)^{1-\psi}$. Twice differentiating h, one gets $h''(s)>0$, so that h is convex. Furthermore $h(0)=\varepsilon\eta_B-\eta(\varepsilon-1)$. Since $\varepsilon\geq 2$, $\frac{\varepsilon-1}{\varepsilon}\geq\frac{1}{\varepsilon}$, so that $\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}\leq\frac{\varepsilon-1}{\varepsilon}$, which ensures that $h(0)<0$. In addition, $h\left(\frac{1}{2}\right)=\varepsilon\left(\eta_B-\eta 2^{\psi-1}\right)<0$ since $\eta_B/\eta<1/\varepsilon$ and $\varepsilon\geq 2>2^{1-\psi}$. Therefore, $\varepsilon\eta_B-\eta s_{ft}^{1-\psi}-\eta(\varepsilon-1)s_{gt}^{1-\psi}<0$ when $s_{gt}\geq 1/2$. This ensures that $f_{t+1}\left(s_{gt}\right)<f_t\left(s_{gt}\right)$ so that $s_{g(t+1)}>s_{gt}$. This establishes Part 1).

Proof of Part 2). To prove Part 2, it suffices to show that an increase in B_{s0} leads to an increase in s_{gt} as long as $t\leq t_{switch}$. We define

$$\widehat{f}_t\left(s_{gt},s_{g(t-1)},\ldots,s_{g1},B_{s0}\right)\equiv\frac{s_{gt}^{\psi}}{\kappa_g^{\varepsilon}C_{g0}^{\varepsilon-1}s_{ft}^{\psi}\gamma^{\eta(\varepsilon-1)\sum_{\tau=1}^{t}s_{g\tau}^{1-\psi}}}\left(\begin{array}{c}\frac{\kappa_c^{\varepsilon}\gamma^{-\eta\sum_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}\left(\frac{\gamma^{-\eta\sum_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}\\+\frac{\kappa_s^{\varepsilon}\gamma^{-\eta\sum_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}\left(\frac{\gamma^{-\eta\sum_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}+\frac{1}{B_{st}}\right)^{-\varepsilon}\end{array}\right),$$

so that the equilibrium innovation allocation is still defined by $\widehat{f}_t\left(s_{gt},s_{g(t-1)},\ldots,s_{g1},B_{s0}\right)=1$ with $\widehat{f}_t$ increasing in s_{gt} and in B_{s0}. We obtain for $\widetilde{\tau}\in[1,t-1)$

$$\frac{\partial\ln\widehat{f}_t}{\partial\ln s_{g\widetilde{\tau}}}=\left[\frac{\begin{array}{c}\frac{\kappa_c^{\varepsilon}}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}\left(1-\varepsilon\frac{\frac{1}{A_{ct}}}{\frac{1}{A_{ct}}+\frac{1}{B_{ct}}}\right)\\+\frac{\kappa_s^{\varepsilon}}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}\left(1-\varepsilon\frac{\frac{1}{A_{st}}}{\frac{1}{A_{st}}+\frac{1}{B_{st}}}\right)\end{array}}{\frac{\kappa_c^{\varepsilon}}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}}s_{f\widetilde{\tau}}^{-\psi}-(\varepsilon-1)s_{g\widetilde{\tau}}^{-\psi}\right]s_{g\widetilde{\tau}}\eta(1-\psi)\ln\gamma.$$

Yet, if $t \leq t_{switch}$, then $s_{f\tilde{\tau}} \geq s_{g\tilde{\tau}}$, so that

$$\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\tilde{\tau}}} \leq -\left[\varepsilon - 2 + \varepsilon \frac{\frac{\kappa_c^\varepsilon}{A_{ct}^2}\left(\frac{1}{A_{ct}}+\frac{1}{B_{ct}}\right)^{-\varepsilon-1} + \frac{\kappa_s^\varepsilon}{A_{st}^2}\left(\frac{1}{A_{st}}+\frac{1}{B_{st}}\right)^{-\varepsilon-1}}{\frac{\kappa_c^\varepsilon}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{\kappa_s^\varepsilon}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}\right] s_{f\tilde{\tau}}^{-\psi} s_{g\tilde{\tau}} \eta (1-\psi) \ln \gamma.$$

Therefore $\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\tilde{\tau}}} < 0$ if $\varepsilon \geq 2$.

Therefore, the natural gas boom reduces $\widehat{f}_1$ leading to a lower value for s_{g1}. It then reduces $\widehat{f}_2$ both directly and because of its negative effect on s_{g1}, leading to a lower value for s_{g2}. By iteration, the natural gas boom will reduce all s_{gt} at least until the switch toward green innovation occurs.

Three Useful Lemmas. We establish three lemmas which are useful to prove part 3.

Lemma B.3 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d\ln A_{st_A} < 0$ and assume that $t_A < \infty$. Then $d\ln A_{gt_A} > d\ln A_{st_A}$.*

Proof. Noting that

$$\ln A_{ct} = \ln A_{c0} + \eta(\ln\gamma)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi} \text{ and } \ln A_{st} = \ln A_{s0} + \eta(\ln\gamma)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi},$$

we obtain

$$d\ln A_{ct} = d\ln A_{st} = \eta(1-\psi)(\ln\gamma)\sum_{\tau=1}^{t} s_{f\tau}^{-\psi} ds_{f\tau}. \qquad \text{(B-7)}$$

By definition of t_A, $d\ln A_{c(t_A-1)} > 0$ and $d\ln A_{ct_A} < 0$, so that we must have $ds_{ft_A} < 0$. Since $ds_{ft} > 0$ for $t \leq t_{switch}$, it must be that $t_A > t_{switch}$. We can similarly write

$$d\ln A_{gt} = -\eta(1-\psi)(\ln\gamma)\sum_{\tau=1}^{t} s_{g\tau}^{-\psi} ds_{f\tau}. \qquad \text{(B-8)}$$

Using (B-7) and (B-8), we get

$$d\ln A_{st_A} - d\ln A_{gt_A} = \eta(1-\psi)(\ln\gamma)\left(\sum_{\tau=1}^{t_A}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau}\right).$$

We know that $ds_{ft} > 0$ for $t \leq t_{switch}$ and that $ds_{ft_A} < 0$, therefore ds_{ft} must change sign as t increases at least once. We index the times where ds_{ft} switches signs by t_{2p} and t_{2p+1}, such that ds_{ft} becomes negative at t_{2p+1} and positive at t_{2p} and p is a weakly positive integer in the integer set $\{0,...,P-1\}$ with $P \geq 1$. We denote by $t_0 = t_{switch}$ and $t_{2P} = t_A + 1$. We get:

$$\begin{aligned}
& d\ln A_{st_A} - d\ln A_{gt_A} \qquad \text{(B-9)} \\
= \; & \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ + \sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \right) \end{array} \right) \\
= \; & \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}-2}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ + \sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(1 - \frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(1 - \frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau} \right) \end{array} \right).
\end{aligned}$$

Using that $s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} < 0$ for $\tau < t_{switch}$, that $\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}$ is decreasing for $\tau \geq t_{switch}$ (as established in the Proof of Part 1), that $ds_{f\tau} > 0$ only on intervals $\left[t_{2p}, t_{2p+1} - 1\right]$, we get

$$d\ln A_{st_A} - d\ln A_{gt_A} < \eta(1-\psi)(\ln\gamma)\sum_{p=0}^{P-1}\left(1 - \frac{s_{f\,t_{2p+1}}^{\psi}}{s_{g\,t_{2p+1}}^{\psi}}\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau}.$$

By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{f\tau}^{-\psi} ds_{f\tau} > 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$. Therefore, we get that

$$\begin{aligned}
& \sum_{p=P-2}^{P-1}\left(1 - \frac{s_{f\,t_{2p+1}}^{\psi}}{s_{g\,t_{2p+1}}^{\psi}}\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau} \\
= \; & \left(1 - \frac{s_{f\,t_{2P-3}}^{\psi}}{s_{g\,t_{2P-3}}^{\psi}}\right)\sum_{\tau=t_{2P-4}}^{t_{2P-2}-1} s_{f\tau}^{-\psi} ds_{f\tau} + \left(1 - \frac{s_{f\,t_{2P-1}}^{\psi}}{s_{g\,t_{2P-1}}^{\psi}}\right)\sum_{\tau=t_{2P-2}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \\
< \; & \left(1 - \frac{s_{f\,t_{2P-3}}^{\psi}}{s_{g\,t_{2P-3}}^{\psi}}\right)\sum_{\tau=t_{2P-4}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau}.
\end{aligned}$$

Iterating, we get

$$d\ln A_{st_A} - d\ln A_{gt_A} < \eta(1-\psi)(\ln\gamma)\left(1 - \frac{s_{f\,t_1}^{\psi}}{s_{g\,t_1}^{\psi}}\right)\sum_{\tau=t_{switch}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \leq 0.$$

Therefore $d\ln A_{gt_A} > d\ln A_{st_A}$. ■

We establish a symmetric lemma:

Lemma B.4 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d\ln A_{gt_A} > 0$ and assume that $t_A < \infty$. Then $d\ln A_{gt_A} > d\ln A_{st_A}$.*

Proof. The proof starts as for the previous lemma: $d\ln A_{gt_A} > 0$ requires that $ds_{ft_A} < 0$, which implies $t_A \geq t_{switch}$ and that ds_{ft} switches sign an odd number of times. We use (B-9) to write:

$$
\begin{aligned}
& d\ln A_{st_A} - d\ln A_{gt_A} \\
= \;& \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ +\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau}\right) \end{array}\right) \\
< \;& \eta(1-\psi)(\ln\gamma)\sum_{p=0}^{P-1}\left(\frac{s_{gt_{2p+1}}^{\psi}}{s_{ft_{2p+1}}^{\psi}}-1\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1}s_{g\tau}^{-\psi}ds_{f\tau},
\end{aligned}
$$

following the same logic as before. By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$, then for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{g\tau}^{-\psi}ds_{g\tau} < 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$. As $ds_{g\tau} = -ds_{f\tau}$, then $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} < 0$. Using the same reasoning as before, we get $d\ln A_{st_A} - d\ln A_{gt_A} < 0$. ■

We can then derive:

Lemma B.5 *For $\ln\gamma$ small, the shale gas boom increases A_{ct}, A_{st} and decreases A_{gt}.*

Proof. We prove this result by contradiction. Assume that A_{gt} does not decrease for all t following the shale gas boom. Denote by t_A the first time that $d\ln A_{gt} > 0$, then if $\ln\gamma$ is small enough, it must be that $d\ln A_{gt_A} \approx d\ln A_{gt_A-1} \approx 0$. According to Lemma B.4, $d\ln A_{gt_A} > d\ln A_{st_A}$, therefore either $d\ln A_{st_A} \approx 0$ or $d\ln A_{st_A} < 0$. Log differentiating f_{t_A}, one obtains:

$$
\begin{aligned}
d\ln f_{t_A} = \;& -(\varepsilon-1)\,d\ln A_{g(t_A-1)} + \frac{\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}\frac{C_{st_A}}{B_{st_A}}\varepsilon\, d\ln B_{st_A} \\
& + \frac{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}\left(\varepsilon\frac{C_{ct_A}}{A_{ct_A}}-1\right)+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}\left(\varepsilon\frac{C_{st_A}}{A_{st_A}}-1\right)}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}\, d\ln A_{s(t_A-1)}.
\end{aligned}
$$

Assume that $d\ln A_{st_A} \approx 0$, then for $\ln\gamma$ small $d\ln A_{s(t_A-1)} \approx 0$, and we get $d\ln f_{t_A} \approx \frac{\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}\frac{C_{st_A}}{B_{st_A}}\varepsilon\, d\ln B_{st_A}$. Following the shale gas boom $d\ln B_s > 0$, in order for A_{gt_A} to increase, it must be that $d\ln s_{ft}$ has been negative for a number of periods before t_A, which requires that $\varepsilon\frac{C_{ct}}{A_{ct}}<1$ for a number of periods. This ensures that $\frac{C_{st_A}}{B_{st_A}}$ is bounded above, so that $\frac{\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon+\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}\frac{C_{st_A}}{B_{st_A}}\varepsilon$ is not too small. As a result, $d\ln f_{t_A}>0$ so that $d\ln s_{ft_A}>0$ which contradicts the fact that $d\ln A_{gt_A}>0>d\ln A_{gt_A-1}$.

Similarly, assume now that A_{st} decreases at some point. We denote by t_B the first time at which $d\ln A_{ct_B}<0$ (t_B could be equal to t_A). Since $d\ln A_{ct_{B-1}}>0>d\ln A_{ct_B}$, then for $\ln\gamma$ small, we have $d\ln A_{ct_{B-1}}\approx d\ln A_{ct_B}\approx 0$. Using Lemma B.3, we get $d\ln A_{g(t_B-1)}<0$ or $d\ln A_{gt_B}\approx 0$. Following the same reasoning as above, we get that $d\ln s_{ft_B}>0$, which contradicts $d\ln A_{ct_{B-1}}>0>d\ln A_{ct_B}$.

Therefore, it must be that A_{ct}, A_{st} increase for all t and A_{gt} decreases for all t. ■

Proof that Emissions Increase Asymptotically. We now show that emissions increase asymptotically. Log-differentiating (A-3), we get:

$$\begin{aligned}
& d\ln P_t \\
= \;& \varepsilon\left(\frac{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{ct}+\frac{\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{st}\right) \\
& +\left(1-\varepsilon+\frac{(\lambda-1)(1-\nu)^{\lambda-1}A_P^{\lambda-1}}{\nu^\lambda\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)\left(\frac{\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}d\ln C_{ct}+\kappa_s^\varepsilon C_{st}^{\varepsilon-1}d\ln C_{st}+\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}d\ln A_{gt}}{\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}+\kappa_s^\varepsilon C_{st}^{\varepsilon-1}+\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}}\right).
\end{aligned} \tag{B-10}$$

As $s_{gt}\to 1$, we get:

$$\begin{aligned}
d\ln P_t \;\sim\;& \varepsilon\left(\frac{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{ct}+\frac{\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{st}\right) \\
& -\left(\varepsilon-1+\frac{(1-\lambda)(1-\nu)^{\lambda-1}A_{Pt}^{\lambda-1}}{\nu^\lambda\widetilde{A}_E^{\lambda-1}C_{Et}^{\lambda-1}+(1-\nu)^{\lambda-1}A_{Pt}^{\lambda-1}}\right)d\ln A_{gt}.
\end{aligned} \tag{B-11}$$

We can rewrite this expression as:

$$\begin{aligned}
d\ln P_t \;\to\;& -\left(\varepsilon-1+\frac{(1-\lambda)(1-\nu)^\lambda A_{Pt}^{\lambda-1}}{\nu^\lambda\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon(\lambda-1)}{\varepsilon-1}}A_{gt}^{\lambda-1}+(1-\nu)^\lambda A_{Pt}^{\lambda-1}}\right)d\ln A_{gt} \\
& +\varepsilon\frac{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon\frac{C_{ct}}{A_{ct}}+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon\frac{C_{st}}{A_{st}}}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln A_{ct}+\varepsilon\frac{\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}\frac{C_{st}}{B_{st}}d\ln B_{st}.
\end{aligned}$$

Since A_{gt} decreases and A_{ct} and A_{st} increase, emissions increase asymptotically following the natural gas boom.

Proof that Gross Output Decreases Asymptotically. Using (A-4), we can write output gross of climate damages $\widetilde{Y}_t \equiv Y_t/(1-D(S_t))$ as:

$$\widetilde{Y}_t = \left((1-\nu)^\lambda A_{Pt}^{\lambda-1} + \nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}\right)^{\frac{1}{\lambda-1}} L.$$

Log-differentiating, one gets

$$d\ln \widetilde{Y}_t = \frac{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}}{(1-\nu)^\lambda A_{Pt}^{\lambda-1} + \nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}} d\ln C_{Et}. \tag{B-12}$$

In return, log-differentiating C_{Et} yields:

$$d\ln C_{Et} = \frac{\kappa_c^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}} d\ln A_{ct} + \kappa_s^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}} d\ln A_{st} + \kappa_s^\varepsilon \frac{C_{st}^\varepsilon}{B_{st}} d\ln B_{st} + \kappa_g^\varepsilon C_{gt}^{\varepsilon-1} d\ln A_{gt}}{C_{Et}^{\varepsilon-1}}. \tag{B-13}$$

Plugging (B-7) and (B-8) in (B-13) and using that A_{gt} grows exponentially but C_{st} and C_{ct} do not, we get for t large enough:

$$\begin{aligned} & d\ln C_{Et} \\ \sim \quad & \eta(1-\psi)(\ln\gamma)\left[\sum_{\tau=1}^{t}\left(\frac{\kappa_c^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}} + \kappa_s^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}}}{\kappa_g^\varepsilon C_{gt}^{\varepsilon-1}} s_{g\tau}^{\psi} s_{f\tau}^{-\psi} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau}\right]. \end{aligned}$$

Further, use (21) to get:

$$\begin{aligned} & d\ln C_{Et} \\ \sim \quad & \eta(1-\psi)(\ln\gamma)\left[\sum_{\tau=1}^{t}\left(\frac{C_{g\tau}^{\varepsilon-1}\left(\kappa_c^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}} + \kappa_s^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}}\right)}{C_{gt}^{\varepsilon-1}\left(\kappa_c^\varepsilon \frac{C_{c\tau}^\varepsilon}{A_{c\tau}} + \kappa_s^\varepsilon \frac{C_{c\tau}^\varepsilon}{A_{c\tau}}\right)} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau}\right]. \end{aligned} \tag{B-14}$$

We want to establish that $d\ln C_{Et} < 0$, but since $ds_{f\tau}$ may not be positive for all τ, we cannot show that directly. As above, we index the times where $ds_{f\tau}$ switches signs by t_{2p} and t_{2p+1}, such that $ds_{f\tau}$ becomes negative at t_{2p+1} and positive at t_{2p}. The first sign switch occurs after t_{switch} and we also define $t_0 = t_{switch}$. We assume that at t, ds_{ft} is negative and denote $t = t_{2P} - 1$ (the reasoning extends easily to the case where $ds_{ft} > 0$). We can then decompose:

$$\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)} \sim \sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}+\sum_{p=0}^{P-1}\left(\begin{array}{c}\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\\+\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\end{array}\right).$$

Using that $ds_{ft}<0$ on $[t_{2P-1},t_{2P}-1]$ and that $\frac{s_{g\tau}}{s_{f\tau}}$ is increasing after t_{switch}, we can write:

$$\begin{aligned}&\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)}\\<&\sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}\\&+\sum_{p=0}^{P-2}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}+\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\right)\\&+\sum_{\tau=t_{2P-2}}^{t_{2P-1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}+\left(1-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{d\ln A_{gt}}{\eta(1-\psi)(\ln\gamma)}.\end{aligned}$$

where we use (B-8). Reiterating the same procedure, one gets:

$$\begin{aligned}&\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)}\\<&\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}\sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_1}}{s_{ft_1}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}\\&+\sum_{p=0}^{P-2}\frac{s_{ft}^{\psi}}{s_{gt}^{\psi}}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}+\frac{\left(\left(\frac{s_{gt_{2p+3}}}{s_{ft_{2p+3}}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)d\ln A_{gt_{2p+2}-1}}{\eta(1-\psi)(\ln\gamma)}\right)\\&+\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}\sum_{\tau=t_{2P-2}}^{t_{2P-1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}+\left(1-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{d\ln A_{gt}}{\eta(1-\psi)(\ln\gamma)}.\end{aligned}$$

The first term is negative because $t_1>t_{switch}$, so $s_{gt_1}>s_{ft_1}$ while for $\tau<t_{switch}$, $s_{g\tau}<s_{f\tau}$ and $ds_{f\tau}>0$. The terms in $\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}$ are negative because over such intervals $ds_{f\tau}>0$ and since $t>t_{switch}$, s_{gt} is increasing so $\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}<0$. In addition, we have established in Lemma B.5 that $d\ln A_{gt}<0$ for all $t's$. Therefore we get that for t large enough $d\ln C_{Et}<0$. This ensures that gross output decreases asymptotically.

B.2 Proofs for the model with endogenous innovation in extraction

Proof of Proposition A.3. Assume first that we have asymptotically positive growth in fossil-fuel power plant technologies A_{st} and A_{ct}. We first establish that there must be growth at the same rate in either B_{st} or B_{ct}. Assume instead that both extraction technologies grow more slowly than A_{st} and A_{ct}. Then using (A-5), we get

$$\left(\frac{s_{Bct}}{s_{Aft}}\right)^{\psi} \sim \frac{\kappa_c^{\varepsilon} B_{ct}^{\varepsilon-1}}{\frac{B_{ct}}{A_{ct}}\kappa_c^{\varepsilon} B_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon}\frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}} \text{ and } \left(\frac{s_{Bst}}{s_{Aft}}\right)^{\psi} \longrightarrow \frac{\kappa_s^{\varepsilon} B_{st}^{\varepsilon-1}}{\frac{B_{ct}}{A_{ct}}\kappa_c^{\varepsilon} B_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon}\frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}}.$$

Assume without loss of generality that $\frac{B_{ct}}{A_{ct}} B_{ct}^{\varepsilon-1}$ grows at least as fast as $\frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}$, then we get

$$\left(\frac{s_{Bc}}{s_{Aft}}\right)^{\psi} = O\left(\frac{A_{ct}}{B_{ct}}\right),$$

so that $s_{Aft} \to 0$. This leads to a contradiction as it implies that B_{ct} cannot grow more slowly than A_{ft}. Hence at least one of the two extraction technologies must grow at least as fast as A_{ct}.

Assume now that B_{ct} grows faster than A_{ct}, then (A-5) implies

$$\left(\frac{s_{Bct}}{s_{Aft}}\right)^{\psi} \sim \frac{A_{ct}}{B_{ct}} \frac{\kappa_c^{\varepsilon} A_{ct}^{\varepsilon-1}}{\kappa_c^{\varepsilon} A_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon}\frac{C_{st}}{A_{st}} C_{st}^{\varepsilon-1}} \leq \frac{A_{ct}}{B_{ct}}.$$

As a result, s_{Bct} tends to 0, which is, again, a contradiction. Therefore, extraction technologies cannot grow faster than A_{ct} on a fossil-fuel path, and at least one extraction technology must grow at the same rate as A_{ct}.

Without loss of generality, assume that B_{ct} grows at the same rate as A_{ct} (while B_{st} grows weakly less fast), using (A-6) we get:

$$\left(\frac{s_{Aft}}{s_{gt}}\right)^{\psi} = O\left(\frac{A_{ct}}{A_{gt}}\right)^{\varepsilon-1}.$$

Then, if A_{ct} grows faster than A_{gt}, $s_{gt} \to 0$. In contrast, if A_{ct} grows more slowly than A_{gt} then $s_{ft} \to 0$, which contradicts the assumption of positive growth in the fossil-fuel sector. Therefore, there is path dependence in innovation and (except for a knifed-edge case) innovation is asymptotically entirely either in the fossil-fuel or in the green sector.

Proof of Proposition A.4. Log-differentiate (A-5) for the natural gas sector (assuming that one can ignore the dependence of the right-hand side on the allocation of innovation) to get:

$$\psi d\ln s_{B_{st}} - \psi d\ln s_{Aft} = \left(\varepsilon \frac{C_{st}}{B_{st}} \frac{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}} - 1 \right) d\ln B_{st}. \tag{B-15}$$

Log-differentiating the ratio of the two equations in (A-5) gives:

$$\psi d\ln s_{B_{st}} - \psi d\ln s_{B_{ct}} = \left(\varepsilon \frac{C_{st}}{B_{st}} - 1 \right) d\ln B_{st}. \tag{B-16}$$

Log-differentiate the ratio of (A-5) for natural gas and (A-6) to get:

$$\psi d\ln s_{B_s} - \psi d\ln s_{gt} = \left(\varepsilon \frac{C_{st}}{B_{st}} - 1 \right) d\ln B_{st}. \tag{B-17}$$

Log-differentiating the scientists market clearing condition gives:

$$s_{Bst} d\ln s_{Bst} + s_{Aft} d\ln s_{Aft} + s_{Bct} d\ln s_{Bct} + s_{gt} d\ln s_{gt} = 0. \tag{B-18}$$

Take the difference between (B-15) and (B-17) to get:

$$d\ln \left(\frac{s_{gst}^{1-\psi}}{s_{Aft}^{1-\psi}} \right) = -\frac{\varepsilon(1-\psi)}{\psi} \frac{C_{st}}{B_{st}} \frac{\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_{ct}^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}} d\ln B_{st}, \tag{B-19}$$

which establishes that a natural gas boom redirects innovation away from green technologies relative to fossil-fuel power plant technologies.

Plugging (B-15), (B-16), and (B-17) in (B-18) implies:

$$d\ln s_{Bst} = \frac{1}{\psi} \left[s_{Aft} \left(\varepsilon \frac{C_{st}}{B_{st}} \frac{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}} - 1 \right) + \left(s_{Bct} + s_{gt} \right) \left(\varepsilon \frac{C_{st}}{B_{st}} - 1 \right) \right] d\ln B_{st}.$$

Then (B-17) gives:

$$d\ln s_{gt} = -\frac{1}{\psi} \left[s_{Bst} \left(\varepsilon \frac{C_{st}}{B_{st}} - 1 \right) + s_{Aft} \varepsilon \frac{C_{st}}{B_{st}} \frac{\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}} \right] d\ln B_{st}.$$

Therefore a natural gas boom decreases green innovation if $\varepsilon \frac{C_{st}}{B_{st}} - 1 \geq 0$.

B.3 Additional Proofs for the Extended Model

B.3.1 Proof of Proposition A.5

We can decompose the change in the emission rate as:

$$\frac{\partial \ln \xi_E}{\partial \ln B_{st}} = \underbrace{\varepsilon \frac{\partial \ln\left(C_{ft}/C_{Et}\right)}{\partial \ln B_{st}}}_{Sub_g:\ \text{substitution effect away from green}} + \underbrace{\frac{\partial \ln\left(\xi_c \kappa_c^{\sigma}\left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^{\sigma} + \xi_s \kappa_s^{\sigma}\left(\frac{C_{st}}{(1+\widetilde{\tau}_s)C_{ft}}\right)^{\sigma}\right)}{\partial \ln(B_{st})}}_{Sub_f:\ \text{substitution within fossil fuels}}.$$

The substitution effect away from green electricity is naturally positive:

$$Sub_g = \varepsilon \frac{\kappa_g^{\varepsilon}}{C_{Et}^{\varepsilon-1}}\left(\frac{A_{gt}}{1+\widetilde{\tau}_g}\right)^{\varepsilon-1} \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1} \frac{C_{st}}{B_{st}}, \tag{B-20}$$

where we use the fact that

$$\frac{\partial \ln C_{ft}}{\partial \ln B_{st}} = \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1} \frac{C_{st}}{B_{st}} \text{ and } \frac{\partial \ln C_{Et}}{\partial \ln B_{st}} = \frac{C_{ft}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}} \frac{C_{st}}{B_{st}}. \tag{B-21}$$

Combining (A-20) and (A-19), we get that the tax-inclusive expenditure share of gas electricity in fossil-fuel electricity obeys:

$$\theta_{sft} = \frac{(1+\tau_{st})p_{st}E_{st}}{p_{ft}E_{ft}} = \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}. \tag{B-22}$$

The tax-inclusive expenditure share on clean energy, using (A-21) is given by:

$$\Theta_{gt} = \frac{\left(1+\tau_{gt}\right)p_{gt}E_{gt}}{p_{Et}E_t} = \frac{\kappa_g^{\varepsilon}}{C_{Et}^{\varepsilon-1}}\left(\frac{A_{gt}}{1+\widetilde{\tau}_g}\right)^{\varepsilon-1}.$$

Then, we can rewrite (B-20) as

$$Sub_g = \varepsilon \Theta_{gt} \theta_{sft} \frac{C_{st}}{B_{st}}. \tag{B-23}$$

Further, we have

$$\begin{aligned} Sub_f &= -\sigma \frac{C_{ft}^{1-\sigma}\kappa_c^{\sigma}\kappa_s^{\sigma}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}\left(\frac{C_{ct}}{1+\widetilde{\tau}_c}\right)^{\sigma-1}}{\left(\xi_c\kappa_c^{\sigma}\left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)}\right)^{\sigma}+\xi_s\kappa_s^{\sigma}\left(\frac{C_{st}}{(1+\widetilde{\tau}_s)}\right)^{\sigma}\right)}\left[\xi_c\frac{C_{ct}}{1+\widetilde{\tau}_c}-\xi_s\frac{C_{st}}{1+\widetilde{\tau}_s}\right]\frac{C_{st}}{B_{st}} \qquad \text{(B-24)} \\ &= -\sigma\theta_{sft}\frac{P_{c,t}}{P_t}\left[1-\frac{\xi_s}{\xi_c}\frac{C_{st}}{1+\widetilde{\tau}_s}\frac{1+\widetilde{\tau}_c}{C_{ct}}\right]\frac{C_{st}}{B_{st}}, \end{aligned}$$

where

$$\frac{P_{ct}}{P_t} = \frac{\xi_c\kappa_c^{\sigma}\left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^{\sigma}}{\xi_c\kappa_c^{\sigma}\left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^{\sigma}+\xi_s\kappa_s^{\sigma}\left(\frac{C_{st}}{(1+\widetilde{\tau}_s)C_{ft}}\right)^{\sigma}} \qquad \text{(B-25)}$$

is the pollution share of coal based electricity. Therefore the substitution effect within fossil-fuel is negative as long as $\xi_c\frac{C_{ct}}{1+\widetilde{\tau}_c} > \xi_s\frac{C_{st}}{1+\widetilde{\tau}_s}$ holds. Combining (B-23) and (B-24), and using (B-22) and (B-25), we obtain equation (A-26).

To compute the scale effect, we log differentiate (A-25) and get:

$$d\ln E_t = d\ln\widetilde{C}_{Et} + d\ln L_{Et}. \qquad \text{(B-26)}$$

Log-differentiating (A-10), we get:

$$d\ln L_{Et} = \frac{L_{Pt}}{L}\left(\lambda d\ln C_{Et} - d\ln\widetilde{C}_{Et}\right). \qquad \text{(B-27)}$$

As long as $d\ln\widetilde{C}_{Et} \approx d\ln C_{Et}$, then an increase in B_{st} is associated with a decline in labor in the energy sector L_{Et}.

From (B-26), we then obtain the change in total energy production:

$$d\ln E_t = \frac{L_{Pt}}{L}\lambda d\ln C_{Et} + \frac{L_{Et}}{L}d\ln\widetilde{C}_{Et}, \qquad \text{(B-28)}$$

which is positive (as long as $d\ln\widetilde{C}_{Et}$ is not largely negative).

Using the definition of $\widetilde{C}_{Et}$ in (A-24), we get:

$$d\ln\widetilde{C}_{ft} = \left(\sigma\theta_{sft} - \frac{(\sigma-1)\kappa_s^{\sigma}(1+\widetilde{\tau}_s)^{-\sigma}C_{st}^{\sigma-1}}{\kappa_c^{\sigma}(1+\widetilde{\tau}_c)^{-\sigma}C_{ct}^{\sigma-1}+\kappa_s^{\sigma}(1+\widetilde{\tau}_s)^{-\sigma}C_{st}^{\sigma-1}}\right)d\ln C_{st}.$$

Using the definition of $\widetilde{C}_{Et}$ in (A-25) and plugging in the previous expression, we can express the change in the productivity variable $\widetilde{C}_{Et}$ as:

$$d\ln\widetilde{C}_{Et}=\left[\varepsilon\Theta_{st}+\frac{C_{ft}^{\varepsilon}\widetilde{C}_{ft}^{-1}\left((\sigma-\varepsilon)\theta_{sft}-\frac{(\sigma-1)\kappa_s^{\sigma}(1+\widetilde{\tau}_s)^{-\sigma}C_{st}^{\sigma-1}}{\kappa_c^{\sigma}(1+\widetilde{\tau}_c)^{-\sigma}C_{ct}^{\sigma-1}+\kappa_s^{\sigma}(1+\widetilde{\tau}_s)^{-\sigma}C_{st}^{\sigma-1}}\right)}{C_{ft}^{\varepsilon}\widetilde{C}_{ft}^{-1}+\kappa_g^{\varepsilon}\left(1+\widetilde{\tau}_g\right)^{-\varepsilon}A_{gt}^{\varepsilon-1}}\right]d\ln C_{st}. \quad \text{(B-29)}$$

For $\widetilde{\tau}_g, \widetilde{\tau}_c$ and $\widetilde{\tau}_s$ small, we get

$$d\ln\widetilde{C}_{Et}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0}\approx d\ln\widetilde{C}_{Et}=\Theta_{st}d\ln C_{st}, \quad \text{(B-30)}$$

which, using (B-28), leads to the same scale effect as in the baseline:

$$\frac{\partial\ln E_t}{\partial\ln B_{st}}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0}\approx\frac{L_{Et}+\lambda L_{Pt}}{L}\Theta_{st}\frac{C_{st}}{B_{st}}. \quad \text{(B-31)}$$

The overall effect on emissions is then given by the sum of (A-26) and (B-31), which we can rewrite as

$$\frac{\partial\ln P_t}{\partial\ln B_{st}}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0}\approx-\frac{C_{st}}{B_{st}}\left[(\sigma-\varepsilon)\theta_{sft}+(\varepsilon-1)\Theta_{st}+\frac{(1-\lambda)L_{Pt}}{L}\Theta_{st}-\sigma\frac{P_{st}}{P_t}\right].$$

For ξ_s/ξ_c small, the term $\frac{P_{st}}{P_t}$ is small, given that $\sigma\geq\varepsilon$ and $\lambda\leq 1$, then emissions decrease following the natural gas boom.

B.3.2 Proof of Uniqueness and Maximal Growth Rate

We show that the equilibrium is unique for $\ln\gamma$ small enough. Using (A-32) and defining $s_{ft}=s_{ct}+s_{st}$, we can write:

$$\begin{aligned} s_{ct} &= \frac{(1-q_s)^{\frac{1}{\psi}}\left(\frac{\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\chi\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}s_{ft}}{(1-q_c)^{\frac{1}{\psi}}\left(\frac{\chi\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}+(1-q_s)^{\frac{1}{\psi}}\left(\frac{\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\chi\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}} \quad \text{(B-32)}\\ s_{st} &= \frac{(1-q_c)^{\frac{1}{\psi}}\left(\frac{\chi\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}s_{ft}}{(1-q_c)^{\frac{1}{\psi}}\left(\frac{\chi\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}+(1-q_s)^{\frac{1}{\psi}}\left(\frac{\kappa_c^{\sigma}(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\chi\kappa_s^{\sigma}(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)^{\frac{1}{\psi}}}. \end{aligned}$$

For $\ln\gamma$ small enough, we can ignore the dependence of the RHS on s_{ct} and s_{st}, so that the previous equations define s_{ct} and s_{st} as increasing (and nearly linear) functions of s_{ft}. We then get that the numerator in the LHS (A-33) is decreasing in s_{ft} (as for $\ln\gamma$ small, we can ignore the dependence of C_{it} and A_{it} on innovation). The denominator is increasing in s_{ft} as $s_{gt} = 1 - s_{ft}$ (and again ignoring the dependence of C_{gt} on the innovation allocation). Therefore the LHS decreases from infinity to 0 in s_{ft}, and the equation defines a unique solution.

We show that the maximal growth rate that can be achieved on a fossil-fuel path corresponds to the growth rate $\gamma^{\eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}} - 1$. The growth rate of C_{Et} is maximized if the growth rate of C_{ft} is maximized which occurs if the growth rates of either C_{st} or C_{ct} are maximized. Without loss of generality, assume that A_{ct} grows faster than A_{st}. Then, the growth rates of C_{ct} and that of A_{ct} are maximized when $s_{ct}^{1-\psi} + \chi s_{st}^{1-\psi}$ is maximized, which occurs if $s_{ct} = s_{ft}/\left(1+\chi^{\frac{1}{\psi}}\right)$. In that case, B_{ct} and B_{st} grow faster than A_{st}, and (B-32) gives $s_{ct} \to s_{ft}/\left(1+\chi^{\frac{1}{\psi}}\right)$ for $q_c = q_s$, so that this optimal growth rate can be achieved. We then get that C_{Et} and $\widetilde{C}_{Et}$ grow asymptotically at the rate $\gamma^{\eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}} - 1$.

B.3.3 Proof of Proposition A.6

Log differentiating (B-32), and assuming that $\ln\gamma$ is sufficiently small that we can ignore the dependence of A_{it} on s_{it}, we can write:

$$d\ln s_{ct} \approx d\ln s_{ft} - \frac{\sigma}{\psi}\frac{s_{st}}{s_{ft}} \frac{\left(1-\chi^2\right)\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}, \tag{B-33}$$

$$d\ln s_{st} \approx d\ln s_{ft} + \frac{\sigma}{\psi}\frac{s_{ct}}{s_{ft}} \frac{\left(1-\chi^2\right)\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}. \tag{B-34}$$

This directly implies that the ratio s_{st}/s_{ct} increases with B_{st}. Log-differentiating (A-33) and using (A-32) and (B-21) (and $\ln\gamma$ small) leads to

$$\left[\begin{array}{c} \dfrac{\left(\left(\chi^2+1\right)\frac{\kappa_c^\sigma}{(1+\widetilde{\tau}_c)^\sigma}\frac{\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{A_{ct}} + 2\chi\frac{\kappa_s^\sigma}{(1+\widetilde{\tau}_s)^\sigma}\frac{\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{A_{st}}\right)\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\sigma\frac{C_{st}}{B_{st}}d\ln B_{st}}{2\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)} \\ +(\varepsilon-\sigma)\frac{\kappa_s^\sigma}{C_{ft}^{\sigma-1}}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}\frac{C_{st}}{B_{st}}d\ln B_{st} - \frac{\psi}{2}d\ln s_{st} - \frac{\psi}{2}d\ln s_{ct} + \psi d\ln s_{gt} \end{array}\right] \approx 0.$$

Noting that $d\ln s_{gt} = -\frac{s_{ft}}{s_{gt}} d\ln s_{ft}$ and plugging in (B-33) and (B-34), we get:

$$
\begin{aligned}
&d\ln s_{ft} \qquad\qquad \text{(B-35)}\\
&\approx \frac{s_{gt}}{\psi}\left[\frac{(\varepsilon-\sigma)\kappa_s^\sigma C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}(1+\tilde{\tau}_s)^{\sigma-1}} + \sigma\frac{\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\left(\left(\frac{s_{st}}{s_{ft}}+\chi^2\frac{s_{ct}}{s_{ft}}\right)\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)}{\left(\chi\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)\left(\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)}\right]\frac{C_{st}d\ln B_{st}}{B_{st}}.
\end{aligned}
$$

The second term in the brackets is positive whereas the first term is weakly negative since $\varepsilon \leq \sigma$. Therefore if $\varepsilon \approx \sigma$, then the first term is small and the shale gas boom increases the mass of scientists in fossil-fuel innovations and decreases green innovation. When $\sigma > \varepsilon$, then green energy is more complementary to natural gas than coal is, this creates a force that pushes toward more green innovation following the shale gas boom.

Combining (B-34) with (B-35), it is also immediate that for $\varepsilon \approx \sigma$, an increase in B_{st} leads to an increase in natural gas innovation. Combining (B-33) with (B-35), we get:

$$
\begin{aligned}
&d\ln s_{ct}\\
&\approx \left[\frac{s_{gt}(\varepsilon-\sigma)\kappa_s^\sigma C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}(1+\tilde{\tau}_s)^{\sigma-1}} + \frac{\sigma\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\left(\left[-s_{st}+\chi^2\left(s_{gt}+s_{st}\right)\right]\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+s_{gt}\chi\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)}{\left(\chi\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)\left(\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\right)}\right]\frac{C_{st}d\ln B_{st}}{\psi B_{st}}.
\end{aligned}
$$

The effect of an increase in B_{st} on s_{ct} is ambiguous even for $\varepsilon = \sigma$: the second term in brackets is positive if χ is close to 1 but negative for χ close to 0. This establishes Part i).

Assume now that $\chi = 1$, then (B-35) gives:

$$
\begin{aligned}
d\ln s_{ft}|_{\chi=1} &\approx \frac{s_{gt}}{\psi}\left[(\varepsilon-\sigma)\frac{\kappa_s^\sigma\left(\frac{C_{st}}{1+\tilde{\tau}_s}\right)^{\sigma-1}}{\kappa_c^\sigma\left(\frac{C_{ct}}{1+\tilde{\tau}_c}\right)^{\sigma-1}+\kappa_s^\sigma\left(\frac{C_{st}}{1+\tilde{\tau}_s}\right)^{\sigma-1}} + \sigma\frac{\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}}{\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}}\right]\frac{C_{st}}{B_{st}}d\ln B_{st}\\
&\approx \frac{s_{gt}}{\psi}\left[\varepsilon + \frac{(\sigma-\varepsilon)\kappa_c^\sigma C_{ct}^{\sigma-1}}{(1+\tilde{\tau}_c)^{\sigma-1}C_{ft}^{\sigma-1}}\left(1-\frac{(1+\tilde{\tau}_s)(1+\overline{\Lambda}_c)A_{st}C_{ct}}{(1+\tilde{\tau}_c)(1+\overline{\Lambda}_s)A_{ct}C_{st}}\right)\right]\frac{\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\tilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\tilde{\tau}_s)^\sigma A_{st}}}.
\end{aligned}
$$

s_{ft} increases following the shale gas boom when $\chi = 1$ provided that $\frac{(1+\overline{\Lambda}_s)C_{st}}{(1+\tilde{\tau}_s)A_{st}} \geq \frac{(1+\overline{\Lambda}_c)C_{ct}}{(1+\tilde{\tau}_c)A_{ct}}$ (or more generally as long as $(\sigma-\varepsilon)\left(1-\frac{(1+\tilde{\tau}_s)(1+\overline{\Lambda}_c)A_{st}C_{ct}}{(1+\tilde{\tau}_c)(1+\overline{\Lambda}_s)A_{ct}C_{st}}\right)$ is not too negative).

B.4 Complementarity between Natural Gas and Renewables

In this Appendix, we present and solve the model sketched in Section 5.3. To capture the notion of greater complementarity between natural gas and renewables, we now assume that energy is produced according to:

$$E_t = \left(\kappa_c E_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_s E_{sat}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{gat}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_b E_{bt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{B-36}$$

E_{bt} is a hybrid energy which uses gas (sb) and green (gb) as inputs according to the Cobb-Douglas technology $E_{bt} = E_{sbt}^{1-\alpha} E_{gbt}^{\alpha}$. E_{sat} and E_{gat} represent natural gas and green technologies which are used "alone" (e.g., nuclear power).

In the following, we solve for the competitive equilibrium and derive the effect of the natural gas boom on emissions. Then, we solve for the dynamic equilibrium and derive the effect of the boom on innovation. The effect is theoretically ambiguous, but we quantify the model and show that for reasonable parameter values, the shale gas boom still decreases green innovation.

B.4.1 Competitive Equilibrium

To solve for the competitive equilibrium, we follow the same strategy as for the baseline model. The Cobb-Douglas structure within the bridge technology implies that the effective productivity of the bridge technology is given by

$$C_{bt} \equiv \frac{C_{st}^{1-\alpha} C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\,\alpha^{\alpha}}, \tag{B-37}$$

so that the price of the bridge technology is given by $p_{bt} = \frac{\gamma w}{C_{bt}}$. Total energy production is still given by $E_t = C_{Et} L_{Et}$ and the price of energy is $p_{Et} = \gamma w / C_{Et}$ with C_{Et} now given by

$$C_{Et} \equiv \left(\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon} C_{st}^{\varepsilon-1} + \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} + \kappa_b^{\varepsilon} C_{bt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{B-38}$$

Similarly to (13), we get

$$\begin{aligned} E_{c,t} &= \kappa_c^{\varepsilon}\left(\frac{C_{ct}}{C_{Et}}\right)^{\varepsilon} E_t \; ; \; E_{sa,t} = \kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} E_t, \\ E_{ga,t} &= \kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon} E_t \; ; \; E_{b,t} = \kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon} E_t. \end{aligned}$$

Using that the bridge technology is produced in a Cobb-Douglas way, we have $p_{st}E_{sbt} = (1-\alpha)\,p_{bt}E_{bt}$ and $p_{gt}E_{gbt} = \alpha p_{bt}E_{bt}$ so that

$$E_{gb,t} = \frac{\alpha C_{gt}}{C_{bt}} E_{bt} \text{ and } E_{sb,t} = \frac{(1-\alpha)\,C_{st}}{C_{bt}} E_{bt}.$$

The aggregate clean and natural gas energy productions are then respectively equal to:

$$E_{gt} = \left(\kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon} + \frac{\alpha C_{gt}}{C_{bt}}\kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right)E_t$$

$$\text{and } E_{s,t} = \left(\kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} + \frac{(1-\alpha)\,C_{st}}{C_{bt}}\kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right)E_t.$$

Total emissions are given by $P_t = \xi_{Et}E_t$, where the emission rate is now:

$$\xi_{Et} = \xi_c\kappa_c^{\varepsilon}\left(\frac{C_{ct}}{C_{Et}}\right)^{\varepsilon} + \xi_{st}\left(\kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} + \frac{(1-\alpha)\,C_{st}}{C_{bt}}\kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right).$$

Labor allocation is still given by (15).

B.4.2 Emission Effects of a Natural Gas Boom

As before we derive the effect of a natural gas boom on emissions (at a constant level of extraction technologies). We get that:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{\partial \ln \xi_E}{\partial \ln C_s}\frac{\partial \ln C_s}{\partial \ln B_s} + \frac{\partial \ln E}{\partial \ln C_s}\frac{\partial \ln C_s}{\partial \ln B_s}.$$

$\frac{\partial \ln \xi_E}{\partial \ln C_s}$ represents the substitution effect and is given by:

$$\begin{aligned}\frac{\partial \ln \xi_E}{\partial \ln C_s} &= -\varepsilon\frac{P_c}{P}\frac{\partial \ln C_{Et}}{\partial \ln C_s} + \varepsilon\frac{P_{sa}}{P}\left(1 - \frac{\partial \ln C_E}{\partial \ln C_s}\right) + \frac{P_{sb}}{P}\left(\frac{\partial \ln\left(C_s C_b^{\varepsilon-1}\right)}{\partial \ln C_s} - \varepsilon\frac{\partial \ln C_E}{\partial \ln C_s}\right)\\ &= \varepsilon\frac{P_{sa}}{P} + (1 + (1-\alpha)(\varepsilon-1))\,\frac{P_{sb}}{P} - \varepsilon\frac{\partial \ln C_E}{\partial \ln C_s};\end{aligned}$$

where

$$\frac{\partial \ln C_E}{\partial \ln C_s} = \frac{\kappa_s^{\varepsilon}C_s^{\varepsilon-1}}{C_E^{\varepsilon-1}} + (1-\alpha)\,\frac{\kappa_b^{\varepsilon}C_b^{\varepsilon-1}}{C_E^{\varepsilon-1}} = \frac{p_{st}E_{sat}}{p_{Et}E_t} + \frac{(1-\alpha)\,p_{bt}E_{bt}}{p_{Et}E_t} = \frac{p_{st}E_{st}}{p_{Et}E_t} \equiv \Theta_s,$$

where as before Θ_s denotes the revenue share of natural gas in the energy sector. We then get that the substitution effect is determined by:

$$\frac{\partial \ln \xi_E}{\partial \ln C_s} = \left(\varepsilon \frac{P_{sa}}{P} + (1 + (1-\alpha)(\varepsilon - 1)) \frac{P_{sb}}{P} - \varepsilon \Theta_s\right),$$

which, for given revenue share and pollution share of natural gas, is lower than in the no bridge technology case. Since $\frac{\partial \ln C_E}{\partial \ln C_s} = \Theta_s$, the scale effect is still determined by:

$$\frac{\partial \ln E_t}{\partial \ln C_{st}} = \frac{\partial \ln C_{Et} L_{Et}}{\partial \ln C_{st}} = \Theta_s (\lambda + (1-\lambda)\Omega_E).$$

Therefore, one gets:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{C_s}{B_s}\left(\varepsilon\left(\frac{P_{sa} + \frac{(1+(1-\alpha)(\varepsilon-1))}{\varepsilon} P_{sb}}{P} - \Theta_s\right) + \Theta_s (\lambda + (1-\lambda)\Omega_E)\right),$$

which is lower than in the baseline case for given observables ($\Theta_{s,}$ Ω_E and P_s/P). We get:

Proposition B.1 *When there is some degree of complementarity between natural gas and the green technology, a natural gas boom leads to a larger reduction in emissions.*

Intuitively, an improvement in the natural gas technology improves the bridge technology which is less polluting than natural gas alone, this tends to make the substitution effect more negative than without the bridge technology.

B.4.3 Innovation Effects of a Natural Gas Boom

We keep the same structure for innovation as in the baseline model, so that again the direction of innovation depends on relative profits from innovating in the various technologies. We now have that expected profits from clean innovations obey:

$$\Pi_{gt} = \eta s_{gt}^{-\psi}\left(1 - \frac{1}{\gamma}\right)(p_{gt}E_{gat} + p_{gt}E_{gbt}),$$

and expected profits from fossil-fuel innovations obey:

$$\Pi_{ft} = \eta s_{ft}^{-\psi}\left(1 - \frac{1}{\gamma}\right)\left(\frac{C_c}{A_c} p_{ct}E_{ct} + \frac{C_{st}}{A_{st}}(p_{st}E_{sat} + p_{st}E_{sbt})\right).$$

The revenue share of green technologies alone is given by:

$$\frac{p_{gt}E_{gat}}{p_{Et}E_t} = \frac{\kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}$$

and the revenue share of green technologies within the bridge technology is given by:

$$\frac{p_{gt}E_{gbt}}{p_{Et}E_t} = \frac{\alpha p_{bt}E_{bt}}{p_{Et}E_t} = \frac{\alpha \kappa_b^{\varepsilon} C_{bt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}} = \frac{\alpha \kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}.$$

With similar expressions for the revenue shares associated with natural gas, and using that $\Pi_{gt} = \Pi_{ft}$ in equilibrium, one gets:

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi} = \frac{\frac{C_c}{A_c}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\left(\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1} + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}\right)}{\kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} + \alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}.$$

To look at the effect of the natural gas boom on the innovation allocation at $t = 1$, we log differentiate the right-hand side of this expression with respect to B_s. If that derivative is positive (and $\ln\gamma$ is sufficiently small that the innovation allocation is unique), then a natural gas boom leads to an increase in fossil-fuel innovations and a decline in green innovations. We get:

$$\frac{\partial \ln\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}}{\partial \ln B_{st}} = \left[\begin{array}{c} \frac{\frac{C_{st}}{A_{st}}\left[\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}\varepsilon + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}((\varepsilon-1)(1-\alpha)+1)\right]}{\frac{C_c}{A_c}\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\left(\kappa_s^{\varepsilon} C_{st}^{\varepsilon-1} + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}\right)} \\ -\frac{(\varepsilon-1)(1-\alpha)\alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}{\kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} + \alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}} \end{array}\right] \frac{\partial \ln C_{st}}{\partial \ln B_{st}}.$$

This expression is not necessarily positive, so that the natural gas boom could lead to an increase in green innovation. Intuitively, the natural gas boom leads to an increase in the hybrid share, which can in return boost innovation. This effect may dominate when the coal technology is very advanced relative to the natural gas and hybrid technologies (C_{ct} is large so that the first term is arbitrarily small): in that case, since most of the revenues of the fossil-fuel power plant sector come from coal, the natural gas boom has a small effect on the incentive to introduce fossil-fuel innovations.

Therefore, one gets

$$\frac{d\ln}{d\ln B_s}\frac{s_{ft}}{s_{gt}} \approx \frac{1}{\psi}\left[\frac{\frac{C_{st}}{A_{st}}\left[\varepsilon\frac{E_{sa}}{E_s}+((\varepsilon-1)(1-\alpha)+1)\frac{E_{sb}}{E_s}\right]\Theta_s}{\frac{C_c}{A_c}\Theta_c+\frac{C_{st}}{A_{st}}\Theta_s}-\frac{(\varepsilon-1)(1-\alpha)E_{ga}}{E_g}\right]\frac{\partial\ln C_{st}}{\partial\ln B_{st}},$$

where the approximation comes from the fact that we ignore the dependence of the $A's$ on the current innovation allocation. In contrast, without the hybrid technology, the corresponding expression is

$$\frac{d\ln}{d\ln B_s}\frac{s_{ft}}{s_{gt}}|_{\kappa_b=0} \approx \frac{1}{\psi}\frac{\frac{C_{st}}{A_{st}}\Theta_{st}}{\frac{C_c}{A_c}\Theta_{ct}+\frac{C_{st}}{A_{st}}\Theta_{st}}\frac{\partial\ln C_{st}}{\partial\ln B_{st}},$$

which is larger for given observables (the revenue shares). However, rearranging terms, we get that the natural gas boom still increases fossil-fuel innovation provided that:

$$\begin{aligned}
&\varepsilon\frac{\kappa_s^\varepsilon\kappa_g^\varepsilon C_{st}^{\varepsilon-1}C_{gt}^{\varepsilon-1}}{\kappa_b^\varepsilon C_{bt}^{\varepsilon-1}}+((\varepsilon-1)(1-\alpha)+1)(1-\alpha)\kappa_g^\varepsilon C_{gt}^{\varepsilon-1} \\
&+[\varepsilon-(\varepsilon-1)(1-\alpha)]\alpha\kappa_s^\varepsilon C_{st}^{\varepsilon-1}+\alpha(1-\alpha)\kappa_b^\varepsilon C_{bt}^{\varepsilon-1} \\
> \;& (\varepsilon-1)(1-\alpha)\alpha\frac{A_{st}}{C_{st}}\frac{C_c}{A_c}\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}.
\end{aligned} \tag{B-39}$$

We then obtain:

Proposition B.2 *When there is a hybrid technology, the increase in fossil-fuel innovation following the natural gas boom is smaller, though it is still positive when (B-39) is satisfied.*

Intuitively, a drop in the price of natural gas may incentivize clean innovation through its effect on the hybrid technology. This counteracting force may dominate if the natural gas and the hybrid shares are small compared to the coal share. In that case, the natural gas boom has little impact on the returns to fossil-fuel innovation (which are dominated by coal), but some positive effect on the returns to clean innovation (through the hybrid technology). For this effect to dominate, however, the coal share needs to be very large (as stipulated in (B-39)) and we now show that for reasonable parameter values, this does not occur so that the natural gas boom still reduces green innovation.

B.4.4 Quantification

This section presents a quantification of the model with complementarity in order to investigate whether condition (B-39) holds in the data. To map (B-36) to the data, we assume that all solar and wind generation is in the hybrid nest E_{gbt}, whereas all other green base period generation (e.g., nuclear, biomass) is in the stand-alone green category E_{gat}. To begin, we solve for the Cobb-Douglas exponent α based on the equilibrium price of the renewable-gas bundle:

$$p_{bt} = \frac{p_{st}^{1-\alpha} p_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\, \alpha^{\alpha}}. \tag{B-40}$$

The University of Chicago Energy Policy Institute (EPIC) has produced recent estimates of the levelized costs of renewables *backed up by natural gas* for both (onshore) wind (p_{bt} = \$54/MWh) and solar photovoltaic energy (p_{bt} = \$61/MWh) (Greenstone and Nath 2021). The corresponding EIA's Annual Energy Report posits levelized costs *without backup* for onshore wind (p_{gt} =\$34/MWh), and for solar (p_{gt} =\$33/MWh).[1] Combined with EPIC's estimate for the levelized cost of natural gas generation (p_{st} = \$42/MWh), we can use (B-40) to back out the implied value of α for wind generation ($\widehat{\alpha} = 0.8457$) and solar ($\widehat{\alpha} = 0.7561$). We take the generation-weighted average between wind and solar for 2011, yielding $\alpha = 0.8446$.

Next, in order to calibrate the distribution parameters in (B-36), we must specify the remaining base year quantities. For natural gas, we proxy stand-alone generation E_{sat} through combined-cycle plant output, and treat all combustion or steam engine gas generation as in the nest with renewables (E_{sbt}). This distinction is motivated by the EIA's observation that combined-cycle plants are "often used as baseload generation" whereas combustion and steam turbines are "generally only run during hours when electricity demand is high."[2]Importantly, this approach almost surely overstates the amount of natural gas that is complementary to renewables since many areas may rely on gas peaker plants to deal with demand fluctuations even in the absence of renewable generation. In 2011, combined cycle accounted for 82% of utility scale net generation from natural gas, with combustion and steam turbines accounting for the remaining 18%.[3]

Applying these assumptions to our base period data (2006-10) and using $E_{b0} = E_{gb0}^{\alpha} E_{sb0}^{1-\alpha}$ to compute the initial E_{b0} (equal to 0.3343 tril. KWh) enables us to back out the $\kappa's$ in (B-36)

[1] For consistency we utilize levelized cost estimates based on the same year assumptions to calibrate α.

[2] U.S. Energy Information Administration, "Today in Energy," Dec. 18, 2017. URL (accessed September 2021): https://www.eia.gov/todayinenergy/detail.php?id=34172#tab1.

[3] EIA "Electricity Power Monthly" Table 1.7.C., Utility Scale Facility Net Generation from Natural Gas by Technology: Total (All Sectors), 2011-October 2021. URL (accessed Septembre 2021): https://www.eia.gov/electricity/monthly/epm_table_grapher.php?t=table_1_07_c.

via the standard profit-maximization conditions,

$$\frac{p_{c0}}{p_{s0}}=\frac{\kappa_c E_{c0}^{-\frac{1}{\varepsilon}}}{\kappa_s E_{sa0}^{-\frac{1}{\varepsilon}}},\ \frac{p_{g0}}{p_{b0}}=\frac{\kappa_g E_{ga0}^{-\frac{1}{\varepsilon}}}{\kappa_b E_{b0}^{-\frac{1}{\varepsilon}}},\ \frac{p_{c0}}{p_{b0}}=\frac{\kappa_c E_{c0}^{-\frac{1}{\varepsilon}}}{\kappa_b E_{b0}^{-\frac{1}{\varepsilon}}},$$

and the condition that $1=\kappa_c+\kappa_s+\kappa_b+\kappa_g$. We note that, in order to ensure time period consistency, we back out the price of the hybrid bundle relevant for the base period (2006-10) based on (B-40) instead of using the aforementioned EPIC estimates. We also note that we now assume the within-fossil nest elasticity of substitution value from the extended model $\sigma=2$ as value for ε since intermittency concerns that were motivating driving the lower benchmark value of ε =1.8561 in the benchmark are now explicitly accounted for. However, the results below are completely robust to using $\varepsilon=1.8561$ here as well. Solving these four equations in four unknowns yields $\kappa_c=0.25$, $\kappa_s=0.30$, $\kappa_b=0.14$, and $\kappa_g=0.31$.

In order to evaluate (B-39), it remains to solve for initial technology levels consistent with equilibrium in the modified model. We do so by solving a modified version of benchmark system of equations (A-16), with equation (B-37) for C_{b0} added and with (B-38) replacing the benchmark condition for C_{E0}. As inputs to this computation, we also calculate the modified model's E_0 from (B-36), p_{E0} based on the equilibrium condition that $p_{ct}=\kappa_c E_{ct}^{-\frac{1}{\varepsilon}} p_{Et} E_t^{\frac{1}{\varepsilon}}$, and $\widetilde{A}_{E0}$ = 2.06e+05 from (A-15) which remains valid. The results are similar to the benchmark:

$A_{g,0}$	$A_{c,0}$	$A_{s,0}$	$B_{c,0}$	$B_{s,0}$	$C_{b,0}$	$C_{E,0}$	$A_{P,0}$	w_0	L_{E0}
100.3	461.7	449.7	337.1	119.4	153.0	32.7	4.79e+03	6.876+03	1.258%

Finally, we evaluate the innovation inequality (B-39), yielding:

$$594.7 >>> 2.9.$$

These results imply that condition (B-39) holds easily, suggesting that the impact of the shale gas boom is to increase incentives for fossil innovation even after accounting for the possibility of complementarity between renewables and natural gas.

Supplementary Material References

Greenstone, M. and I. Nath (2021). "U.S. Energy & Climate Roadmap". Energy Policy Instiute at the University of Chicago.