The saturation of spending diversity and the truth about Mr Brown and Mrs Jones

Andreas Chai*, Christian Kiedaisch†, Nicholas Rohde‡

August 21, 2017

DRAFT - PLEASE DO NOT QUOTE WITHOUT AUTHOR CONSENT

Abstract

Several cross country studies find that rising household income leads to consumption spending being spread more evenly across different spending categories(Clements et al., 2006). We argue that this result is likely due to aggregation. Using more disaggregated UK household level spending data, we show that the spending diversity of households only rises up to a certain income level and then starts to decline as households concentrate more of their spending on particular expenditure categories that differ across households. It is precisely because of this growing heterogeneity on the household level that the average spending diversity of the population can nevertheless always rise in income. We build a model to capture this observed pattern and use it to show that ignoring preference heterogeneity across households and focusing on a model with representative households leads to an underestimation of the value of product variety.

JEL classification: D12, C14, O33.

Keywords: Demand for variety, Engel's Law, Spending Diversity.

---

*Corresponding author: Griffith Business School, Gold Coast Campus Griffith University, Australia QLD 4222. a.chai@griffith.edu.au.

†University of Zürich, Switzerland,christian.kiedaisch@econ.uzh.ch

‡Griffith Business School, Gold Coast Campus Griffith University, Australia QLD 4222. n.rohde@griffith.edu.au

We would like to thank comments receid at the 2017 EPAP workshop on demand analysis (Griffith University, Gold Coast). A. Chai would like to acknowledge funding support from the Griffith Business School and the Griffith Asia Institute. C.Kiedaisch gratefully acknowledges financial support by the Swiss National Science Foundation. The usual disclaimer applies.

‘The preference hypothesis only acquires prima facie plausibility when it is applied to the *statistical average*. To assume that the representative consumer acts like an ideal consumers is a hypothesis worth testing; to assume that an actual person, the Mr. Brown or Mrs. Jones, who lives around the corner, does in fact act in such a way does not deserve a moment’s consideration.’
J.R. Hicks- A Revision of Demand Theory (1956) -

1 Introduction

One of the most salient features of modern economies is the wide range of goods and services available to consumers in markets. While much has been said about how firm behavior generates product variety, less has been said about how demand may also contribute to this phenomenon (Gronau and Hamermesh, 2008). In standard product variety models with homothetic preferences (Dixit and Stiglitz, 1977), the demand for variety is independent of income in the sense that the expenditure shares on particular consumption items are the same for rich and poor households. In random utility models that incorporate heterogeneity in consumer preferences (McFadden, 1984; Calvet and Common, 2003), this heterogeneity usually does not depend on economic factors like household income, either. Yet much evidence suggest that the demand for variety increases in household inocme (Prais, 1952; Theil, 1967; Theil and Finke, 1983; Jackson, 1984; Falkinger and Zweimüller, 1996; Bils and Klenow, 2001). The growth in the range of goods consumed is widely recognized to have vital implications for a range of economic issues: when the demand for different final goods changes with the level of income, this can lead to changes in the industrial composition and structural change (Pasinetti, 1981; Saviotti, 2001; Metcalfe et al., 2006; Foellmi and Zweimüller, 2008), impact the incentives to innovate (Foellmi and Zweimuller, 2006), as well as influence the realization of economies of scale (Bresnahan and Gambardella, 1998; Lipsey et al., 2008) and international trade flows (Hallak, 2010).

If variety demand does change with income, a major question is whether the direction of variety demand growth is similar or different across the population of consumers. In other words, as households get rich, do the types of new varieties they demand fundamentally differ across the population of consumers?

Here previous studies suggest that the answer is negative as spending patterns are found to become more similar as income rises. Several studies have used entropy measures to calculate the distribution of spending across different expenditure categories, which we dub the 'diversity of spending' (Theil and Finke, 1983; Clements and Chen, 1996; Clements et al., 2006). These cross-country studies of spending patterns suggest that this diversity always increases when income rises. In other words, as their income grows, consumers appear to spread their spending more evenly across all available goods and services. This suggests that, as income rises, differences in spending pattern across households will decline.

We argue that this literature has ignored the possibility that heterogeneity in spending patterns is masked by high levels of aggregation present in cross country data. Many recognize that it is crucial to study the precise relationship between aggregate and individuals behavior (Grandmont, 1987, 1992; Hildenbrand, 1994; Quah, 1997; Blundell and Stoker, 2005). A number of researchers have begun considering how behavioural heterogeneity can be modelled (Calvet and Common, 2003; Beckert and Blundell , 2008). This represents a departure from the main paradigm of postwar demand analysis that has concentrated on studying aggregates to verify representative agent models of behavior, even though these aggregates may not reflect actual household behavior. This paradigm is reflected in the above quote by J. R. Hicks, who argued that rather than attempting to account for actual household behavior, scholars should restrict their focus on *average* household behavior. It also underpins many commonly used models of demand analysis such as AIDS (Deaton and Muellbauer, 1980).

In the case of spending diversity, whether to focus on average rather than actual behavior is particularly important. In this paper, we argue that as households shift their spending from basic necessities towards more discretionary categories, heterogeneity in spending patterns is likely to increase in income as consumers concentrate their spending into different consumption areas once incomes are sufficiently large. It is a well known fact that amongst the poorest, spending patterns are highly homogeneous across households as food spending tends to dominate household outlays (Banerjee and Duflo, 2007; Clements et al., 2006). The notion that heterogeneity of spending grows with income is consistent with Engel's Law as well as evidence that Engel curves are highly heteroskedastic (Blundell and Stoker, 2005; Lewbel, 2008). Using UK household level spending data, we find evidence suggesting that the diversity of household spending tends to fall at high income levels and that the overall differences in household spending patterns tend to grow (rather than fall) for high income levels. In other words, the truth about Mr. Brown and Mrs. Jones is that they not only possess different spending patterns, but that the differences in these patterns increase in income when they are sufficiently rich.

This emergent nature of consumption heterogeneity is worth taking into account. We develop a model that accounts for the fact that demand heterogeneity increases in income at high income levels and that can explain why there can be a hump shaped relation between spending diversity and income at the individual level and a positive relation at the aggregate level. A key characteristic of the model is that differences between household spending patterns increase in income for high income levels. This pattern does not arise in previous models (?Theil and Finke, 1983; Gronau and Hamermesh, 2008) that study how variety demand using the representative consumer approach.

Within this model setup, we analyze how much an increase in product variety is valued by individual households and by representative households the preferences of which are such that the resulting aggregate demand for each good is the same as in the case of consumer heterogeneity. We find that the representative households values an increase in product variety less than individual households with heterogeneous tastes do. As it is widely believed that the welfare effects of increasing product variety are substantial, this finding therefore calls for more sophisticated welfare analyses that take individual heterogeneity into account.

In terms of methodology, this paper studies the relationship between income and variety demand using cross sectional data. It may be tempting to study household spending patterns over time. However, the main obstacle in doing so is that one cannot control for exogenous changes in variety demand over time. There has been rapid growth in the number of good available over time, which fundamentally affect the measurement of spending diversity. For this reason the main focus of this paper is on cross sectional results.

2 Stylized facts about spending diversity

We begin by reporting some stylized facts about how the diversity of consumption spending changes with income, looking at both individual household consumption and at the aggregated consumption of several households with similar income levels. We use the following notation in our analysis: There are n households indexed by i and k expenditure categories (or goods) indexed by j. Total expenditures on all k categories by household i are denoted by x_i (and also referred to as income). The expenditure share of household i on good j is denoted by s_{ij}, such that $s_i = (s_{i1}, s_{i2}..., s_{ik})$ denotes the vector of expenditure shares for household i. The overall expenditures of household i on each good j are consequently given by $x_i \times s_{ij}$.

We are concerned with modelling the cross-sectional distribution of expenditures across expenditure categories. In order to do so, we follow (Theil, 1967; Theil and Finke, 1983; Clements et al., 2006) and use the following entropy measure of the expenditure shares:

$$E_i = -\sum_{j=1}^{k} \phi(s_{ij}) \qquad \begin{cases} \phi(s_{ij}) = s_{ij} \ln s_{ij} & s_{ij} > 0 \\ \phi(s_{ij}) = 0 & s_{ij} = 0 \end{cases} \tag{1}$$

The entropy E_i is an index number that measures the extent to which spending is dispersed across expenditure categories. It takes on a value of zero when all the expenditure is concentrated on a single item, and is equal to $\ln(k)$ (> 0) when the expenditure shares on all items are equal. As entropy rises when expenditures are distributed more smoothly across different goods, it can be used as a measure of spending diversity.[1]

[1]There are several other other ways to measure spending diversity, like the Hirschmann-Herfindahl

In order to replicate the cross-country studies cited above, we order our sample of households according to their expenditure levels ($x_1 < x_2 < ...x_n$) and partition them into ten subgroups, i.e. into income deciles.[2] The expenditure shares are then averaged within these subgroups in order to derive a measure of the diversity of aggregated spending at the decile level. To do so, the average expenditure shares at the decile level are denoted by $\hat{s}_{jd} = [10/n] \sum_{i \in d} s_{ij}$, where d is the decile under consideration. The entropy $\hat{E}$ of these shares $\hat{E}_d(\hat{s}_{jd})$ is then calculated as a function of the average income level of households within a decile and denoted as the spending diversity at the decile level.[3] As the expenditure distributions within the richer (poorer) deciles are likely to be similar to the distributions of aggregate expenditures in richer (poorer) countries, we can compare our results to those derived in the cross country studies cited above.

In order to extract the cross-sectional relations that individual household and decile entropies E_i and $\hat{E}$ have with income, we use kernel regressions based on Nadarya (1964) and Watson (1964). These are non-parametric regressions for which it is not necessary to assume a specific functional form for the relationship between E and x. We use second order polynomial terms and choose the bandwidth that minimizes the mean integrated squared error.[4] As there is a smaller number of observations at the decile level (i.e. for $\hat{E}$), a larger bandwidth (50) is used in this case in order to avoid discontinuities in the kernel regression.

In terms of data, we use annual household data sourced from the UK Family Expenditure Survey (FES) from 1990 to 2000. Over this time period the classification method for expenditure categories has been subject to change. To ensure consistency across sample periods, the classification method specified

or the Gini index. In a different analysis based on the same data,Chai et al. (2015) show that using such measures of consumption diversity instead of the level of entropy does not affect the shape of the spending diversity curve, i.e. the qualitative relation between diversity and income.

[2]Figure 8 shows the results when households are instead partitioned into 20 or 50 subgroups

[3]As we consider households' equivalence-scale-adjusted expenditures x_i and not their actual expenditures, we base our analysis on the average of the budget shares of all households within a decile (i.e. on $\hat{s}_{jd} = [10/n] \sum_{i \in d} s_{ij}$), instead of basing it on the share of the total (non-equivalence-scale-adjusted) expenditures on good j by households falling into decile d.

[4]This is done using the **lpoly** command in Stata

by the Office of National Statistics in 2000 featuring $k = 12$ categories (see Table 1) was selected and retrospectively applied to the data. In addition, we also study the case of three goods in which the 12 categories are aggregated into 'Food', 'Goods' and 'Services', and the case of 200+ aggregation categories in which no aggregation procedure is used.

We exclude housing expenditures because of well-known problems with this data (Tanner, 1999; Blow et al., 2004). Savings are also excluded as we focus on consumption expenditures. We censor the data by removing Northern Ireland and households with more than two adults (which mainly affects shared houses), but keep all households with two adults and any number of children. OECD equivalence scales are used to control for differences in household composition.

Household spending on major durable spending items (e.g. automobile purchases) is converted into weekly expenditure equivalents as provided by the UK Office for statistics. Inflation is accounted for by using the Retail Price Index (RPI) percentage change over 12 months.[5]Appendix ? shows that the trends of total expenditure in our data set are broadly consistent with other data sets devised by Blow et al. (2004) and the UK National Accounts. Some differences are likely due to the fact that we have dropped household with more than two adults and excluded recall categories from 1986. Across the thirty year period, the average annual sample size is about 6000 observations but drops to 5000 between 1998 to 2000.

As most household expenditure surveys have less observations at high levels of household income, a common problem is sample bias. However, Tanner (1999) finds that the ratio of non-housing total expenditure in the FES to non-housing total expenditure in the National Accounts was around 90 per cent between 1974 and 1992. This instills us with some confidence that the magnitude of the sampling bias is not too large as the FES expenditure match the National accounts relatively well in this earlier period. Moreover, the problem of sample bias is also mitigated by the fact that our sample sizes

[5]This is calculated using data from the UK Office of National statistics on all consumption items except for housing and mortgage payments (CDKG).

are relatively large (between 2,000 to 3,000 observations per year). We also removed all observations that were more than three standard deviations above the average household income.

Table 2 provides an overview of how the average decile budget shares $\hat{s}_{jd}$ for the three broad categories food, goods, and services evolve across income levels and time. Income x is measured by real weekly total expenditures. This table reveals a relatively stable pattern that is consistent with Engel;s law: as household income rises, the average budget share dedicated to food declines. Also consistent with other studies is the fact that poor households on average spend a considerable fraction of their budget on food(Banerjee and Duflo, 2007; Clements et al., 2006), while spending on average tends to become more widely dispersed across different expenditure categories when income rises.

FIGURE 1 ABOUT HERE

Figure 1 depicts the estimated cross-sectional relation between the individual household spending diversity E_i and household income and also that between decile spending diversity $\hat{E}$ and average decile income (income is measured by real weekly total household expenditure). The first row depicts the case where consumption items are aggregated into three broad categories (food, goods and services), the middle row depicts the 12 good aggregation (See Table 1), and the last row the case where goods are highly disaggregated (200+ expenditure categories). The column on the left depicts spending diversity E_i for individual households, while the right column depics spending diversity $\hat{E}$ on the decile level. Each figure depicts curves for three years: 1990, 1995 and 2000.[6]

As there are some robust pattern across all levels of aggregation, we derive the following stylized facts:

[6]The choice of years does not seem to affect the results. We conducted tests in other years between 1987 and 2000 and found similar results. due to major changes in the expenditure categories used by the UK Family expenditure survey, years after 2001 are not used.

- **Stylized fact 1**: There is an inverse-U relation between household spending diversity E_i (measured by the the level of entropy) and household income
- **Stylized fact 2**: There is either a positive relation or an inverse-U relation between the diversity (entropy) $\hat{E}$ of (aggregated) decile spending and average decile income.

An interesting pattern of the data is that the decile entropy $\hat{E}$ seems to keep rising in income at income levels at which individual entropy E_i already falls in income and that $\hat{E}$ consequently reaches its maximum (in case there is one) at higher levels of income than E_i does. Moreover, entropies fall more rapidly in income at high income leves in the case of 200+ aggregation categories.

Beyond differences in the shape, there are also important differences in the levels of spending diversity between the household and the aggregate (decile) level. This can be seen in Figure 2, in which the case of three expenditure categories is considered and in which E_i and $\hat{E}$ are depicted together for the years 1990 (left), 1995 (middle) and 2000 (right).From this figure, as well as from Figures 4 and 6 that consider the case of 12 and 200+ expenditure categories, the following stylized fact emerges:

- **Stylized fact 3**: The diversity (entropy) $\hat{E}$ of (aggregated) decile consumption spending exceeds the diversity (entropy) E_i of household consumption spending for each level of household income x.

As $\hat{E} = E_i$ would hold for a given level of x if each household would spend its income in exactly the same fashion, the observed pattern must stem from the fact that different households allocate their spending differently across different goods. Figure 3 shows the calculated difference $\hat{E} - E_i$ between decile and household level spending diversities for the case of three consumption categories in each year. Figures 5 and 7 show the same for the cases of 12 and 200+ categories. From these figures, we obtain our last stylized fact

- **Stylized fact 4**: The difference $\hat{E} - E_i$ between the (aggregated) decile and the household spending diversities tends to either rise in income or to first fall and to then rise in income (U-relation).

As can be inferred from figures 2, 4 and 6, this stylized fact results from the following shapes of the entropy curves: at low income levels, both $\hat{E}$ and E_i rise and $\hat{E} - E_i$ can either rise or fall. At high levels of income, household spending diversity E_i falls, while $\hat{E}$ either rises or falls less strongly, implying that $\hat{E} - E_i$ increases.

It should be noted that, unlike in Figure 1, the E_i curves are shortened to the length of the $\hat{E}$ curves in Figures 2-7. In these figures, both curves therefore begin at the average income of the lowest income decile and end at the average income of the highest income decile as those are the values for which the $\hat{E}$ curve is properly defined.[7] In Figure 8, the $\hat{E}$ curves are plotted for the cases where households are grouped into 20 groups (middle) and 50 groups (right) and where averages are formed within these smaller groups (the case of three consumption categories is considered). The $\hat{E}$ curves can then be draws for a larger income range, but their shapes do not change much in the range considered in graphs 2-7.

3 Model setup

In the following, the model setup that is used to explain the stylized facts and to undertake a welfare analysis, is introduced. The utility of household i is given by the generalized Stone Geary form:

$$U_i = \left[\sum_{j=1}^{k} \beta_{ij}^{\frac{1}{\varepsilon}} \left(q_{ij} - \gamma_j \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{2}$$

The terms $q_{ij} \geq 0$ denote the quantity of good j consumed by household i and $\gamma_j \gtreqless 0$ the "subsistence consumption" level of good j. This utility function is only defined if $q_{ij} \geq \gamma_j$ holds, i.e. if the household is rich enough to consume the subsistence level of all goods with $\gamma_j > 0$. The parameter $\varepsilon > 0$ determines

[7]We abstain from artificially extending this curve to lower and higher values of x in order to avoid that for the lowest and highest values of x E_i "mechanically" falls short of $\hat{E}$ simply due to the fact that E_i rises (falls) in x when x is small (large) and that this trend is averaged out in the $\hat{E}$ curve.

the degree of substitutability between goods: when $\varepsilon \to 0$, goods become perfectly complementary (utility is then given by $\underset{\varepsilon \to 0}{lim} U_i = \underset{j}{min}\{\beta_{ij}(q_{ij} - \gamma_j)\}$), while they become perfectly substitutable when $\varepsilon \to \infty$. When $\varepsilon = 1$, utility is given by the standard Stone-Geary (cite Stone-Geary???) form: $U_i = \prod_{j=1}^{k} (q_{ij} - \gamma_j)^{\beta_{ij}}$. The degree of substitutability therefore increases in ε.

It is assumed that $\sum_{j=1}^{k} \beta_{ij} = 1$ holds. In order to explain the empirically observed heterogeneity of consumption pattern of households with similar incomes, some preference heterogeneity is needed[8] : It turns out that all pattern observed in the data can be explained by assuming that only the parameters $\beta_{ij} \geq 0$ can vary across households while the parametes γ_j are the same for all of them. It is therefore assumed that the subsistence consumption levels γ_j are the same for all households (as they might reflect "biological" needs for food, shelter etc.), while households might differ with respect to the relative importance that they attribute to consumption exceeding these levels (and that is reflected by the size of the parameters β_{ij}).

Total income (or expenditures) of household i is denoted by x_i and the price of one unit of good j by p_j. The budget constraint of household i is therefore given by[9]:

$$x_i = \sum_{j=1}^{k} p_j q_{ij} \tag{3}$$

The following analysis focuses on the case in which income of household i lies (weakly) above the threshold income level $\underline{x}$ which is required to purchase positive quantities of all goods ($q_{ij} > 0$), i.e. in which $x_i \geq \underline{x}$ (**Condition A**) holds[10]. Setting up the Lagrangian $L_i = U_i + \lambda_i \left[x_i - \sum_{j=1}^{k} p_j q_{ij} \right]$ and deriving with respect to q_{ij} gives the first order conditions:

$$\frac{\partial L_i}{\partial q_{ij}} = U_i^{\frac{1}{\varepsilon - 1}} \beta_{ij}^{\frac{1}{\varepsilon}} (q_{ij} - \gamma_j)^{-\frac{1}{\varepsilon}} - \lambda_i p_j = 0 \tag{4}$$

[8] Alternatively, one could assume that households face different relative prices, which, however, seems less plausible.

[9] As utility is strictly increasing in q_{ij}, this budget constraint is always satisfied with equality

[10] In the case where $\gamma_j \geq 0 \; \forall j$, Condition A is given by $x_i > \sum_{j=1}^{k} p_j \gamma_j = \underline{x}$.

Dividing the first order conditions for goods j and $l \neq j$ by each other gives the equation:

$$\frac{q_{ij} - \gamma_j}{(q_{il} - \gamma_l)} = \frac{\beta_{ij}}{\beta_{il}} \left(\frac{p_l}{p_j} \right)^{\varepsilon} \tag{5}$$

Combining equations 5 and 3 then allows us to solve for the optimal quantity q^*_{ij} of good j by household i:[11]

$$q^*_{ij} = \frac{x_i - \sum_{l \neq j} \left[p_l \gamma_l - p_l \gamma_j \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} \right]}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon}} \tag{6}$$

The optimal quantity q^*_{ij} is a linear function of income x_i, implying linear Engel curves. As q_{ij} increases by $\frac{1}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon}}$ $(= \frac{\beta_{ij}}{p_j}$ if $\varepsilon = 1)$ units for each unit that x_i increases, the slope of the Engel curve for good j increases in β_{ij} and decreases in p_j. Differences in the taste parameters β_{ij} across households can therefore generate the heteroskedasticity of Engel curves that is observed in the data[12].

The Engel curve of household i for good j shifts up when γ_j increases and the size of this shift does not depend on x_i or γ_l $(l \neq j)$[13]. The income elasticity of demand for good j by individual i is given by

$$\epsilon_{jx}(i) = \frac{\partial q^*_{ij}}{\partial x_i} \frac{x_i}{q^*_{ij}} = \frac{x_i}{x_i - \sum_{l \neq j} \left[p_l \gamma_l - p_l \gamma_j \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} \right]} > 0 \tag{7}$$

and therefore decreases if γ_j increases. goods with a high value γ_j therefore represent basic need goods on which poor households concentrate their expenditures, while goods with a lower (or even negative) value γ_j are more luxurious and are only purchased in substantive amounts by rich households.

[11]Equation 5 can be rewritten as $q_{il} = \gamma_l + \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} (q_{ij} - \gamma_j)$ and equation 3 as $q_{ij} = \frac{x_i - \sum_{l \neq j} p_l q_{il}}{p_j}$. Inserting the first into the latter then gives the result.

[12]Another way to generate such heteroskedasticity within the model setup would be to assume that different households face different prices p_{ij} for different goods j. We, however, focus on the case of preference heterogeneity which we believe to be an important driver of this empirically observed heteroskedasticity.

[13]While the Engel curve of household i is only defined for $x_i \geq \underline{x}$ (i.e. when $q^*_{ij} \geq 0$), they all "originate" at the values $q_{ij} = \gamma_j \gtreqless 0$ that are reached when $x_i = \sum_{j=1}^{k} p_j \gamma_j$ holds.

The share of the budget that household i allocates to good j is given by $s_{ij} = \frac{q_{ij}^* p_j}{x_i}$ and increases in β_{ij} as q_{ij}^* increases in β_{ij} if Condition A holds (as a strict inequality).

3.1 An example with three goods

In order to show which mechanisms can generate the four stylized facts observed in the data, the following simple example is considered: There are three goods consisting of one basic need good $j = 1$ for which $\gamma_1 > 0$ holds and two more luxurious goods $j = 2$ and $j = 3$, for which $\gamma_2 = \gamma_3 \gtreqless 0$ holds. While the price of good 1 is normalized to one ($p_1 = 1$), the prices of goods 2 and 3 are given by $p_2 = p_3 = p$. While β_{i1} (the welfare weight on good 1) is assumed to be the same for all households and equal to the constant $\beta_{i1} = 1 - \bar{\beta}$, the degree to which household i prefers good 2 over good 3 is allowed to vary within the range in which the household still purchases positive quantities of all available goods and in which $\beta_{i2} + \beta_{i3} = \bar{\beta}$ holds.

From equation 6 we can infer that q_{i1} and also the sum $q_{i2} + q_{i3}$ then only depend on on the aggregate welfare weight $\bar{\beta}$ for goods 2 and 3, but not on how much goods 2 and 3 are liked by a particular household. This allows to study the role of individual heterogeneity in the following simple setup:

There are two households ($i = 1$ and $i = 2$) with the same income $x_i = x$ that have opposing preferences with respect to the otherwise identical goods 2 and 3, so that $\beta_{12} = \beta_{23}$ and $\beta_{13} = \beta_{22}$ holds in addition to $\beta_{i2} + \beta_{i3} = \bar{\beta}$ (implying that $\beta_{12} + \beta_{22} = \beta_{13} + \beta_{23} = \bar{\beta}$). The aggregated demand $Q_j = q_{1j} + q_{2j}$ for goods $j = 1$, $j = 2$ and $j = 3$ then only depends on x, γ_j, p and $\bar{\beta}$, but not on the individual values β_{i2} and β_{i3} as individual preference heterogeneity washes out in the aggregate. Aggregated demand for good j and also the elasticity of aggregated demand with respect to the relative price p is therefore the same as in the case where both households value both goods equally ($\beta_{i2} = \beta_{i3} = \frac{\bar{\beta}}{2}$) and can also be derived from the utility maximization problem of two "average" households with preference parameters $\beta_{a1} = 1 - \bar{\beta}$ and

$\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$ and (per household) expenditures $x_a = x$.[14]

Using equation 6 and the parameter assumptions from above, the optimal budget shares can be derived as

$$s_{11}(x) = s_{21}(x) = \frac{q^*_{11}(x)}{x} = \frac{\left(1-\bar{\beta}\right)\left(x - 2p\gamma_2\right)p^{\varepsilon} + \gamma_1\bar{\beta}p}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{8}$$

$$s_{12}(x) = s_{23}(x) = \frac{pq^*_{12}(x)}{x} = \frac{p\left[\beta_{12}\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{9}$$

$$s_{13}(x) = s_{22}(x) = \frac{pq^*_{13}(x)}{x} = \frac{p\left[\left(\bar{\beta} - \beta_{12}\right)\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{10}$$

Without loss of generality, it is assumed that $\beta_{12} = \beta_{23} > \beta_{13} = \beta_{22}$ holds, implying that $s_{12}(x) = s_{23}(x) > s_{13}(x) = s_{22}(x)$ if $x > \underline{x}$, i.e. that household 1 prefers good 2 over good 3, while household 3 prefers good 3 over good 2.

The levels of household spending diversities $E_i(x)$ measured by the entropies of consumption spending of households are given by

$$E_1(x) = -s_{11}lns_{11} - s_{12}lns_{12} - s_{13}lns_{13} = E_2(x) = -s_{21}lns_{21} - s_{22}lns_{22} - s_{23}lns_{23} \tag{11}$$

For a given income level x, these entropies are therefore the same for both households as their consumption shares coincide for good 1, and are simply reversed for goods 2 and 3.

When aggregated consumption is considered, the share $\hat{s}_1(x) = s_{11}(x) = s_{21}(x)$ of the aggregated income (which equals $2x$) is spent on good 1 and the shares $\hat{s}_2(x) = \hat{s}_3(x) = \frac{p\left(q^*_{12}(x) + q^*_{22}(x)\right)}{2x} = \frac{s_{12}(x) + s_{22}(x)}{2} = \frac{s_{12}(x) + s_{13}(x)}{2}$ on goods 2 and 3. These shares are of equal size as the heterogeneity of individual consumption washes out in the aggregate. The entropy of aggregated consumption spending when the spending of each of the two households is equal to x is therefore given by

$$\hat{E}(x) = -\hat{s}_1 ln\hat{s}_1 - \hat{s}_2 ln\hat{s}_2 - \hat{s}_3 ln\hat{s}_3 = -\hat{s}_1 ln\hat{s}_1 - 2\hat{s}_2 ln\hat{s}_2 \tag{12}$$

[14] The analysis would be similar in a setting with more than two households as long as there is an equal number of households of each type.

Lemma 1. *Suppose that* $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ ***(Condition B)*** *holds, implying that the spending shares on the basic need good* 1 *fall in income* x *(i.e. that* $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{\partial(\hat{s}_1(x))}{\partial x} < 0$ *holds). Then, the entropy of aggregated consumption spending* $\hat{E}$ *continuously rises in* x *when* $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ *holds (Case i), while it first rises in* x *(for* $\underline{x} \leq x < \check{x}$*) and then falls in* x *(for* $\check{x} < x < \infty$*) when* $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$ *holds and when* γ_1 *is sufficiently large (Case ii).*
(In Case ii, γ_1 *is sufficiently large if* $\gamma_1 > p\gamma_2$ *and* $\gamma_2 \geq 0$ ***(Condition C1)*** *or if* $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{12}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{12}\right)}$ *and* $\gamma_2 < 0$ *hold* ***(Condition C2)****).*

Proof. See Appendix A1. □

The parameter conditions in this Lemma guarantee that poor households (for which x is close to $\underline{x}$) spend more than one third of their budget on the basic need good 1 and that the budget share of this good falls as income grows, implying that the shares $\hat{s}_2(x) = \hat{s}_3(x)$ rise in x. At low levels of income, an increase in income therefore always leads to a rise in the entropy of aggregated consumption spending $\hat{E}$. This is due to the fact that it leads to a smoother allocation of consumption spending over the three goods (note that entropy is maximal if one third of the budget is spent on each of the goods). If the budget share of good 1 still exceeds one third when income becomes infinitely large (Case i), $\hat{E}$ therefore always rises in x. When the budget share of good 1 falls below one third at a finite income threshold $\check{x} > \underline{x}$, there is an inverse-U relation between $\hat{E}$ and x. $\hat{E}$ then first rises in x, but falls in x once $x > \check{x}$ holds. The model can therefore generate **stylized fact 2**.

While the model is not designed to exactly fit the data in the case of three goods, but to rather provide qualitative insights that can also be applied to the case with more than three goods, the assumptions about the shares of the aggregated expenditures $\hat{s}_j$ made in Lemma 1 do indeed match the data quite well in the case of three goods: Table 2 shows that in this case, the average budget share of food exceeds one third for all but the richest income decile and that it falls in income (Engel's law). Moreover, the average budget shares for goods and services initially lie below one third and tend to rise in income[15]. Figure 1 (the top right figure) shows that the decile entropy $\hat{E}$ tends to always rise in income in 1995 and in 2000, and only falls in income for high income levels in 1990. This pattern is therefore in line with Lemma 1.

When $x > \underline{x}$ holds, the entropy E_i of individual consumption spending falls short of that of aggregated consumption spending (i.e. $E_i < \hat{E}$ holds) as the budget shares are more unequal at the individual level, implying a lower level of entropy and therefore of consumption diversity[16]. This is in line with **stylized fact 3**. The following proposition analyzes the relation between E_i and $\hat{E}$:

Proposition 1. *Suppose that $\gamma_1 > -2\gamma_2 p$ (**Condition D**) and that the conditions from Lemma 1 (leading to either Case i or ii) hold, implying that $\frac{\partial(s_{i1}(x))}{\partial x} < 0$, $\frac{\partial(s_{12}(x))}{\partial x} > 0$ and that $\frac{\partial(s_{12}(x))}{\partial x} > \frac{\partial(s_{13}(x))}{\partial x}$ hold. Then, the (non-negative) difference $\hat{E} - E_i$ between the individual and the aggregated consumption entropy increases in income x if $\gamma_2 > 0$ holds, while it first decreases and then increases in income when $\gamma_2 < 0$ holds.*
Formally, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ when $\gamma_2 > 0$, while $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ for $\underline{x} \leq x < \tilde{x}$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ for $\tilde{x} < x < \infty$ when $\gamma_2 < 0$ (when $\gamma_2 = 0$, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ $(= 0)$ holds for $x > \underline{x}$ $(x = \underline{x})$).

Proof. See Appendix A2. □

This proposition shows under which conditions the model can generate **stylized fact 4**. Whether the entropy difference $\hat{E} - E_i$ continuously rises in x or is U-shaped in x therefore depends on whether γ_2 is positive or negative. In the following, both cases are discussed separately:

When $\gamma_2 \geq 0$, $E_i(x) = \hat{E}(x)$ holds at the minimal income level $x = \underline{x}$ as all households then consume the same quantities $q_{ij} = \gamma_j$. When income exceeds the level $\underline{x}$, individual households allocate their spending in more uneven

[15] Unlike in the model, these shares are, however, not of equal size

[16] If $s_{12}(x) > s_{13}(x)$ and $s_{12}(x) + s_{13}(x) = 2\hat{s}_2(x)$, the term $-s_{12}(x) ln s_{12}(x) - s_{13}(x) ln s_{13}(x)$ falls in $s_{12}(x)$ and is therefore maximal if $s_{12}(x) = s_{13}(x)$ holds. This implies that E_i is maximal and that $E_i = \hat{E}$ holds if $s_{12}(x) = s_{13}(x)$.

ways across goods 2 and 3 than households do on average. $\hat{E}$ then exceeds E_i, and the more so the more heterogeneous individual tastes are, i.e. the more $\beta_{12} = \beta_{23}$ exceed the value $\frac{\bar{\beta}}{2}$ of the average consumer. As the consumption of individual households becomes more specialized when income rises, $\hat{E} - E_i$ then continuousely rises in x. The heterogeneity of demand is therefore emergent in the sense that differences in spending patterns between different household types grow when household income rises. Due to the fact that individual consumption pattern closely reflect average consumption pattern at low levels of income, the assumptions (from Lemma 1) that guarantee that $\frac{\partial \hat{E}}{\partial x} > 0$ holds for low income levels also guarantee that E_i rises in x when x is low. E_i can, however, fall as x rises when x is sufficiently high and when the share of the budget that a household allocates to either good 2 or 3 becomes disproportionally large. There can therefore be an inverse-U relationship between individual consumption entropy E_i and x as observed in the data (**stylized fact 1**). Given that the model generates stylized fact 1, the finding that $\hat{E} - E_i$ rises in x when $\gamma_2 > 0$ holds implies that there can be the following relations between $\hat{E}$ and x in this case: $\hat{E}$ either continuously rises in x (see **Figure 10a**???), or that there is also an inverse-U relation between $\hat{E}$ and x and $\hat{E}$ reaches its maximum for larger values of x than E_i does (see **Figure 10b**???).

When $\gamma_2 < 0$ holds, $\hat{E} > E_i$ holds even at the minimal income level $x = \underline{x}$, as individual households then do not purchase any units of either good 2 or 3 (i.e. as $s_{13} = s_{22} = 0$ holds), while the aggregate spending shares $\hat{s}_2 = \hat{s}_3$ are positive for these goods. As $\frac{\partial E_i}{\partial x} = -\frac{\partial s_{i1}}{\partial x}(lns_{i1} + 1) - \frac{\partial s_{i2}}{\partial x}(lns_{i2} + 1) - \frac{\partial s_{i3}}{\partial x}(lns_{i3} + 1)$ and as $\frac{\partial s_{i2}}{\partial x}$ and $\frac{\partial s_{i3}}{\partial x}$ are positive in the case considered in proposition 1 when $\gamma_2 < 0$ holds (this is shown at the beginning of the proof of Proposition 1) the derivative $\frac{\partial E_i}{\partial x}$ gets infinitely large when s_{13} or s_{22} go to zero. This implies that the spending diversity of a householdind increases substantially when a household starts consuming positive quantities of an good that it has not consumed before. When x is close to $\underline{x}$, $\hat{E} - E_i$ therefore falls in x when $\gamma_2 < 0$ holds as $\frac{\partial E_i}{\partial x}$ exceeds the value of $\frac{\partial \hat{E}}{\partial x}$ which is finite even at the point where $x = \underline{x}$[17]. When income is so large that all consumption shares are sufficiently distinct from zero, the mechanisms that are already at work in the case where $\gamma_2 \geq 0$ become dominant again and a further increase in income induces households to devote an ever increasing share of their budget towards their preferred consumption good. This reduces individual consumption entropy relative to aggregated consumption entropy as consumption heterogeneity washes out in the aggregate. Consequently, $\hat{E} - E_i$ again rises in x when x is sufficiently large (i.e. when $\tilde{x} < x < \infty$) and increasing spending diversity at the aggregate level can again go along with declining diversity at the household level[18]. Given that parameters are such that there is an inverse-U relation between E_i and x (stylized fact 1), the fact that there is a U-shaped relation between the entropy difference $\hat{E} - E_i$ and x when $\gamma_2 < 0$ holds therefore implies that the relation between $\hat{E}$ and x can again be of two forms in this case: $\hat{E}$ can continuously rise in x (see **Figure 10c**???), or there is an inverse-U relation between $\hat{E}$ and x and the inverse-U of $\hat{E}$ reaching its maximal level at a higher level of income than the inverse-U of E_i (see **Figure 10d**???).

As figures 3, 5, and 7 show that $\hat{E} - E_i$ can either rise or be U-shaped in x in our data, both the case where $\gamma_2 > 0$ holds and the case where $\gamma_2 < 0$ holds (with the latter implying a larger income elasticity for goods 2 and 3 then the former) therefore seem to be relevant cases in order to explain the observed empirical pattern.

4 The value of product variety

In this section it is shown that the insights obtained about how heterogeneity in consumption pattern depends on the level of income can have important welfare consequences. This is done by analyzing the value of product variety.

[17]This argument can be generalized to the case of more than three consumption goods.

[18]Condition D, which can only be binding if $\gamma_2 < 0$ holds, is imposed to ensure that $\frac{\partial(s_{12}(x))}{\partial x} > \frac{\partial(s_{13}(x))}{\partial x}$ holds. If this condition is violated, $\frac{\partial(\hat{E}(x) - E_i(x))}{\partial x} < 0$ holds for all values of x (this is shown in the proof of Proposition 1). As this case is not in line with the empirical observations, Condition D is imposed in Proposition 1.

This is a key issue when it comes to designing optimal innovation, trade, and antitrust policies as these policies affect how large the set of goods is that households can consume. The analysis is carried out within the three-good example from Subsection 3.1.

It is assumed that initially only the "basic need" good 1 exists and that goods 2 and 3 can be introduced through innovation or can be made available through a free trade agreement. While good 1 is always sold at price $p_1 = 1$, goods 2 and 3 are now only sold at price $p_2 = p_3 = p$ when they are available, but have an infinite price when not. In order to allow to compare the welfare levels with and without goods 2 and 3, the case is considered in which $\gamma_2 = \gamma_3 < 0$ holds, i.e. in which there is no required positive subsistence consumption level for goods 2 and 3.

As above, the case in which $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{12} = \beta_{23} > \frac{\bar{\beta}}{2}$ and $\beta_{13} = \beta_{22} = \bar{\beta} - \beta_{12} \geq 0$ is considered in which household 1 prefers good 2 over good 3 and household 2 has exactly the opposite preferences and prefers good 3 over good 2. As aggregated demand does in this case not depend on the extent of preference heterogeneity (i.e. on β_{12}) and can also be derived from the utility maximization problem of two households with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$), we can analyze whether a household with heterogeneous preferences ($\beta_{i2} \neq \frac{\bar{\beta}}{2}$) values an increase in product variety in a different way than a household with average preferences does. This is an interesting question as it allows to evaluate whether and how ignoring the preference heterogeneity that we have identified as the driving force behind our empirical observations and instead focusing on a simpler model with hypothetical average consumers leads to biased welfare results. It should be noted that such a simpler model does not only allow to correcty derive aggegate demand, but would also allow to correctly determine the incentives to innovate in an environment with endogenous innovation by profit seeking firms, at least when the inventors of goods 2 and 3 charge the same endogenous (monopoly) prices in equilibrium. While preference heterogeneity does not affect demand and the profits to innovate in our setting, it might, however, nevertheless affect the value that households attribute to an increase in product variety.

While it is obvious that a household benefits more from the introduction of a good that it likes a lot than from the introduction of a good that it does not like, the question considered here is whether a household benefits more or less from the joint introduction of both goods 2 and 3 when it puts a larger relative welfare weight β_{ij} on one of them, keeping $\beta_{i2}+\beta_{i3}=\bar{\beta}$ and therefore the total quantity of the two goods that it consumes constant[19]. The extent of preference heterogeneity is then increasing in β_{ij} when $\beta_{ij}>\frac{\bar{\beta}}{2}$ holds for an good $j\epsilon\{2;3\}$. To which extent a household values variety is measured by the amount F_i of income x_i (or by the quantity F_i of good 1) that it is maximally willing to give up in order to be able to not only purchase good 1 at price 1, but to in addition purchase goods 2 and 3 at price p. The value that a household with average preferences ($\beta_{a2}=\beta_{a3}=\frac{\bar{\beta}}{2}$) attributes to variety is denoted by F_a (so that $F_i|_{\beta_{ij}=\frac{\bar{\beta}}{2}}=F_a$ holds) and the extent to which an individual and an average household disagree about the value of product variety is measured by the term $D\equiv\frac{F_i-F_a}{x_i}$ (we divide by x_i as both F_i and F_a depend positively on x_i). As before, the case is considered in which $x_i\geq\underline{x}$ holds and in which households consume positive quantities of all available goods.

Proposition 2. *When $\gamma_2=\gamma_3<0$ and $\varepsilon\neq1$, the following holds:*

a) A household with heterogeneous preferences ($\beta_{ij}\neq\frac{\bar{\beta}}{2}$ for $j\epsilon\{2;3\}$, but $\beta_{i2}+\beta_{i3}=\bar{\beta}$) values variety more than a household with average preferences ($\beta_{a2}=\beta_{a3}=\frac{\bar{\beta}}{2}$) does and the more so, the more heterogeneous these preferences are (i.e. $F_i>F_a$ holds, with $\frac{\partial F_i}{\partial\beta_{ij}}>0$ when $\beta_{ij}>\frac{\bar{\beta}}{2}$ holds for a good $j\epsilon\{2;3\}$).

Further results:

b) When γ_2 becomes more negative, implying a higher income elasticity for goods 2 and 3, the increase in the value of variety induced by an increase in preference heterogeneity, $\frac{\partial F_i}{\partial\beta_{ij}}$, gets larger when goods are substitutable ($\varepsilon>0$), but smaller when goods are complementary ($\varepsilon<1$), i.e. $sign\frac{\partial^2F_i}{\partial\beta_{i2}\partial\gamma_2}=sign\left(1-\varepsilon\right)$ holds when $\beta_{ij}>\frac{\bar{\beta}}{2}$ for $j\epsilon\{2;3\}$. (Furthermore, assuming that $\beta_{ij}>\frac{\bar{\beta}}{2}$ holds for a good $j\epsilon\{2;3\}$, $\lim\limits_{\gamma_2\to-0}\frac{\partial F_i}{\partial\beta_{ij}}=0$ and $\lim\limits_{\gamma_2\to-\infty}\frac{\partial F_i}{\partial\beta_{ij}}=\infty$ hold

[19] By looking at the joint introduction of two goods, one does not need to consider individual risk preferences that might play a role when instead the welfare consequences of the introduction of only one good of ex ante unknown desirability were studied.

when $\varepsilon > 1$*, and* $\lim_{\gamma_2 \to -0} \frac{\partial F_i}{\partial \beta_{ij}} = \infty$ *and* $\lim_{\gamma_2 \to -\infty} \frac{\partial F_i}{\partial \beta_{ij}} = 0$ *when* $\varepsilon < 1$*. See Figure ???).*

c) When $\underline{x} < x_i < \gamma_1 + \frac{1}{\varepsilon}$ *(*$x_i > \gamma_1 + \frac{1}{\varepsilon}$*) holds, increasing preference heterogeneity leads to a larger (lower) increase in the disagreement* $D \equiv \frac{F_i - F_a}{x_i}$ *about the value of product variety when income* x_i *becomes larger (when* $\beta_{ij} > \frac{\bar{\beta}}{2}$ *holds for an good* $j \epsilon \{2; 3\}$*,* $\frac{\partial^2 D}{\partial \beta_{ij} \partial x_i} > 0$ *therefore holds when* $\underline{x} < x_i < \gamma_1 + \frac{1}{\varepsilon}$ *and* $\frac{\partial^2 D}{\partial \beta_{ij} \partial x_i} < 0$ *when* $x_i > \gamma_1 + \frac{1}{\varepsilon}$*).*

Proof. See Appendix A3. □

Even though aggregated consumption can be derived from the utility maximization problem of households with average preferences, such hypothetical households therefore value variety less than households with heterogeneous preferences do when $\gamma_2 < 0$ holds. Under this parameter condition, newly introduced goods have a relatively high income elasticity, which is is a highly relevant case when it comes to various applications in the areas of innovation and trade. Moreover, the observation that there is in many cases a U-shaped relation between the entropy difference $\hat{E} - E_i$ and x in our data can be explained by our model when $\gamma_2 < 0$ holds. Therefore, this case also seems to be relevant for the sample of goods that we look at in our empirical study.

Studying the welfare of households with average preferences without taking the empirically observed heterogeneity into account consequently leads to an underestimation of the true value that households attach to product variety, and the more so, the larger the extent of preference heterogeneity is.

Parts b) and c) of the proposition analyze how the size of γ_2 (determining the income elasticity of goods 2 and 3), the parameter ε (that determines whether goods are substitutable or complementary to each other), and the level of income x_i affect the effect of preference heterogeneity on the value of product variety. It turns out that these variables can have strong effects, implying that the effect of preference heterogeneity on the disagreement about the value of variety might be quite different for different consumption goods and different economic environments[20]. Interestingly, there are cases in which the disagreement between a household with heterogeneous and a household with average preferences about the value of product variety can become very large: When γ_2 is sufficiently close to zero and $\varepsilon < 1$ holds, the derivative $\frac{\partial F_i}{\partial \beta_{ij}}$ (with $j \epsilon \{2;3\}$) becomes very large as $\underset{\gamma_2 \to -0}{lim} \frac{dF_i}{d\beta_{i2}} = \infty$ holds for any value $\beta_{i2} > \frac{\bar{\beta}}{2}$, implying that even small degrees of preference heterogeneity can lead to large levels of disagreement $D = \frac{F_i - F_a}{x_i}$. The same holds true for the case where γ_2 is sufficiently negative and where $\varepsilon > 1$ holds. The analysis therefore suggests that simple "representative household" models as advocated by Hicks might not be very useful to determine the welfare effects of product variety when heterogeneity of household consumption pattern is a prevalent feature of the data.

4.1 Accounting for variety demand

The above analysis focused on the case in which households are rich enough to purchase non-negative quantities of all goods, i.e. in which $x_i \geq \underline{x}$ holds. For smaller incomes $x_i < \underline{x}$, the model can, however, also account for the empirically observed fact that richer households demand a larger variety of goods (like for example documented by Jackson (1984) and Falkinger and ZweimÃŒller (1996)): when $\gamma_j > 0$ holds for some goods and $\gamma_j < 0$ for others, all households purchase positive quantities of the goods for which $\gamma_j > 0$ holds, while only households with sufficient income purchase positive quantities of goods for which $\gamma_j < 0$ holds (as the marginal utility of the first unit of such goods is finite while that of goods with $\gamma_j > 0$ is infinite). The variety of goods consumed therefore increases in income x_i when there are several goods for which $\gamma_j < 0$ holds. Even when the parameters γ_j are the same for all households, households that differ with respect to the parameters β_{ij} might then increase the variety of goods that they consume in a different order. This becomes clear by looking at the example with three

[20] In order to properly derive the value of product variety in a particular context, one therefore needs to take all these things into account. As our data is not detailed enough to allow us to estimate all these parameters (we do not have information on relative prices), we do not try to quantify the extent of disagreement for particular goods as we fear that the results would not be very robust.

goods from section 3.1, assuming that $\gamma_2 < 0$, $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{12} = \beta_{23} > \frac{\bar{\beta}}{2}$ and $\beta_{13} = \beta_{22} = \bar{\beta} - \beta_{12} \geq 0$ hold: In this case, $\underline{x} = \gamma_1 + 2p\gamma_2 - \gamma_2 \left[\frac{(1-\bar{\beta})p^{\varepsilon} + \bar{\beta}p}{\bar{\beta} - \beta_{12}} \right]$ holds (see the proof of Lemma 1), implying that household 1 (2) stops consuming good 3 (2) when income falls below the level $\underline{x}$. Applying equation 6 to the case where households only purchase the two remaining goods, it can be shown that households stop consuming two goods and spend all their income on the basic need good 1 when there is a further fall in x_i below the threshold $\dot{x} \equiv \gamma_1 - \gamma_2 \frac{1-\bar{\beta}}{\beta_{12}} p^{\varepsilon} < \underline{x}$. When incomes increase from a level $x_i < \dot{x}$ to a level $x_i > \underline{x}$, households therefore expand the variety of goods that they consume from one to three, but in a different order: while household 1 purchases goods 1 and 2 when income lies in the range $\dot{x} < x_i < \underline{x}$, household 2 purchases goods 1 and 3 in this range as tastes are heterogeneous with respect to goods 2 and 3.

Applying these insights to a more general setting with many goods j, the direction in which variety demand grows can then vary across the population when households differ with respect to the parameters β_{ij}. At low income levels, households with different preferences then not only purchase different quantities of different goods but also spend their money on different goods. This implies that the average consumption basket comprises a larger variety of goods than individual consumption baskets do. When individual goods are grouped into broader consumption categories, different households then, moreover, pick the goods which they consume in a more uneven way from these categories than households do on average. This implies that the "diversity of the variety demand" of a household with income x_i is lower than the diversity of the average consumption basket of all households with income x. This is indeed the case when we look at the data: Figure 10 presents the diversity of variety demand across 12 expenditure categories at the household level using data from the year 2000. This figure is derived in the following way: goods are grouped into 12 broader categories indexed by h, with the total number of goods in category h given by N_h. Denoting the number of different goods (i.e. the varieties) that household i consumes within category h by n_{ih}, we then determine the fractions $\frac{n_{ih}}{N_h}$ for all households and categories. The entropy measure described in Section 2 is then applied to these fractions in order to estimate the diversity of household variety demand $D_i = \sum_{h=1}^{12} -\left(\frac{n_{ih}}{N_h} ln\left(\frac{n_{ih}}{N_h}\right)\right)$ across the 12 expenditure categories.

Figure 11 presents the diversity of the average variety demand $\hat{D} = \sum_{h=1}^{12} -\left(\frac{\hat{n}_{dh}}{N_h} ln\left(\frac{\hat{n}_{dh}}{N_h}\right)\right)$ for the same year. In order to determine $\hat{D}$, households are grouped into deciles and the individual varieties n_{ih} are replaced by the variety $\hat{n}_{dh}$ of goods of category h consumed by decile d (i.e. by the number of all goods of which positive quantities are consumed by at least one household falling into the decile). These two figures show that the diversity of variety demand at the household level D_i is lower than the diversity of variety demand $\hat{D}$ at the representative (decile) level and that both D_i and $\hat{D}$ rise in income x. As different households grow their variety demand, the consumption baskets therefore become more diverse in terms of varieties consumed across different expenditure categories.

5 Conclusion

The truth about Mr Brown and Mrs Jones is that they not only possess different spending patterns, but that the differences between these patterns tend to grow in income when income is sufficiently high. In this paper we have highlighted how this 'emergent' aspect of consumption heterogeneity has important implications for the extent to which the behavior of representative consumers reflects the actual behavior and preferences of individual consumers.

While at the aggregate level the spread of household expenditure across categories, i.e. the diversity of spending, continues to rise as income grows, this is not the case when the diversity of expenditures is examined at the household level. Rather, household spending patterns on the more disaggregated level show that rich households tend to concentrate their spending patterns into particular expenditure categories. Because each household concentrates into different types of expenditure categories, diversity of household expenditure can nevertheless increase at the aggregate level while it declines at the individual level.

These findings, in combination with the results obtained in the theoretical analysis, highlight the pitfalls of adopting representative agent models when a considerable extent of heterogeneity across households can be observed in the data. Paying attention to what Mr Brown and Mrs Jones do instead of only focusing on average behavior should therefore become a priority for future research.

6 Appendix A

Proof. Let us define $x_i \equiv \tilde{x}_i + F_i$. Individual i must be indifferent between only consuming item 1 and having income/spending x_i and consuming all three items and having income/spending $x_i - F_i = \tilde{x}_i$. Using equation 2, this implies the following equation:

$$\left(1-\bar{\beta}\right)^{\frac{1}{\epsilon}}\left(\tilde{x}_i + F_i - \gamma_1\right)^{\frac{\epsilon-1}{\epsilon}} + \beta_{i2}^{\frac{1}{\epsilon}}\left(-\gamma_2\right)^{\frac{\epsilon-1}{\epsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\epsilon}}\left(-\gamma_2\right)^{\frac{\epsilon-1}{\epsilon}} = \left(1-\bar{\beta}\right)^{\frac{1}{\epsilon}}\left(q_{i1}(\tilde{x}_i) - \gamma_1\right)^{\frac{\epsilon-1}{\epsilon}} + \left(\beta_{i2}\right)^{\frac{1}{\epsilon}}\left(q_{i2}(\tilde{x}_i) - \gamma_2\right)^{\frac{\epsilon-1}{\epsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\epsilon}}\left(q_{i3}(\tilde{x}_i) - \gamma_2\right)^{\frac{\epsilon-1}{\epsilon}}$$

Subtracting the right hand side (RHS) from the left hand side (LHS) and defining $Q \equiv LHS - RHS$, we can implicitly differentiate this equation and obtain $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$. We therefore analyze how F_i depends on β_{i2}, taking $\tilde{x}_i$ as given (and x_i to be variable), as this simplifies the analysis. This yields the same qualitative results as studying how F_i depends on β_{i2}, taking x_i as given. We obtain $\frac{\partial Q}{\partial F_i} = \frac{\epsilon-1}{\epsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\epsilon}}$. Moreover, $\frac{\partial Q}{\partial \beta_{i2}} = \frac{1}{\epsilon}\left(-\gamma_2\right)^{\frac{\epsilon-1}{\epsilon}}\left[\left(\beta_{i2}\right)^{\frac{1-\epsilon}{\epsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\epsilon}{\epsilon}}\right]$ holds (in order to show this, the condition $\frac{q_{i2}(\tilde{x}_i)-\gamma_2}{\beta_{i2}} = \frac{q_{i3}(\tilde{x}_i)-\gamma_2}{\bar{\beta}-\beta_{i2}}$, which can be derived from the consumers first order conditions, was used).

When $\gamma_2 = \gamma_3 = 0$ or when $\beta_{i2} = \frac{\bar{\beta}}{2}$, we obtain $\frac{\partial Q}{\partial \beta_{i2}} = 0$. Given that $\epsilon \neq 1$, this implies that $\frac{dF_i}{d\beta_{i2}} = 0$ and that (by symmetry) $\frac{dF_i}{d\beta_{i3}} = 0$ must hold. When $\beta_{i2} > \frac{\bar{\beta}}{2}$, $sign\frac{\partial Q}{\partial \beta_{i2}} > 0$ holds, implying that $sign\frac{dF_i}{d\beta_{i2}} = sign(1-\epsilon)$ when $\epsilon \neq 1$. Due to symmetry, also $sign\frac{dF_i}{d\beta_{i3}} = sign(1-\epsilon)$ holds if $\beta_{i3} > \frac{\bar{\beta}}{2}$.

Differentiating $\frac{dF_i}{d\beta_{i2}}$ with respect to $\tilde{x_i}$ gives $sign\frac{\partial\left(\frac{dF_i}{d\beta_{i2}}\right)}{\partial\tilde{x_i}} = sign\left[\frac{\partial Q}{\partial\beta_{i2}}\frac{\partial^2 Q}{\partial F_i\partial\tilde{x_i}}\right] = sign\frac{\partial\left(\frac{dF_i}{d\beta_{i2}}\right)}{\partial x_i}$. As $sign\frac{\partial^2 Q}{\partial F_i\partial\tilde{x_i}} = sign(1-\epsilon)$ and as $\frac{\partial Q}{\partial\beta_{i2}} > 0$ when $\beta_{i2} > \frac{\bar{\beta}}{2}$ and $\gamma_2 < 0$ hold, we therefore get $sign\frac{\partial\left(\frac{dF_i}{d\beta_{i2}}\right)}{\partial x_i} = sign(1-\epsilon)$ under these conditions, implying that an increase in x_i makes $\frac{dF_i}{d\beta_{i2}}$ more positive if $\epsilon < 1$ holds and more negative if $\epsilon > 1$ holds. When $\beta_{i2} = \frac{\bar{\beta}}{2}$, $F_i = F_r$ and household i values variety in the same way as the representative household. When $\epsilon < 1$ ($\epsilon > 1$), the condition $sign\frac{\partial\left(\frac{dF_i}{d\beta_{i2}}\right)}{\partial x_i} = sign(1-\epsilon)$ therefore implies that raising β_{i2} above the level $\frac{\bar{\beta}}{2}$ increases (decreases) $\frac{F_i-F_r}{x_i}$ more, the larger x_i is (note that $F_i < F_r$ when $\epsilon > 1$). $\square$

References

Andrews, D. (1993) Tests for parameter instability and structural change with unknown change point, Econometrica, 61, 821-856.

Aitchison, J. and J.A.C. Brown (1954), A Synthesis of Engel Curve Theory, The Review of Economic Studies, 22(1), 35-46.

Banerjee, A. V. and E. Duflo (2007) "The Economic Lives of the Poor," Journal of Economic Perspectives, 21(1): 141-168.

Banks, J., Blundell, R., and Lewbel, A. (1997). Quadratic Engel curves and consumer demand. Review of Economics and statistics, 79(4), 527-539.

Blow, L., A. Leicester and Z. Oldfield (2004) “Consumption Trends in the UK: 1975-99,". Institute for Fiscal Studies, London.

Beckert, W. and Blundell, R.(2008). “Heterogeneity and the Non-Parametric Analysis of Consumer Choice: Conditions for Invertibility." Review of Economic Studies, 75(4), 1069-1080.

Bertola, G., Foellmi, R., and ZweimÃŒller, J. (2014). Income distribution in macroeconomic models. Princeton University Press.

Bils, M., and P.J. Klenow (2001), Quantifying Quality Growth, *The American Economic Review*, 91 (4), 1006-1030.

Blow, L., Leicester, A., Oldield, Z., 2004. Consumption Trends in the UK 1975-99. Institute for Fiscal Studies, London.

Blundell, R., and M. Stoker (2005) Heterogeneity and Aggregation, *Journal of Economic Literature*, 43 (2), 347-391.

Bresnahan, T., and Gambardella, A. (1998). The Division of Inventive Labor and the Extent of the Market. In E. Helpman (Ed.), Genral Purpose Technologies and Economic Growth (pp. 253-282). Cambridge, M.A.: MIT Press.

Chai, Andreas, Nicholas Rohde, and Jacques Silber. Measuring the diversity of household spending patterns. Journal of Economic Surveys 29, no. 3 (2015): 423-440.

Calvet, L. and E. Common (2003) Behavioral Heterogeneity and the Income Effect, Review of Economics and Statistics 85(3): 653-669.

Clements, K. and Chen, D. (1996) Fundamental similarities in consumer behavior. Applied Economics 28: 747-757.

Clements, K. and Selvanathan, S. (1994) Understanding consumption patterns. Empirical Economics 19: 69-110.

Clements, K.W., Selvanathan, A. and Selvanathan, S. (1996) Applied demand analysis: a survey. Economic Record 72(216):63-81.

Clements, K.W., Yanrui, W. and Zhang, J. (2006) Comparing international consumption patterns. Empirical Economics 31(1):1-30.

Deaton, A., amd Muellbauer, J. (1980). An almost ideal demand system. The American economic review, 70(3), 312-326.

Dixit, A. K., and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. The American Economic Review, 67(3), 297-308.

Earl, P. (1983), *The Economic Imagination*, Wheatsheaf Books, Brighton.

Engel, E. (1857), Die Produktions- und Consumtionsverhältnisse des Königreichs Sachsen. (Reprinted in *Bulletin de Institut International de Statistique* 9: 1-54 (1895).)

Engel, E. (1895), Das Lebenskosten belgischer Arbeiterfamilien früeher und jetzt, *Bulletin de Institut International de Statistique* 9: 1-124.

Falkinger, J. and J. Zweimüller (1996) The cross-country Engel curve for product diversification, Structural Change and Economic Dynamics 7: 79-97.

Foellmi, R. and J. Zweimüller (2008) Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. Journal of Monetary Economics 55: 1317-1328

Foellmi, R., Zweimüller, J., 2006. Income Distribution and Demand-Induced Innovation. Review of Economic Studies 63 (2), 187-212.

Fry, J. M., Fry, T. R., and McLaren, K. R. (2000). Compositional data analysis and zeros in micro data. Applied Economics, 32(8), 953-959.

Galtung, J. (1980), The Basic Needs Approach, in K. Lederer [ed.]*Human Needs*. Oelgeschlager, Gunn and Hain, Cambridge.

GarcÃa, J., and Labeaga, J. M. (1996). Alternative approaches to modelling zero expenditure: an application to spanish demand for tobacco. Oxford Bulletin of Economics and statistics, 58(3), 489-506.

Georgescu-Roegen, N. (1966), *Analytical Economics*, Cambridge University Press, Cambridge.

Grandmont, J. M. (1987) Distribution of preferences and the law of demand, Econometrica 55, 155-161.

Grandmont, J. M. (1992) Transformations of the Commodity Space, Behavioral Heterogeneity, and the Aggregation Problem, Journal of Economic Theory 57: 1-35.

Gronau, R. and D.S. Hamermesh (2008) The Demand for Variety: A Household Production Perspective. Review of Economics and Statistics 90(3): 562-572.

Härdle, W., Applied Nonparametric Regression (Cambridge, U.K.: Cambridge University Press, 1990).

Hallak, J. (2010) "A Product-Quality View of the Linder Hypothesis," Review of Economics and Statistics 92(3): 453-466.

Heckman, J. J. (2001), Micro data, heterogeneity, and the evaluation of public policy: Nobel lecture. *Journal of Political Economy*, 109(4), 673-748.

Heckman, N. E. and R. H. Zamar (2000), Comparing the shapes of regression functions, *Biometrika*, 87(1), 135-144.

Herrmann, E. (1997), Local Bandwidth Choice in Kernel Regression Estimation, *Journal of Computational and Graphical Statistics*, 6(1), 35-54.

Hildenbrand, W. (1994) Market Demand: Theory and Empirical Evidence Princeton University Press.

Jackson, L. (1984), Hierarchic Demand and The Engel Curve for Variety. Review of Economics and Statistics 66 (1): 8-15. Manski, C. F. (1977). The structure of random utility models. Theory and decision, 8(3), 229-25

Moneta, A. and Chai, A. (2014), The Saturation of Engel Curves and its Implications for Structural Change Theory, *Cambridge Journal of Economics*. vol 38(4), pp. 895-923.

McFadden, D. L. (1984). Econometric analysis of qualitative response models. Handbook of econometrics, 2, 1395-1457. Chicago

Metcalfe, S., J. Foster, and R. Ramlogan (2006), Adaptive Economic Growth, *Cambridge Journal of Economics*, 30:7-32.

Lewbel, A., and Pendakur, K. (2009). Tricks with Hicks: The EASI demand system. The American Economic Review, 99(3), 827-863.

Lewbel, A. (2008). Engel Curves, New Palgrave Dictionary of Economics, 2nd edition, Palgrave Macmillan.

Lipsey, R. G., Carlaw, K. I., and Bekar, C. T. (2005). Economic transformations: general purpose technologies and long-term economic growth. Oxford University Press: Oxford.

Nadaraya, E. (1964). Some new estimates for distribution functions, Theory of Probability and Its Applications, 15, 497-500.

Pasinetti, L. (1981), Structural Change and Economic Growth, Cambridge University Press, Cambridge.

Prais, S. J. (1953), Non-Linear Estimates of the Engel Curves. The Review of Economic Studies, 20(2):87-104.

Prais, S. J., and H. S. Houthakker, The Analysis of Family Budgets (Cambridge: Cambridge University Press, 1955).

Quah, J. (1997) The Law of Demand when Income is Price Dependent, Econometrica 65(6): 1421-1442.

Salop, S. (1977). The noisy monopolist: imperfect information, price dispersion and price discrimination. The Review of Economic Studies, 393-406.

Saviotti, P. (2001), Variety, Growth and Demand, pp. 115-138 In U. Witt (ed.), Escaping Satiation. Springer, Berlin.

Tanner, S., 1999. How Much Do Consumers Spend? Comparing the FES and National Accounts. In: Banks, J., Johnson, P. (Eds.), How reliable is the Family Expenditure Survey?. London: Institute for Fiscal Studies.

Theil, H. (1967) Economics and Information Theory, Amsterdam: North Holland.

Theil, H, and R. Finke (1983) The Consumers Demand For Diversity, European Economic Review 23: 395-400.

Theil, H. and K.W. Clements (1987). Applied Demand Analysis: Results from System-wide Approaches. Cambridge: Ballinger.

Tables

Table 1: Categories of the UK Family Expenditure Survey,2000

Category	Examples of spending
Food	Milk, Eggs, vegetables, meats, sweets, non-alcoholic beverages. Take away meals, food bought and consumed at work and school.
Fuel Light and Power	Gas, Electricity, Coal, bottled gas, paraffin, wood.
Alcoholic Drinks	Beer, Lager, Cider, Spirits Liqueurs.
Tobacco	Cigarettes, Pipe tobacco, cigars
Clothing and Footwear	Outerwear, Underwear, Clothing accessories, Footwear, Haberdashery and clothing materials
Household goods	Furniture and Furnishings, Electrical and gas appliances. Hardware, decorative goods. Toilet paper, Pet and garden expenditure.
Domestic and Paid services	Childcare, domestic help, laundry, postage and telephones, subscriptions and stamp duty.
Personal Goods and Services	Hairdressing, cosmetic requisites. Baby goods, medicines and medical goods. Personal effects and travel goods.
Motoring Expenditure	Accessories, parts, repairs and servicing of motor vehicles. Petrol and oil. Insurance, driving lessons and other payment.
Travel	Fares, other transport costs, Purchase and maintenance of non-motor vehicles.
Leisure Goods	TV, video and Audio equipment. Sports, camping and outdoor good and equipment. Newspapers, magazines, books and stationary. Toy, hobbies and photography.
Entertainment and Education Services	Cinema, spectator sports, TV rental and subscription, hotels and holiday expenses, betting stakes, educational fees and maintenance, Ad hoc school expenditure, betting stakes.

Table 2: Average Budget shares - 3 Expenditure Categories

2000				1995				1990			
Income	Food	Goods	Services	Income	Food	Goods	Services	Income	Food	Goods	Ser
31.23	0.608	0.210	0.182	25.40	0.644	0.193	0.163	20.49	0.681	0.164	0.15
51.89	0.552	0.261	0.187	39.57	0.584	0.229	0.187	32.93	0.612	0.206	0.18
67.40	0.495	0.305	0.201	50.63	0.546	0.256	0.198	42.49	0.563	0.252	0.18
83.35	0.470	0.344	0.186	62.04	0.516	0.293	0.191	52.30	0.527	0.279	0.19
100.53	0.432	0.385	0.184	73.98	0.484	0.322	0.194	63.37	0.497	0.311	0.19
119.36	0.415	0.406	0.179	86.80	0.460	0.340	0.200	74.80	0.466	0.351	0.18
140.54	0.380	0.437	0.183	101.81	0.434	0.367	0.200	89.49	0.441	0.358	0.20
166.62	0.365	0.435	0.200	121.41	0.417	0.375	0.207	108.55	0.391	0.387	0.22
203.71	0.330	0.474	0.197	150.26	0.377	0.417	0.207	138.44	0.359	0.393	0.24
292.12	0.277	0.483	0.240	219.75	0.309	0.424	0.267	215.15	0.267	0.442	0.29

Figures

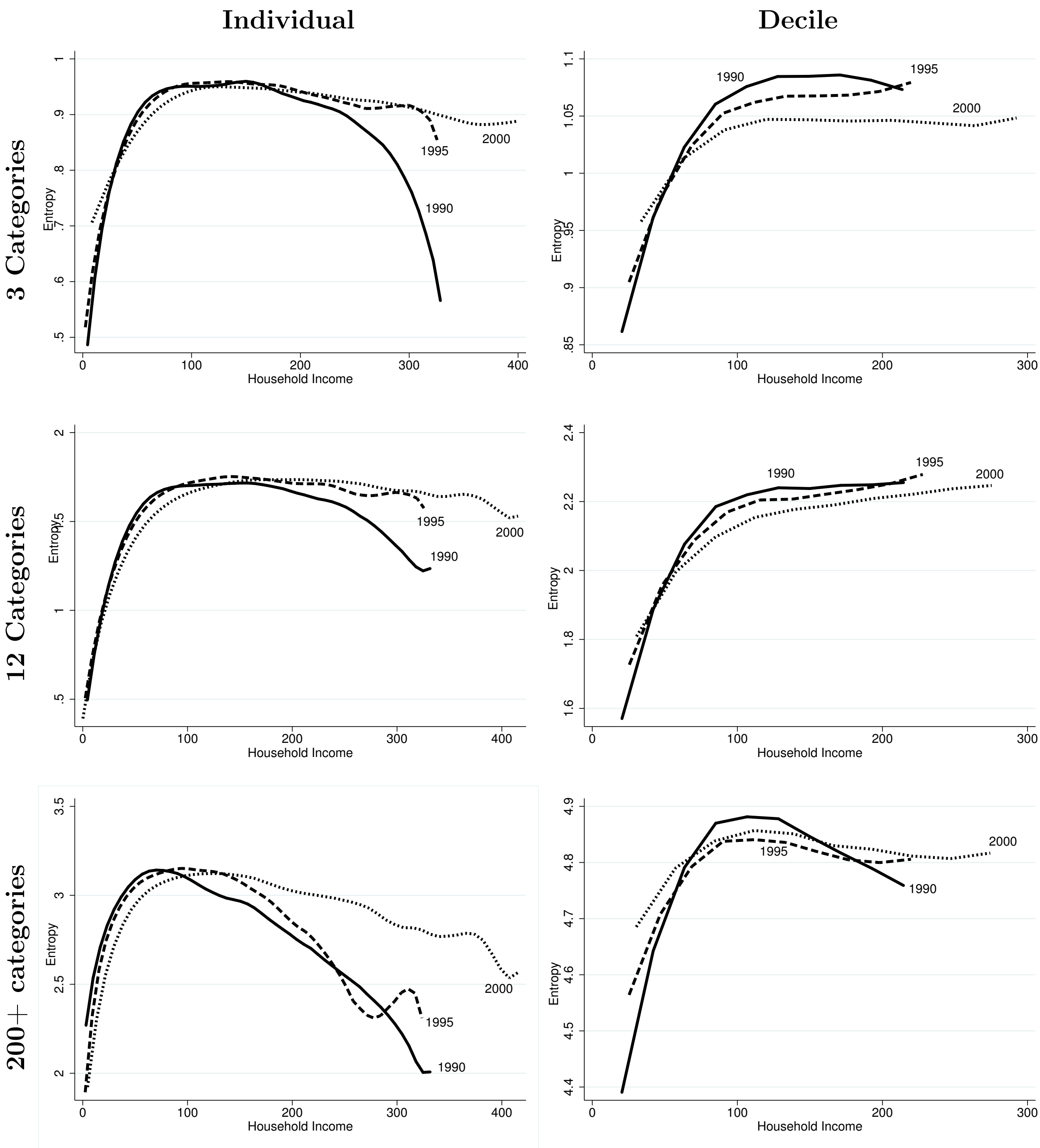

Figure 1: *Notes:* The Figures on the left show the entropy of consumption spending of individual households, while the Figures on the right depict the entropy of aggregated consumption spending for different income deciles. Each row represents a different level of aggregation across consumption items. In the first row, three broad categories are used: food, goods and services. The middle row uses the 12 expenditure categories listed in Table 1 of the Appendix, and the bottom row uses the maximum level of disaggregation of 200+ categories. The number of observations was 6,047 in 1990, 5,984 in 1995 and 5,865 in 2000.

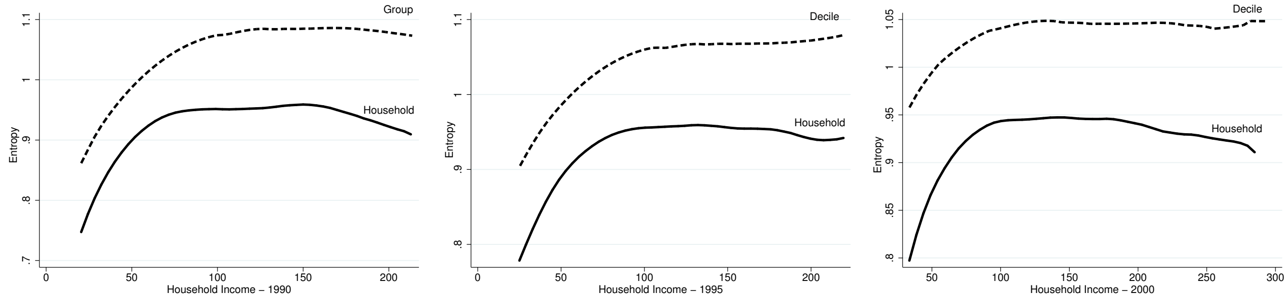

Figure 2: Household and decile entropies (3 expenditure categories)

Notes: The figures depict spending diversity on the household level (solid line) and on the decile level (dashed line) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories are aggregated into 3 categories - food, goods and services. Note that the individual spending diversity curves are shortened to omit observations below the average income of the lowest decile and above the average income of the highest decile. As a result these curves are shorter than those displayed in Figure 1

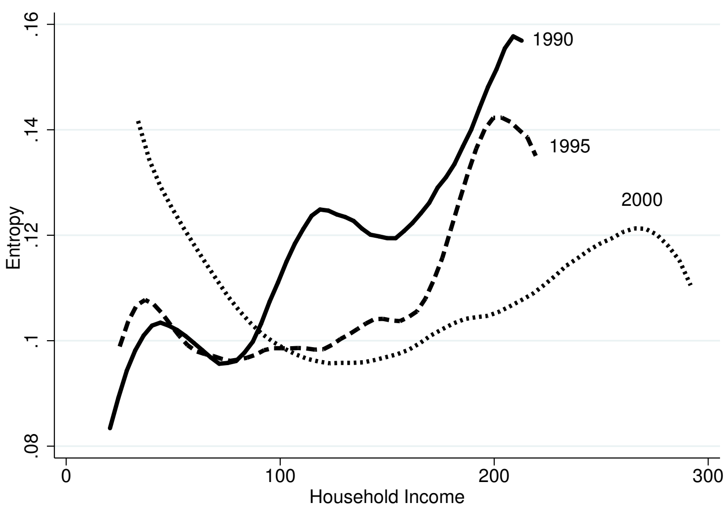

Figure 3: Difference between decile and household entropies (3 categories)

Note: The curves depict the difference $\hat{E} - E_i$ between (aggregated) decile and household level spending entropies.

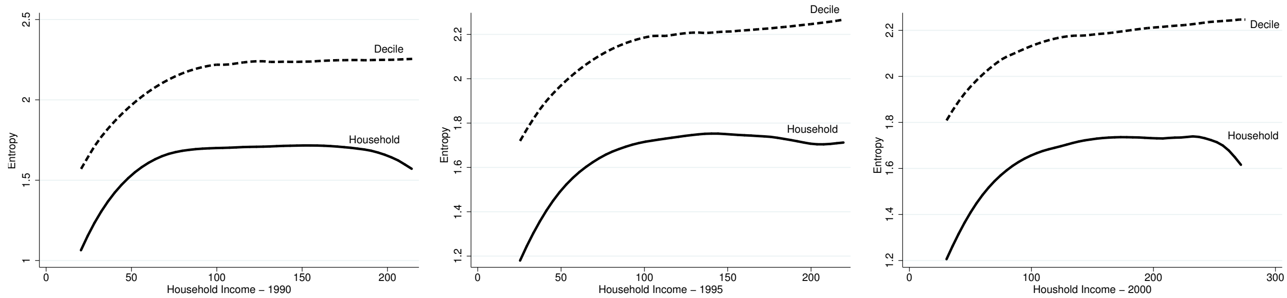

Figure 4: Household and decile entropies (12 expenditure categories)

Notes: The figures depict spending diversity on the household level (solid line) and on the decile level (dashed line) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories were aggregated into 12 categories - see Table 1. Note that the individual spending diversity curves was shortened to omit observations below the average of the lowest decile and above the average of the highest decile. As a result these curves are shorter than those displayed in Figure 1

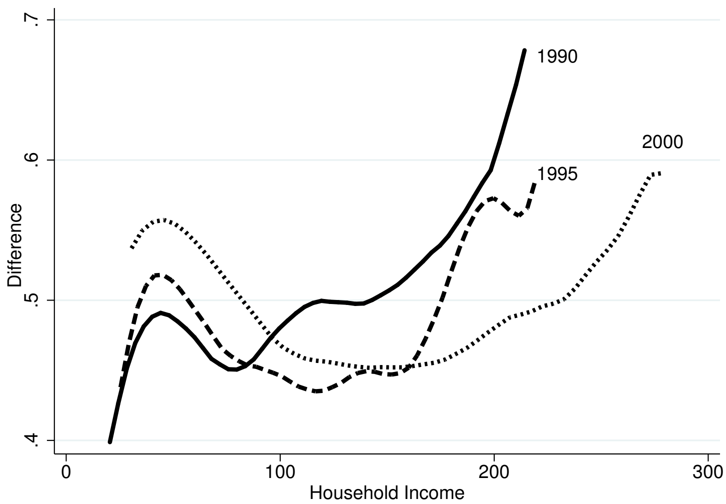

Figure 5: Difference between decile and household entropies (12 categories)

Note: The difference between decile level spending (Group) and household level (individual) entropy of spending. This shows that differences between the household level and the decile level tends to grow as income rises.

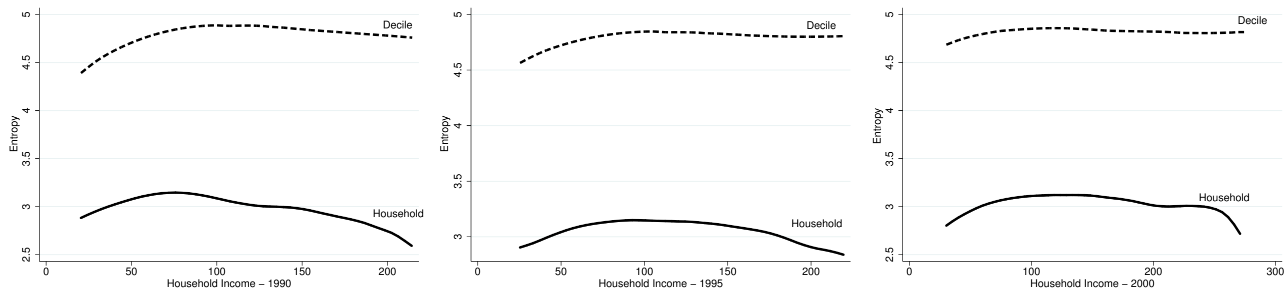

Figure 6: Household and decile entropies (200+ expenditure categories)

Notes: The figures depict spending diversity on the household level (solid line) and on the decile level (dashed line) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories were not aggregated. Note that the individual spending diversity curves was shortened to omit observations below the average of the lowest decile and above the average of the highest decile. As a result these curves are shorter than those displayed in Figure 1

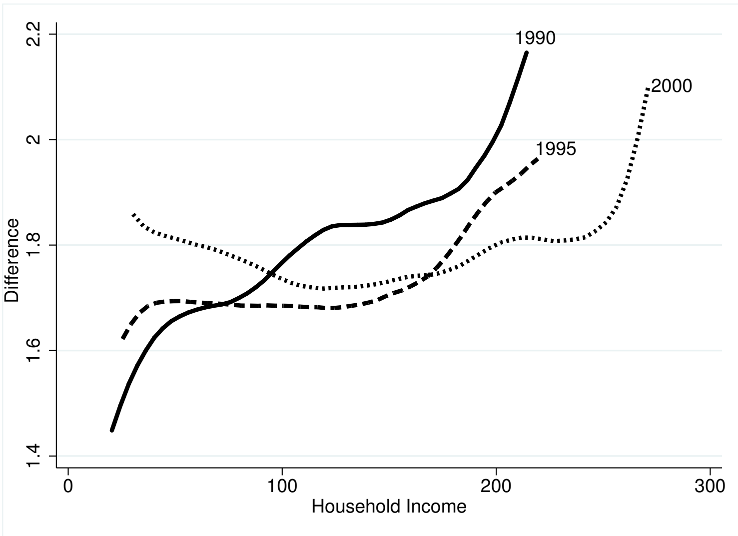

Figure 7: Difference between decile and household entropies (200+ Categories)

Note: The difference between decile level spending (Group) and household level (individual) entropy of spending. This shows that differences between the household level and the decile level tends to grow as income rises.

6.1 A3 Comparison across representative household aggregation levels

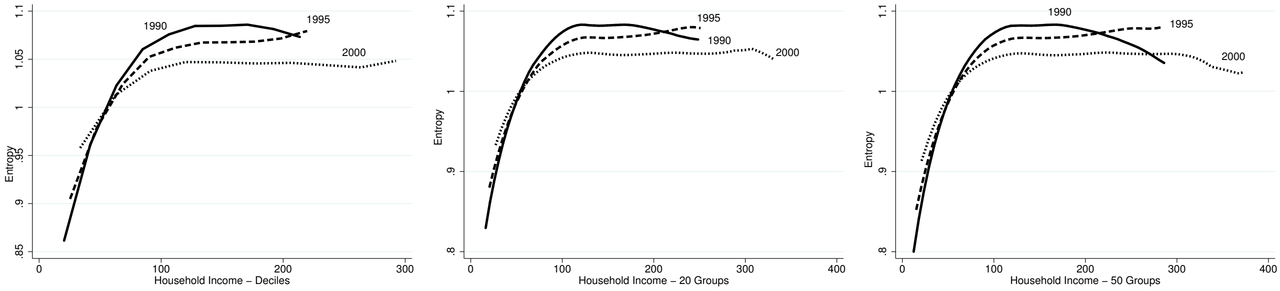

Figure 8: Aggregation of representative groups

Notes: This Figure depicts the the entropy of spending across different levels of representative groups, including deciles (left) level, 20 groups (middle) and 50 groups (right).

6.2 A3 Zeroes removed

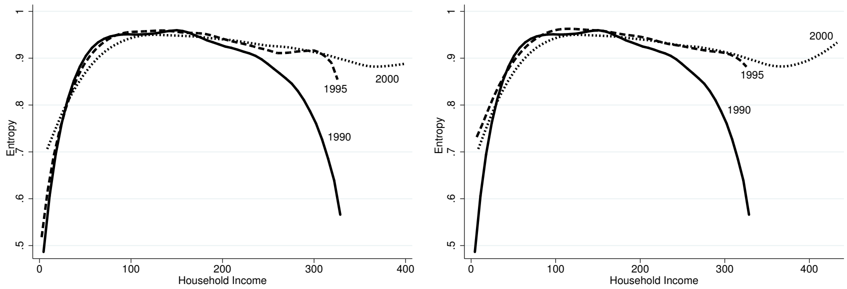

Figure 9: Zeroes removed

Notes: This Figure compares the entropy of spending for 3 expenditure categories between the base case where household with zero expenditure in one or two of the three expenditure categories have been included (left) and the case where they have been removed (right) The number of observation fell by around 90 households observations per year as result of excluding the zero expenditures.

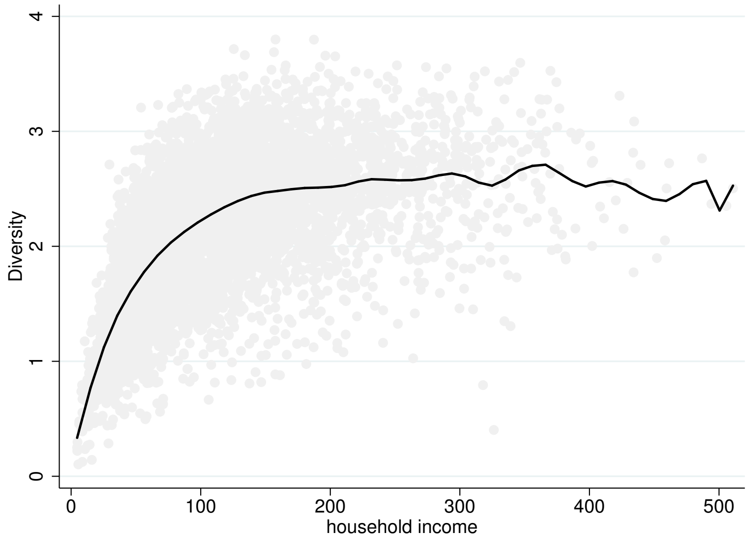

Figure 10: Diversity of variety demand on the household level (2000)

Note: This figure reports how evenly the varieties consumed by a household are distributed across the 12 categories (see Table 1). This figure shows that initially this diversity increases and then flattens out.

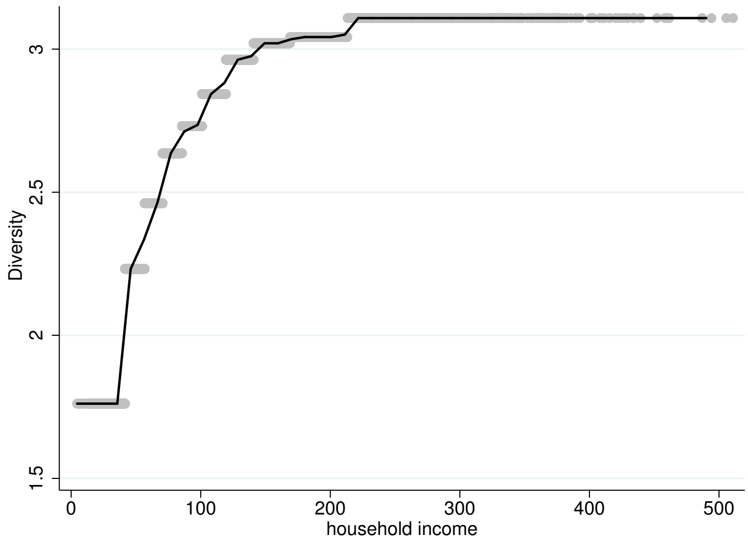

Figure 11: Diversity of variety demand on the decile level (2000)

Note: This figure reports how evenly the varieties consumed by a representative household (decile level) are distributed across the 12 categories (see Table 1).

Appendix A: Proofs

A1: Proof of Lemma 1

Proof. Differentiating equation 8, we obtain that $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{2p\gamma_2\left(1-\bar{\beta}\right)-\gamma_1\bar{\beta}p^{1-\varepsilon}}{x^2\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)}$, so that $\frac{\partial(s_{i1}(x))}{\partial x} < 0$ holds when $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ (Condition B). Differentiating equation 12 gives

$$\frac{\partial \hat{E}}{\partial x} = -\frac{\partial \hat{s}_1}{\partial x}\left(ln\hat{s}_1+1\right) - 2\frac{\partial \hat{s}_2}{\partial x}\left(ln\hat{s}_2+1\right) = -\frac{\partial s_{11}}{\partial x}\left(lns_{11}\right) - \left(\frac{\partial s_{12}}{\partial x}+\frac{\partial x_{13}}{\partial x}\right)\left(ln\hat{s}_2\right)$$

$$= -\frac{\partial s_{11}}{\partial x}\left(lns_{11} - ln\hat{s}_2\right) = -\frac{\partial s_{11}}{\partial x}\left(lns_{11} - ln\left(\frac{1-s_{11}}{2}\right)\right)$$

where the conditions $\hat{s}_1 = s_{11}$, $\hat{s}_2(x) = \frac{s_{12}(x)+s_{13}(x)}{2}$, $s_{11}+s_{12}+s_{13}=0$ and $\frac{\partial s_{11}}{\partial x}+\frac{\partial s_{12}}{\partial x}+\frac{\partial x_{13}}{\partial x}=0$ were used for the transformations. As $\frac{\partial s_{11}}{\partial x}<0$, $sign\frac{\partial \hat{E}}{\partial x} = sign\left(lns_{11} - ln\left(\frac{1-s_{11}}{2}\right)\right) = sign\left(s_{11}-\frac{1}{3}\right)$ holds. As $\lim_{x\to\infty} s_{11} = \frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}}$ and as (by assumption) s_{i1} continuously falls in x, $\frac{\partial \hat{E}}{\partial x}>0$ therefore always holds if $\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}} > \frac{1}{3}$ holds, i.e. if $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ holds. If $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$, $\frac{\partial \hat{E}}{\partial x}<0$ holds for large values of x (i.e. $x>\breve{x}$), while $\frac{\partial \hat{E}}{\partial x}>0$ still holds for lower values ($\underline{x} < x < \breve{x}$) when $s_{11}|_{x=\underline{x}} > \frac{1}{3}$ (**Condition C**) holds. The minimum income level $\underline{x}$ is given by $\underline{x} = \gamma_1 + 2p\gamma_2$ when $\gamma_2=\gamma_3\geq 0$ holds (as this income is required to purchase the positive subsistence consumption level of each good) and by $\underline{x} = \gamma_1+2p\gamma_2-\gamma_2\left[\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}+\bar{\beta}p}{\bar{\beta}-\beta_{12}}\right]$ when $\gamma_2=\gamma_3<0$ (in this case, $\underline{x}$ is pinned down by the condition $s_{13}(\underline{x}) = s_{22}(\underline{x}) = 0$). This implies that $s_{11}|_{x=\underline{x}} = \frac{\gamma_1}{\gamma_1+2p\gamma_2}$ when $\gamma_2\geq 0$, and $s_{11}|_{x=\underline{x}} = \frac{\left(\bar{\beta}-\beta_{12}\right)\gamma_1-\left(1-\bar{\beta}\right)\gamma_2}{\left(\bar{\beta}-\beta_{12}\right)\gamma_1-\gamma_2\left(\left(1-\bar{\beta}\right)p^{\varepsilon}+2p\beta_{12}-p\bar{\beta}\right)}$ when $\gamma_2<0$. Plugging these values into Condition C, we obtain that Condition C is satisfied if either $\gamma_1 > p\gamma_2 \geq 0$ (**Condition C1**) holds, or if $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{12}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{12}\right)}$ and $\gamma_2<0$ (**Condition C2**) holds. □

A2: Proof of Proposition 1

Proof. Differentiating equations 9 and 10 gives $\frac{\partial(s_{12}(x))}{\partial x} = \frac{\gamma_1\beta_{12}-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}-p\gamma_2\left(\bar{\beta}-2\beta_{12}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$ and $\frac{\partial(s_{13}(x))}{\partial x} = \frac{\gamma_1\left(\bar{\beta}-\beta_{12}\right)-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}+p\gamma_2\left(\bar{\beta}-2\beta_{12}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$, implying that $\frac{\partial(s_{12}(x))}{\partial x} > \frac{\partial(s_{13}(x))}{\partial x}$ holds when $\gamma_1 > -2\gamma_2 p$ (**Condition D**) holds. As $s_{i1}+s_{i2}+s_{i3}=1$ and therefore $\frac{\partial(s_{i1})}{\partial x}+\frac{\partial(s_{i2})}{\partial x}+\frac{\partial(s_{i3})}{\partial x}=0$, the conditions $\frac{\partial(s_{i1}(x))}{\partial x}<0$ (implied by Condition B) and $\frac{\partial(s_{12}(x))}{\partial x} > \frac{\partial(s_{13}(x))}{\partial x}$ imply that $\frac{\partial(s_{12}(x))}{\partial x}>0$ needs to hold. The derivative $\frac{\partial(s_{13}(x))}{\partial x}$ can be either positive or negative, where the latter is only possible if $\gamma_2>0$ holds ($\frac{\partial(s_{13}(x))}{\partial x}$ falls in β_{12} and is therefore most likely negative when $\beta_{12}=\bar{\beta}$ holds. As $sign\left.\frac{\partial(s_{13}(x))}{\partial x}\right|_{\beta_{12}=\bar{\beta}} = sign\{-\gamma_2\}$, $\frac{\partial(s_{13}(x))}{\partial x}<0$ can only hold if $\gamma_2>0$).

Subtracting equation 11 from equation 12 and differentiating with respect to x, we obtain that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x}>0$ holds when the following **Condition E** is satisfied:

$$\frac{\partial\left(s_{12}(x)\right)}{\partial x}\left(lns_{12}-ln\hat{s}_2\right) > \frac{\partial\left(s_{13}(x)\right)}{\partial x}\left(ln\hat{s}_2-lns_{13}\right)$$

As $\hat{s}_2(x) = \frac{s_{12}(x)+s_{13}(x)}{2}$ and $s_{12}(x) > s_{13}(x)$, the terms in brackets are positive, implying that Condition E always holds when $\frac{\partial(s_{13}(x))}{\partial x} < 0$ holds. When $\frac{\partial(s_{13}(x))}{\partial x} < 0$, which is only possible if $\gamma_2 > 0$ (see above), $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ therefore holds. In the following, the remaining case where $\frac{\partial(s_{13}(x))}{\partial x} > 0$ holds is considered. This is done by rewriting Condition E as follows:

$$Z \equiv \frac{\frac{\partial(s_{12}(x))}{\partial x}}{\frac{\partial(s_{13}(x))}{\partial x}} > Q \equiv \frac{ln\hat{s}_2 - lns_{13}}{lns_{12} - ln\hat{s}_2} \tag{13}$$

Due to the concavity of the ln function, $Q > 1$ holds. The proposition studies the case in which $\frac{\partial(s_{12}(x))}{\partial x} > \frac{\partial(s_{13}(x))}{\partial x}$, i.e. in which $Z > 1$ holds. The reason for this is that in the case where $\frac{\partial(s_{12}(x))}{\partial x} < \frac{\partial(s_{13}(x))}{\partial x}$, $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds for all values of x, which would not be in line with the empirical observations. Inserting the corresponding expressions, Z can be derived as:

$$Z = \frac{\gamma_1\beta_{12} - \gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon} - p\gamma_2\left(\bar{\beta} - 2\beta_{12}\right)}{\gamma_1\left(\bar{\beta} - \beta_{12}\right) - \gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon} + p\gamma_2\left(\bar{\beta} - 2\beta_{12}\right)} \tag{14}$$

Z is therefore independent of income x. The proof (for the case in which $\frac{\partial(s_{13}(x))}{\partial x} > 0$) proceeds as follows: In part i) it is shown that $sign\frac{\partial Q}{\partial x} = sign\gamma_2$. In part ii) it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ always holds when $\gamma_2 > 0$ and the case where $\gamma_2 = 0$ is discussed. In part iii), the case where $\gamma_2 < 0$ is analyzed and it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ ($Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$) then holds if $\tilde{x} < x < \infty$ ($\underline{x} < x < \tilde{x}$).

i) Deriving Q with respect to x yields

$$sign\frac{\partial Q}{\partial x} = sign\left\{\frac{\partial s_{12}}{\partial x}\left[\frac{1}{s_{12}+s_{13}}\left(lns_{12} - lns_{13}\right) - \frac{1}{s_{12}}\left(ln\left(\frac{s_{12}+s_{13}}{2}\right) - lns_{22}\right)\right] + \frac{\partial s_{13}}{\partial x}\left[\frac{1}{s_{12}+\right.\right.$$

Bringing all terms to a common denominator gives

$$sign\frac{\partial Q}{\partial x} = sign\left\{s_{12}lns_{12} + s_{13}lns_{13} - 2\left(\frac{s_{12}+s_{13}}{2}\right)ln\left(\frac{s_{12}+s_{13}}{2}\right)\right\}\left[s_{13}\frac{\partial s_{12}}{\partial x} - s_{12}\frac{\partial s_{13}}{\partial x}\right]$$

As the term in curly brackets is equal to $\hat{E}(x) - E_i(x)$ and therefore positive (see above),

$$sign\frac{\partial Q}{\partial x} = sign\left[s_{13}\frac{\partial s_{12}}{\partial x} - s_{12}\frac{\partial s_{13}}{\partial x}\right]$$

Inserting $\frac{\partial s_{12}}{\partial x} = \frac{p\beta_{12} - s_{12}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ and $\frac{\partial s_{13}}{\partial x} = \frac{p\left(\bar{\beta}-\beta_{12}\right) - s_{13}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ (note that both derivatives are assumed to be positive here) and then s_{12} and s_{13}, we get that $sign\frac{\partial Q}{\partial x} = sign\left[\beta_{12}s_{13} - \left(\bar{\beta} - \beta_{12}\right)s_{12}\right] = sign\left\{\gamma_2\left[\left(1-\bar{\beta}\right)p^{\varepsilon}\left(2\beta_{12} - \bar{\beta}\right) + p\left(\left(\beta_{12}\right)^2 - \right.\right.\right.$ $sign\gamma_2$

$ii)$ When $\gamma_2 > 0$, Q continuously rises in x. $Z > Q$ therefore holds for all values of x if it holds for $x \to \infty$ (Z does not depend on x). Inserting the corresponding budget shares into the expression of Q and solving for the limit gives

$$\lim_{x\to\infty} Q = \frac{ln\left(\frac{\bar{\beta}}{2}\right) - ln\left(\bar{\beta} - \beta_{12}\right)}{ln\beta_{12} - ln\left(\frac{\bar{\beta}}{2}\right)}$$

As Z continuously rises in γ_2 (using equation 14 it can be shown that $sign\frac{\partial Z}{\partial \gamma_2} = sign\left(2\beta_{12} - \bar{\beta}\right)\left(\left(1-\bar{\beta}\right)p^{\varepsilon} + p\bar{\beta}\right) > 0$), $Z > Q$ therefore always holds if $Z|_{\gamma_2=0} = \frac{\beta_{12}}{\bar{\beta}-\beta_{12}} > \lim_{x\to\infty} Q$ holds. This inequality is satisfied if $\beta_{12}ln\beta_{12} + \beta_{22}ln\beta_{22} = E_i > 2\frac{\bar{\beta}}{2}ln\left(\frac{\bar{\beta}}{2}\right) = \hat{E}$ holds. $E_i > \hat{E}$ holds if $\beta_{12} > \frac{\bar{\beta}}{2}$ and $x > \underline{x}$ (see footnote 16). Consequantly, $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ hold when $\gamma_2 > 0$. When $\gamma_2 = 0$, $Q = \frac{ln\left(\frac{\bar{\beta}}{2}\right)-ln\left(\bar{\beta}-\beta_{12}\right)}{ln\beta_{12}-ln\left(\frac{\bar{\beta}}{2}\right)}$ holds independently of x, so that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ still holds for $x > \underline{x}$. At the point where $\gamma_2 = 0$ and $x = \underline{x}$, $s_{i2}(x) = s_{i3}(x)$ and therefore $E_i = \hat{E}$, implying that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} = 0$.
$iii)$ When $\gamma_2 < 0$, $Z > Q$ still holds when x is sufficiently large (see part $ii)$ of the proof), implying that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ still holds in this case. When $\gamma_2 < 0$ and x approaches the lower bound $\underline{x}$, $s_{13} = s_{22}$ approaches zero (while $s_{12} = s_{23}$ remains positive), implying that $Q \equiv \frac{ln\hat{s}_2 - lns_{13}}{lns_{12} - ln\hat{s}_2}$ becomes infinitely large and that $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds. As Q continuously falls in x when $\gamma_2 < 0$ holds (see part $i)$ of the proof), $Z < Q$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ therefore holds in this case when $\underline{x} \leq x < \tilde{x}$ and $Z > Q$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ when $\tilde{x} < x < \infty$, where $\tilde{x}$ ($\underline{x} < \tilde{x} < \infty$) is a positive parameter. □

A3: Proof of Proposition 2

Proof. a) Let us define $x_i \equiv \tilde{x}_i + F_i$. Individual i must be indifferent between only consuming good 1 and having income x_i and consuming all three goods and having income $x_i - F_i = \tilde{x}_i$. Using equation 2, this implies the following equation:

$$\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}\left(\tilde{x}_i + F_i - \gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \beta_{i2}^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} =$$
$$\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}\left(q_{i1}(\tilde{x}_i) - \gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q_{i2}(\tilde{x}_i) - \gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q_{i3}(\tilde{x}_i) - \gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

Subtracting the right hand side (RHS) from the left hand side (LHS) and defining $Q \equiv LHS - RHS$, we can implicitly differentiate this equation and obtain $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$. We therefore analyze how F_i depends on β_{i2}, taking $\tilde{x}_i$ as given (and x_i to be variable), as this simplifies the analysis. This yields the same qualitative results as studying how $F_i = x_i - \tilde{x}_i$ depends on β_{i2}, taking

x_i as given. We obtain $\frac{\partial Q}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$, so that $sign\frac{\partial Q}{\partial F_i} = sign\,(\varepsilon - 1)$ holds as the term $\tilde{x}_i + F_i - \gamma_1 = x_i - \gamma_1$ is positive when $x_i > \underline{x}$ holds. Moreover, $\frac{\partial Q}{\partial \beta_{i2}} = \frac{1}{\varepsilon}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]$ holds (in order to show this, the condition $\frac{q_{i2}(\tilde{x}_i)-\gamma_2}{\beta_{i2}} = \frac{q_{i3}(\tilde{x}_i)-\gamma_2}{\bar{\beta}-\beta_{i2}}$, which can be derived from the consumers first order conditions, and the condition $\frac{\partial q_{i2}(\tilde{x}_i)}{\partial \beta_{i2}} + \frac{\partial q_{i3}(\tilde{x}_i)}{\partial \beta_{i2}} = 0$, which holds as $\beta_{i2} + \beta_{i3} = \bar{\beta}$, were used). When $\gamma_2 < 0$ and $\beta_{i2} > \frac{\bar{\beta}}{2}$, $(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} > 0$ (< 0) holds when $\varepsilon < 1$ ($\varepsilon > 1$), implying that $sign\frac{\partial Q}{\partial \beta_{i2}} = sign\,(1-\varepsilon)$ holds. Assuming $\varepsilon \neq 1$, we consequently obtain:

$$\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}} = \frac{(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{(1-\varepsilon)(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}} > 0$$

Due to symmetry, also $sign\frac{dF_i}{d\beta_{i3}} > 0$ holds if $\beta_{i3} > \frac{\bar{\beta}}{2}$ (if $\beta_{ij} = \frac{\bar{\beta}}{2}$, $\frac{\partial F_i}{\partial \beta_{ij}} = 0$ holds). F_i therefore increases in β_{ij} when $\gamma_2 < 0$ and when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds.

b) In order to determine the sign of $\frac{\partial^2 F_i}{\partial \beta_{i2} \partial \gamma_2}$, we again take $\tilde{x}_i$ as given (and x_i to be variable), as this simplifies the analysis compared to the case where x_i as given and leads to the same qualitative results. Taking into account that $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$, that $\frac{\partial Q}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$ is independent of γ_2, and that $\frac{\partial^2 Q}{\partial \beta_{i2}\partial \gamma_2} = -\frac{(\varepsilon-1)}{\varepsilon^2}(-\gamma_2)^{-\frac{1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]$, we obtain (assuming $\varepsilon \neq 1$):

$$\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = -\frac{\frac{\partial^2 Q}{\partial \beta_{i2}\partial \gamma_2}}{\frac{\partial Q}{\partial F_i}} = \frac{(-\gamma_2)^{-\frac{1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{\varepsilon(1-\bar{\beta})^{\frac{1}{\varepsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}}$$

As $sign\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right] = sign\,(1-\varepsilon)$ when $\beta_{i2} > \frac{\bar{\beta}}{2}$ holds and as $\tilde{x}_i + F_i - \gamma_1 = x_i - \gamma_1$ is positive when $x_i > \underline{x}$ holds, $sign\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = sign\,(1-\varepsilon)$ holds when $\gamma_2 < 0$ (and $sign\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = 0$ when $\beta_{i2} = \frac{\bar{\beta}}{2}$). As $\frac{\partial F_i}{\partial \beta_{i2}} = \frac{(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{(1-\varepsilon)(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i+F_i-\gamma_1)^{-\frac{1}{\varepsilon}}}$, $\lim_{\gamma_2 \to -0}\frac{\partial F_i}{\partial \beta_{i2}} = 0$ and $\lim_{\gamma_2 \to -\infty}\frac{\partial F_i}{\partial \beta_{i2}} = \infty$ hold when $\varepsilon > 1$ holds, while $\lim_{\gamma_2 \to -0}\frac{dF_i}{d\beta_{i2}} = \infty$ and $\lim_{\gamma_2 \to -\infty}\frac{\partial F_i}{\partial \beta_{i2}} = 0$ hold when $\varepsilon < 1$ holds. Due to symmetry, the same results apply when β_{i2} is replaced by β_{i3} and when $\beta_{i3} > \frac{\bar{\beta}}{2}$ holds.

c) Instead of directly calculating the sign of $\frac{\partial^2 D}{\partial \beta_{i2}\partial x_i}$, we simplify the analysis by making use of the fact that analyzing $\frac{F_i - F_a}{\tilde{x}_i}$, taking $\tilde{x}_i$ as given (and $x_i = \tilde{x}_i + F_i$ as endogenous) leads to the same qualitative results as analyzing $\frac{F_i - F_a}{x_i}$, taking x_i as given (and $\tilde{x}_i = x_i - F_i$ as endogenous), i.e. that $sign\frac{\partial^2 D}{\partial x_i \partial \beta_{i2}} = sign\frac{\partial^2 \tilde{D}}{\partial \tilde{x}_i \partial \beta_{i2}}$, where $\tilde{D} \equiv \frac{F_i - F_a}{\tilde{x}_i}$. Taking into account that F_a does not depend on β_{i2}, that $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$, $\frac{\partial Q}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$, and that $\frac{\partial^2 Q}{\partial \beta_{i2}\partial \tilde{x}_i} = 0$, we obtain

$$\frac{\partial^2\left(\frac{F_i-F_a}{\tilde{x}_i}\right)}{\partial\beta_{i2}\partial\tilde{x}_i}=\frac{\frac{\partial^2 F_i}{\partial\beta_{i2}\partial\tilde{x}_i}\tilde{x}_i-\frac{\partial F_i}{\partial\beta_{i2}}}{\tilde{x}_i^2}=\frac{1}{\tilde{x}_i}\frac{\frac{\partial Q}{\partial\beta_{i2}}}{\frac{\partial Q}{\partial F_i}}\left[1+\frac{\frac{\partial^2 Q}{\partial F_i\partial\tilde{x}_i}}{\frac{\partial Q}{\partial F_i}}\right]=\frac{1}{\tilde{x}_i}\frac{\frac{\partial Q}{\partial\beta_{i2}}}{\frac{\partial Q}{\partial F_i}}\left[1-\frac{1}{\varepsilon\left(\tilde{x}_i+F_i-\gamma_1\right)}\right]$$

The term $\tilde{x}_i+F_i-\gamma_1=x_i-\gamma_1$ is positive when $x_i>\underline{x}$ holds. As $sign\frac{\partial Q}{\partial\beta_{i2}}=sign\left(1-\varepsilon\right)$ holds if $\beta_{i2}>\frac{\bar{\beta}}{2}$ and if $\gamma_2<0$ hold, and as $sign\frac{\partial Q}{\partial F_i}=sign\left(\varepsilon-1\right)$, we therefore obtain that

$$sign\frac{\partial^2\left(\frac{F_i-F_a}{\tilde{x}_i}\right)}{\partial\beta_{i2}\partial\tilde{x}_i}=sign\left[\frac{1}{\varepsilon}-\left(\tilde{x}_i+F_i-\gamma_1\right)\right]=\begin{cases}>0 & if\ \ x_i<\gamma_1+\frac{1}{\varepsilon}\\ <0 & if\ \ x_i>\gamma_1+\frac{1}{\varepsilon}\end{cases}$$

Using the fact that $sign\frac{\partial^2 D}{\partial x_i\partial\beta_{i2}}=sign\frac{\partial^2\tilde{D}}{\partial\tilde{x}_i\partial\beta_{i2}}$ then gives the result that $\frac{\partial^2 D}{\partial x_i\partial\beta_{i2}}>0$ when $\underline{x}\leq x_i<\gamma_1+\frac{1}{\varepsilon}$ holds (Case 1) and that $\frac{\partial^2 D}{\partial x_i\partial\beta_{i2}}<0$ holds if $x_i>\gamma_1+\frac{1}{\varepsilon}$ (Case 2). Due to symmetry, the same results apply when β_{i2} is replaced by β_{i3} and when $\beta_{i3}>\frac{\bar{\beta}}{2}$ holds.

As $\underline{x}=\gamma_1+2p\gamma_2-\gamma_2\left[\frac{\left(1-\bar{\beta}\right)p^\varepsilon+\bar{\beta}p}{\bar{\beta}-\beta_{12}}\right]$ when $\gamma_2=\gamma_3<0$ and $\beta_{12}=\beta_{23}>\frac{\bar{\beta}}{2}$ (see the proof of Lemma 1), there is a non-empty parameter range $\underline{x}\leq x_i<\gamma_1+\frac{1}{\varepsilon}$ for which Case 1 results if $\gamma_2>-\frac{\left(\bar{\beta}-\beta_{12}\right)}{\varepsilon\left(\left(1-\bar{\beta}\right)p^\varepsilon+2p\left(\beta_{12}-\frac{\bar{\beta}}{2}\right)\right)}$ holds. □