Transition to Green Technology along the Supply Chain*

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Abstract

We develop a dynamic model of green technological transition along supply chains, with a unique equilibrium and multiple steady-states. Even with Pigouvian environmental taxation, targeted sectoral subsidies are generally needed to reach the social optimum. A government constrained to small subsidies or below-social-cost carbon prices should target downstream sectors. With strategic complementarity in greenification, subsidies (weakly) raise welfare; under strategic substitutability, subsidizing greenification in a sector whose output mainly feeds dirty downstream production can derail the transition. Calibrating the model to the long-range heavy-duty transport sector, we find that Pigouvian carbon pricing alone is insufficient to escape a low-greenification trap.

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1 Introduction

There is a growing consensus worldwide for the need to speed up the transition away from fossil fuels to slow down and eventually curb global warming. However, there is no unanimity among policymakers when it comes to the best choice of climate policy instrument. While Europe's policies center around carbon pricing, China and the U.S. have prioritized industrial policy. In parallel, the green transition often requires building new supply chains, which need to become greener over time. For instance, while electric vehicles can replace fossil fuel vehicles, they rely on a CO_2-intensive upstream input, batteries, so that green innovation in battery production will be needed to make the whole supply chain clean. Until recently the climate economics literature has emphasized carbon pricing as the main policy instrument to reduce emissions without paying as much attention to the industrial policy leg, and it has largely sidestepped supply chain considerations.

In this paper, we develop a dynamic model of technological transition along the supply chain. We show that strategic complementarities in green innovation naturally arise in that context and argue that the coordination of green innovation incentives along the supply chain provides a new rationale for the use of industrial policies (i.e., sector-specific R&D subsidies) on top of carbon pricing. While our model is set in the context of the green transition, its insights generalize to situations where the economy may switch from one technology to another superior one that requires the development of its own supply chain: cross-sectoral strategic complementarities may lead to inefficiently low adoption without policy intervention.[1]

We start in Section 2 by providing suggestive evidence of i) sectors changing their input mix as they become greener, and ii) of strategic complementarity in green innovation along the supply chain.

We then lay out the baseline model of a vertical supply chain in Section 3. Each sector is a layer in the supply chain and produces a "good" which is a Cobb-Douglas aggregate of a mass of industrial processes or varieties. Each variety can be produced either in a dirty way using labor only or, if that variety has been "greenified", in a clean way using labor and the immediate upstream good. To move from dirty to clean, a variety in a given sector needs to undergo "greenification".[2] We assume heterogeneous fixed costs of greenification across varieties within

[1] For instance, one could re-cast our model from being about the green transition to being about the adoption of a modern technology that relies on complex inputs versus a traditional technology that uses a simpler production process. In that sense, our model introduces supply chains in Zeira (1998).

[2] Greenification is a stand-in here for a process that allows replacing fossil fuels as production inputs with other

a sector, so that some varieties may be greenified at one point in time while others still have to greenify. We also assume that taking into account environmental taxation, producing using the clean technology is cheaper than using the dirty technology, so that a variety producer always chooses the former once the variety has been greenified. A producer may choose to incur the sunk greenification cost in exchange for a one-period exclusive right to use the clean technology and Bertrand-compete with competitive producers that use the dirty technology. One period after the greenification of a variety, the clean technology becomes available to all producers, and the production of that variety becomes competitive.

Under these assumptions, the incentive to greenify for any variety producer depends on the degree of greenification downstream (more greenification downstream, which uses the variety as inputs, increases the demand for that variety, and thereby revenues for the innovator) and on greenification upstream (more greenification upstream, which supplies inputs to the variety, reduces the cost of producing the variety once it has been greenified, and thereby allows the innovator to charge a higher mark-up). The resulting cross-sectoral strategic complementarities in greenification generate a coordination problem: insufficient greenification in other sectors reduces the private incentives for variety producers in a given sector to greenify. We characterize the equilibrium and show that it is unique for given initial conditions because, while the effect of downstream greenification on innovation incentives is contemporaneous, that of upstream greenification is delayed by a period (i.e. until the patent has expired). Yet, cross-sectoral strategic complementarities in greenification generally lead to a multiplicity of steady-states.

We then characterize the social optimum and show that it generally differs from the decentralized solution, even once emissions are optimally priced through a Pigouvian tax and the difference in the time horizon of the Social Planner and private agents is corrected for. The key source of inefficiency is the complementarity in greenification that generates increasing returns to scale in innovation. As a result, the decentralized economy with Pigouvian emission taxes may be stuck in a steady-state which differs from the social optimum, even though the private incentives to greenify are locally in line with the social incentives. We show that the socially optimal steady-state can be uniquely implemented through the combination of a Pigouvian tax with a set of temporary sector-specific greenification subsidies.

Section 4 introduces a general network where both clean and dirty technologies rely on upstream inputs, albeit with potentially different intensities. We characterize the equilibrium and

inputs that do not directly generate emissions. In practice, this often, but not always, involves replacing fossil fuels with electricity or hydrogen.

the social optimum in that setting as well, and generalize the vertical chain results. We show that strategic complementarity arises when, within a sector, the clean production process relies more on greenifiable inputs than the dirty production process does.

Even though we have an environment that displays complementarities as in other models, e.g. the Big Push model of Murphy et al. (1989), the fact that our complementarities occur along the supply chain leads to drastically different insights. First, strategic complementarities lead to multiple steady-states instead of multiple equilibria.

Second, our model generates the possibility that a small and temporary subsidy to greenification that targets key sectors can be sufficient to achieve large welfare gains by moving the decentralized equilibrium just a little out of an inefficient steady-state and then relying on market forces to complete the transition towards the socially efficient steady-state: large, sustained interventions across all sectors are not needed.

Third, our framework has implications for how to prioritize public intervention between upstream and downstream when the government is constrained to implement marginal subsidies. In a vertical supply chain or when dirty technologies do not rely much on greenifiable inputs, the government should prioritize downstream sectors. Intuitively, greenification propagates proportionally upstream through the demand channel but less than proportionally downstream through the cost channel—a rise in demand from all downstream buyers raises a sector's revenue proportionally, whereas a fall in upstream input prices reduces only part of the sector's marginal cost, since production also uses labor. This intuition need not generalize when dirty technologies also rely significantly on greenifiable inputs. Then, we find that greenification incentives propagate more when subsidies target either (i) sectors whose production draws heavily on greenifiable inputs, directly or through the supply chain, and whose clean technologies use more of these inputs than their dirty counterparts; or (ii) sectors that are central suppliers to downstream sectors with this latter property.

Fourth, we show that a second realistic policy restriction, where carbon prices are set below the true social cost of carbon, provides additional justification for targeting downstream greenification. With a suboptimal carbon price, industrial policy should incorporate the effect of greenification on emissions. In a vertical supply chain, this emission-reduction motive for greenification favors downstream greenification when initial greenification is low because in that case, greenifying upstream sectors has little effect on equilibrium emissions.

Fifth, we analyze the consequences of industrial policies when the government may be misinformed. We show that with strategic complementarity, industrial policy cannot backfire in the sense that greenification subsidies weakly increase the long-run utility flow. Yet, with strategic substitutability, a government that boosts greenification in a sector whose output mainly feeds dirty downstream production processes (e.g. oil refining) risks derailing the overall transition towards greenification. Thus, our model provides a robust set of reasons for prioritizing downstream greenification in the face of restricted policy instruments or information.

Finally, we provide a quantitative application of our model to the decarbonization of long-range and heavy-duty (LRHD) transportation in the United States in Section 5. We consider "greenification" via a switch to hydrogen, a key technology to help decarbonize LRHD transportation (IPCC 2022) with emerging commercial developments in trucking, rail, and passenger aircraft. Importantly, hydrogen can itself be produced using fossil fuels—currently the dominant method—or from green electricity, highlighting the importance of full value chain greenification in this context (Rapson and Muehlegger 2023). First, we find that the multiplicity of steady-states is empirically relevant: even a carbon tax set equal to a high social cost of carbon (\$300/tCO_2 in \$2022) is consistent with multiple steady-states, including the current low greenification state and a high greenification state. Second, very high carbon prices (around \$440/tCO_2) would be required to eliminate the multiplicity of steady-states. Third, a policy that complements emissions taxes with a temporary downstream subsidy toward the development of LRHD transportation technologies is sufficient to induce high levels of greenification and yields large welfare gains estimated at \$3.2 trillion in present value.

Our paper relates to several strands of literature. First, we relate to the literature on the macroeconomics of climate change, starting with integrated assessment models (Nordhaus, 1994, Golosov, Hassler, Krusell, and Tsyvinski, 2014, among many others). However, this literature often takes technological change as given, and its emphasis is on the optimal design of carbon tax policies. More closely related to our analysis is the literature on directed technical change and the environment, in particular Acemoglu, Aghion, Bursztyn, and Hémous (2012), henceforth AABH, who show that optimal climate policy in the presence of endogenous directed innovation requires combining a carbon tax and green research subsidy. The model in AABH relies on knowledge externalities, not on cross-sectoral strategic complementarities and coordination, as it does not model clean innovations along the supply chain.[3]

Second, a set of recent contributions study the positive properties of environmental policies

[3] Acemoglu et al. (2016) present another model of directed technical change in the environmental context. In their model as in ours, intermediates are aggregated in a Cobb-Douglas way and can be produced with a clean or a dirty input, but there is no supply chain or coordination issue.

in static production networks with carbon emissions, a setting where the optimal intervention remains a uniform carbon tax (e.g. see King, Tarbush, and Teytelboym 2019; Devulder and Lisack 2020; Martin, Muuls, and Stoerk, 2023; Mahen 2025). We depart from these papers by introducing endogenous technological change and by analyzing dynamic strategic complementarities in technology adoption along the supply chain.[4]

Third, we relate to papers on strategic complementarities in technology adoption, including the seminal work of Murphy et al. (1989), recent work by Sturm (2023), and in an environmental setting, Greaker and Midttømme (2016), Dugoua and Dumas (2021, 2024), and Smulders and Zhou (forthcoming). While these papers feature multiple equilibria, our dynamic model has multiple steady-states but a unique equilibrium for given initial conditions, thereby generating unambiguous predictions on the impact of industrial policy.[5] This feature enables our model to maintain tractability—despite rich strategic interactions along the supply chain—and generate new insights on how industrial policy should target key sectors to aid the technological transition.

A fourth related strand is the literature on industrial policy, in particular Greenwald and Stiglitz (2006), who model the infant industry argument based on "learning by doing" externalities, and Murphy et al. (1989), and Buera, Hopenhayn, Shin, and Trachter (2026), who model Rosenstein-Rodan (1943)'s Big Push idea (see Juhasz, Lane, and Rodrik, 2023, for a recent survey). In the environmental context, van der Ploeg and Venables (2025) argue for a green big push based on peer effects in demand and technological externalities. However, none of these papers considers supply chains and the associated motives for sectoral policies.[6]

2 Evidence for the Supply Chain's Role in Decarbonization

Before describing our model, we present reduced form evidence for why explicitly considering a supply chain is important for studying the green transition. We document two facts. First, reductions in emissions at the sector level are associated with changes in the sector's supplier composition. Second, sectors engage in more greenification, as proxied by emission intensity

[4] Wang (2026) shows that the structure of the EV market distorts the direction of upstream innovation in batteries.

[5] Also see Crouzet, Gupta, and Mezzanotti (2023) for a dynamic model of strategic complementarity in the context of adopting electronic payments.

[6] Closest to our paper is Liu (2019), who analyzes sector-specific policy in a production network with market imperfections, showing that targeting upstream sectors—as done in South Korea and China—can boost aggregate welfare. Liu and Ma (2024) examine optimal cross-sector R&D allocation within an innovation network featuring knowledge spillovers. Donald (2023) incorporates Liu and Ma's innovation network into AABH's directed innovation framework, showing that such spillovers prevent the need for a big push to switch to clean innovation. Buera and Trachter (2024) contemporaneously study industrial policy in a static network with endogenous technology adoption.

reductions and green patenting, when the sectors connected to them through the production network greenify.

We draw on three data sources: the BLS input-output table, which records how much each sector buys from every other sector; the EPA's greenhouse gas inventory, which records sector-level CO_2e emissions; and PatentsView, which provides patent grants and citations. We classify a patent as "green" if its technology classification flags it as a clean-energy, clean-production, or clean-transport innovation, or as an electric vehicle patent. We then match patents to firms and firms to sectors using standard crosswalks.[7]

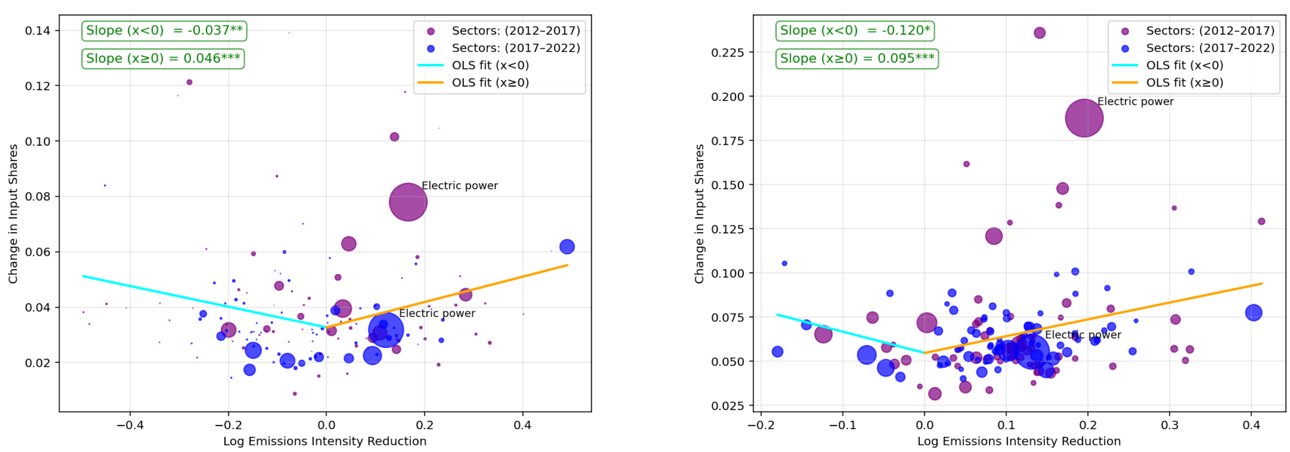
When we measure emissions and greenification at the sector level, we exclude service sectors with minimal emissions and focus on agriculture, mining, utilities, manufacturing, construction, transportation and warehousing, and wholesale and retail trade. This restriction applies only to the sectors whose outcomes we track: input sectors are unrestricted. So when we compute a sector's upstream or downstream connections, including its Leontief-inverse-based exposure to the rest of the economy, all sectors still enter on the supplier side.

Figure 1 asks whether sectors that cut their emissions intensity did so by reshuffling their supply chain. We split the decade 2012–2022 into two five-year windows so that each sector contributes two observations to our sample. We focus on five-year rather than annual frequencies because green technology adoption and supply-chain reorientation are slow processes; annual variation would largely reflect measurement noise rather than the structural reallocation we aim to capture. For each sector in each window, we plot how much its emissions intensity fell (horizontal axis) against how much its mix of suppliers shifted (vertical axis). The supplier-mix measure is one-half the sum of absolute changes in input cost shares: it is zero when a sector keeps buying in the same proportions from the same suppliers as before, and rises as input spending migrates to new suppliers. Each dot is one sector in one window, sized in proportion to that sector's emissions, and standard errors are clustered by sector.[8]

[7] We define green patents as those carrying CPC codes Y02E, Y02P, Y02T, or B60L, except for a subset of Y02T patents that deal with energy efficiency improvements to fossil-fuel-based transportation. The patent-to-firm crosswalks follow Kogan et al. (2017) and Arora et al. (2021), and firm-to-sector assignment uses Compustat. See Appendix A for details, notably on our approach to clean the model-attributed sectoral emissions in the EPA data.

[8] In the left panel we exclude fossil fuel extraction (oil, gas, and coal mining) from the supplier side, so the pattern cannot be driven mechanically by a sector using less fossil fuels as its emissions fall. In Appendix Figure A.1, we weight the regressions underlying the fitted lines by the cube root of sector emissions, which keeps the fit sensitive to the ordering of high-emission sectors without letting a few large ones dominate. These weighted regressions, as well as specifications with time fixed effects (Appendix Figure A.2) and with Euclidean distance for the supplier-mix measure (Appendix Figure A.3), all yield very similar results.

Our hypothesis predicts a V-shape around zero: sectors whose emissions intensity barely moved should show little change in their supplier mix, while sectors moving far from zero in either direction should show more reshuffling, but for opposite reasons: sectors cutting their emissions should be building new links to cleaner suppliers, while sectors whose emissions rose should be shifting toward dirtier ones. We therefore fit separate slopes above and below zero, expecting a positive slope on the right (deeper emissions cuts associated with larger supplier-mix changes) and a negative slope on the left (larger emissions increases associated with larger supplier-mix changes). The two panels differ only in how far upstream we look: the left panel counts each sector's direct suppliers, while the right panel includes all changes in higher-order suppliers (e.g. the change in a sector's suppliers' suppliers, and so on) by taking the Leontief inverse of the input-output matrix.[9] The V-shape is apparent in both panels: changes in the supplier mix grow with the absolute size of the emissions-intensity change.



(a) IO Table

(b) Leontief Inverse

Figure 1. Reductions in CO2e emissions are associated with changes in the supply chain

Our second piece of evidence concerns the propagation of greenification incentives through the production network. For each sector and five-year period, Table 1 regresses a sector's own greenification on a Leontief-weighted aggregate of greenification in the sectors it is connected to, with time fixed effects and standard errors clustered by sector. The network aggregate is the sum

[9]The Leontief inverse also enters the emissions measure in the right panel: the emissions embodied in a dollar of a sector's output include those generated throughout its upstream supply chain.

Table 1. Supply Chain Incentives for Greenification

	Dependent Variable:						
	Log Emissions Intensity Reduction			Green Patent Share		Green Citation Share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Network Log Emissions Intensity Reduction	0.202*			0.068		0.047	
	(0.112)			(0.060)		(0.063)	
Network Green Patents		0.183**			0.098**		
		(0.088)			(0.039)		
Network Green Citations			0.181**				0.078**
			(0.084)				(0.038)
R^2	0.069	0.068	0.070	0.027	0.071	0.010	0.044
Obs	144	144	144	75	273	75	273

Notes: Standard errors clustered by sector are reported in parentheses. Columns 4-7 only consider sectors with at least one green patent in a period. All variables are winsorized to the 90th percentile, and all specifications include time fixed effects. † $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

of the sector's upstream and downstream exposures, so that both direct and indirect neighbors contribute.[10] The dependent variable is a sector's log emissions intensity reduction in columns 1–3, its green-patent share in 4–5, and its green-citation share in 6–7. The network variable on the right-hand side varies across columns, as indicated by the row headings. Appendix Table A.1 lists summary statistics for all of the variables used in these regressions.

All seven coefficients are positive, as strategic complementarity would predict. The emissions reduction regressions (columns 1–3) imply that a sector's own emissions intensity reduction responds to network greenification with an elasticity around 0.2. Own patenting and citation outcomes (columns 4–7) move in the same direction, with coefficients around 0.1. We emphasize that these relationships are correlational: they are consistent with our model's strategic-complementarity channel, but also with common shocks to connected sectors. What the pattern does establish is that the supply-chain dimension of greenification is quantitatively large enough to take seriously. We note that recent empirical work also provides causal evidence on strategic complementarity in greenification of the automotive sector (Gessner, 2026).[11]

[10] We construct the aggregate using the Leontief inverse of the input-output matrix (minus the identity); Appendix Table A.2 reports upstream and downstream separately. For emissions, we count only reductions in connected sectors' emissions (setting increases to zero) so that the aggregate measures network greening. Appendix Table A.3 shows the relationship largely persists at a one-period lag, and Appendix Table A.4 reports the weighted version.

[11] Gessner (2026) documents that stricter downstream environmental regulations cause car manufacturers to form more supply chain links and their upstream suppliers to increase green innovation.

3 A Green Supply Chain

We start with a simple model of a vertical supply chain. Section 3.1 presents the model, section 3.2 derives the decentralized economy, and section 3.3 solves for the social optimum. Sections 3.4 and 3.5 both consider limited policy intervention, where either subsidies are limited to key sectors or the carbon tax is suboptimal.

3.1 Model

Preferences. Time is discrete and denoted by t. The consumer demand side of the economy consists of a continuum of mass one of agents with the same intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \kappa \ell_t - a_t\right), \tag{1}$$

where c_t denotes the consumption flow, ℓ_t denotes labor supply, κ parameterizes the disutility of labor, a_t denotes the disutility of pollution, and β is the discount factor.

Production. The production side is a vertical supply chain consisting of N layers or "sectors" which we rank from the most upstream, namely $i = 1$, to the most downstream, $i = N$, which is the consumption good. Production y_{it} in each sector i at time t is a Cobb-Douglas aggregate of a continuum of mass one of sector-specific varieties (ν):

$$\ln y_{it} = \int_0^1 \ln y_{it}(\nu)\, \mathrm{d}\nu. \tag{2}$$

"Varieties" should be understood here as tasks, production processes, or subsectors, rather than representing horizontal differentiation of the same good.

Each variety ν in any sector i can be produced using either a dirty or clean technology. Three features distinguish clean and dirty technologies. First, the dirty technology is associated with pollution while the clean one is not. Second, the dirty technology is always available, while the clean technology is only available for a variety ν in sector i that is "greenified"—greenification occurs through a process described below. And third, the dirty technology only uses labor (one for one), while the clean technology uses good $i-1$ as an intermediate input together with labor in a Cobb-Douglas fashion. More specifically, the most upstream input is produced according to:

$$y_{1t}(\nu) = \ell_{d1t}(\nu) + \gamma_{1t}(\nu)\, e^{z} \ell_{c1t}(\nu),$$

and good i ($i > 1$) is produced according to:

$$y_{it}(\nu) = \ell_{dit}(\nu) + \gamma_{it}(\nu) \left(\frac{e^z \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \left(\frac{m_{it}(\nu)}{1-\alpha_i} \right)^{1-\alpha_i}, \tag{3}$$

where: (i) $\gamma(\nu)$ is an indicator function, equal to 1 for greenified varieties and to 0 for non-greenified varieties; (ii) $\alpha_i \in (0,1)$ for all $i > 1$; (iii) $\ell_{dit}(\nu)$ denotes the labor input used by variety ν in sector i using the dirty technology; (iv) $\ell_{cit}(\nu)$ and $m_{it}(\nu)$ denote respectively the labor input and the amount of intermediate input from sector $i-1$ used for producing variety ν with the clean technology; (v) e^z is the relative (labor-augmenting) productivity of using the clean, rather than dirty, technology.[12] Therefore, for non-greenified varieties, only the dirty production process is available, while for greenified varieties, the producer has access to two perfect substitute technologies.

Our initial focus is therefore on a vertical supply chain where greenification involves the use of new inputs that are themselves only clean upon further greenification. From a theoretical standpoint, this is interesting because it is the simplest case that enables us to discuss strategic complementarity in greenification along the supply chain. But, this is also an empirically relevant situation: For instance, electric vehicles rely on a new type of batteries, the production of which is currently CO_2-intensive, but amenable to green innovation. In Section 4, we extend our analysis to a generic network with both clean and dirty supply chains as well as heterogeneous z, derive conditions under which strategic complementarity persists, and analyze the consequences of strategic substitutability.

Emissions. We assume that producing with the dirty technology generates 1 unit of emissions per unit of labor, so that aggregate emissions are given by $E_t = \ell_{dt}$, where ℓ_{dt} is the total labor input used for dirty production. We denote by ξ the utility cost of emissions so that the total disutility of pollution is $a_t = \xi \ell_{dt}$.[13] Note that a_t can be interpreted as the present value of damages from emissions at time t, so equation (1) need not imply that carbon emissions only generate contemporaneous damages. In fact, our formulation is isomorphic to the carbon cycle

[12]For final good consumers, z could also represent a preference shifter toward cleaner varieties.

[13]This setting is equivalent to one where production with the dirty technology uses a free and inexhaustible fossil fuel resource together with labor in a Leontief way. Alternatively, the model can accommodate extraction costs for the resource if ξ includes both environmental damages and the extraction costs and the carbon tax τ introduced below includes both the tax itself and the extraction cost. For simplicity, we assume a constant ξ but the analysis can be straightforwardly generalized to a time-varying ξ.

of Golosov, Hassler, Krusell, and Tsyvinski (2014).[14]

Market Clearing. Labor market clearing requires that, at any time t:

$$\ell_{ct} = \sum_{i=1}^{N} \ell_{cit} \equiv \sum_{i=1}^{N} \int_0^1 \ell_{cit}(\nu)\, \mathrm{d}\nu, \quad \ell_{dt} = \sum_{i=1}^{N} \ell_{dit} \equiv \sum_{i=1}^{N} \int_0^1 \ell_{dit}(\nu)\, \mathrm{d}\nu, \quad \ell_t = \ell_{ct} + \ell_{dt} + \ell_{et},$$

where ℓ_{cit} and ℓ_{dit} are the labor inputs used for clean and dirty production in sector i (integrating across varieties within each sector), ℓ_{ct} and ℓ_{dt} are the total labor used for clean and dirty production (summing across all sectors), and ℓ_{et} is the total labor employed for greenification (we specify the greenification technology below).

Market clearing in the upstream sectors $i = 1, \ldots, N-1$ equalizes total output of sector i with its use as an intermediate input across varieties in sector $i+1$:

$$y_{i,t} = \int_0^1 m_{i+1,t}(\nu)\, \mathrm{d}\nu,$$

whereas market clearing for the downstream consumption good N simply boils down to:

$$c_{Nt} = y_{Nt}.$$

Greenification and Market Power. To operate the clean technology, a variety must be "greenified". A producer of variety ν in sector i must incur a one-time sunk cost which is sector-variety specific to greenify production. We order varieties in each sector i by increasing cost of greenification and let $\phi_i(s)$ denote the cost of greenification associated with the variety quantile s in sector i, where that cost is expressed in labor units.[15]

We let $F_i(\cdot)$ denote the CDF cost distribution, which we assume to be continuously differen-

[14]Golosov et al. (2014) assume that net-of-damages consumption is given by $\tilde{c}_t = \exp(-DS_t)\, c_t$, where D is a damage coefficient, and S_t is excess carbon concentration. Further, they assume that a share φ_L of emissions stay in the atmosphere, a fraction φ_T decays geometrically at rate φ, while the remaining emissions $(1 - \varphi_T - \varphi_L)$ are quickly reabsorbed. Therefore, excess carbon concentration is given by $S_t = \sum_{\tau=0}^{t} \left(\varphi_L + \varphi_T (1-\varphi)^{t-\tau}\right) E_\tau$. Then, if the utility function is $U \equiv \sum_{t=0}^{\infty} \beta^t (\ln \tilde{c}_t - \ell_t)$, we can rewrite it after some algebra as:

$$U = \sum_{t=0}^{\infty} \beta^t (\ln c_t - \ell_t - DS_t) = \sum_{t=0}^{\infty} \beta^t (\ln c_t - \ell_t - \xi \ell_{dt}),$$

with $\xi \equiv D\left(\frac{\varphi_L}{1-\beta} + \frac{\varphi_T}{1-\beta(1-\varphi)}\right)$ a constant, exactly as in our formulation.

[15]Our model of greenification is directly inspired by the automation literature where after an innovation or payment of a fixed cost, labor can be replaced by capital in the production process (see e.g. Zeira 1998). Colmer et al. (2025) provide empirical backing for such a model of greenification.

tiable (or a single mass point in some examples). That is, for any cost ϕ, $F_i(\phi)$ is the measure of the set of variety quantiles s in sector i with costs $\phi_i(s)$ less than ϕ. Let χ_{it} denote the fraction of greenified varieties at time t in sector i, which also corresponds to the cut-off quantile s beyond which varieties cease to be greenified. The sum of all greenification fixed costs up to χ_{it} is given by $\mathcal{F}_i(\chi_{it}) \equiv \int_0^{\chi_{it}} \phi_i(s)\, \mathrm{d}s$.[16] The collection of χ_{it}'s across sectors forms the key state variables of the economy. Starting from an initial condition $\{\chi_{i0}\}_{i=1}^N$ at $t = 0$, greenification raises χ_{it}'s monotonically over time, until χ_{it}'s converge to a steady-state.

We assume that the dirty technology is operated competitively, but that, if she greenifies her variety, the producer of that variety earns one period of monopoly profit upon greenification, with fringe producers operating the dirty technology as competitors.[17] Starting from the subsequent period, the clean technology becomes freely available to all producers so that the variety is again competitively produced.

Policy Instruments. We assume that the government can: (i) impose a carbon tax τ; or (ii) impose a cap-and-trade limit $\bar{\ell}_d$ on the amount of dirty input used (leading to a carbon price τ); and (iii) implement an industrial policy that is a set of sector-specific, time-varying greenification subsidies q_{it}. In what follows, we take the carbon tax τ as constant for simplicity but the analysis generalizes straightforwardly to a time-varying τ.

3.2 Equilibrium

We now characterize the equilibrium of the economy in laissez-faire and when the government only resorts to a carbon tax ($\tau \geq 0$).

Equilibrium Price. In each period, the representative consumer solves

$$\max_{c_t, \ell_t} \ln c_t - \kappa \ell_t - a_t \qquad \text{s.t. } p_t c_t = w_t \ell_t + \pi_t + T_t,$$

where p_t is the price index of the consumption good, π_t are profits, T_t is a lump-sum transfer from the government, and w_t is the wage rate which we normalize to one.[18] Consumer optimization implies that the total expenditure on the consumption good is constant $p_t c_t = 1/\kappa$. Without loss

[16] There is an obvious relationship between F_i and $\mathcal{F}_i$. Namely, for any s : $F_i(\phi_i(s)) = s$ or equivalently $\phi_i(s) = F_i^{-1}(s)$. This directly leads to $\mathcal{F}_i(\chi_{it}) = \int_0^{\chi_{it}} F_i^{-1}(s)\, \mathrm{d}s$.

[17] With this market structure, it is never profitable for more than one producer of the same variety to spend the sunk cost to greenify.

[18] For simplicity, investment in greenification and its outcome occur within the same period, so that there is no meaningful intertemporal decision. Households may have access to a bond in zero net supply, but will end up maximizing flow consumption.

of generality, we set $\kappa = 1$ from here on, so that $p_t c_t = 1$.

Given that dirty production uses only labor, its marginal cost is $1+\tau$. We assume that τ and/or z are large enough to ensure that $1+\tau > e^{-z}$. This condition ensures that a producer always uses the clean technology once her variety has been greenified. For notational simplicity, we define $Z \equiv \ln(1+\tau) + z$, as the tax-adjusted relative productivity of clean versus dirty technology, so we assume that $Z > 0$.[19]

The equilibrium price index p_{it} for good i satisfies

$$\ln p_{it} \equiv \int_0^1 \ln p_{it}(\nu) \, \mathrm{d}\nu, \tag{4}$$

where, for each variety ν in sector i, $p_{it}(\nu)$ is determined as follows. If variety ν has been greenified by the previous period, then it is priced at the marginal cost of the clean technology. If variety ν has not yet been greenified in the previous period, then it is priced at the marginal cost of the dirty technology. This is trivial for producers using the dirty technology, but it is also true for a newly greenified variety. The producer of a newly greenified variety is a monopolist who faces a fringe that uses the dirty technology. Given the unit demand elasticity, she charges a price equal to the marginal cost of the fringe.

The market structure implies that in the most upstream sector (sector 1), the price of a variety that has been greenified in the previous period is equal to $p_{1t}(\nu) = e^{-z}$, whereas the price of a non-greenified or of a newly greenified variety is $p_{1t}(\nu) = 1+\tau$. Next, in sectors $i > 1$, we have:

$$p_{it}(\nu) = \begin{cases} e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i} & \text{if the variety has been greenified by time } t-1, \\ 1+\tau & \text{otherwise,} \end{cases} \tag{5}$$

as the marginal cost of dirty production is $1+\tau$, while that of clean production is $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$.

To get more explicit expressions for these equilibrium prices, it is helpful to consider, for all i, the network-adjusted share μ_{it} of greenified content in the production of any greenified variety in sector i. In the most upstream sector 1, we have $\mu_{1t} = 1$. In more downstream sectors $i > 1$, μ_{it} is recursively determined by:

$$\mu_{it} = \alpha_i + (1-\alpha_i) \chi_{i-1,t} \mu_{i-1,t}. \tag{6}$$

In words, the greenified content of a greenified variety in sector i, is equal to the direct share of

[19] $Z > 0$ implies that the labor portion of the cost of the clean technology is lower than the cost of the dirty technology. This is a necessary and sufficient condition to ensure that the clean technology is cheaper than the dirty one (regardless of the share of greenified varieties) because the intermediate-input part of the costs is a weighted average between clean and dirty labor costs.

clean labor input α_i plus $(1-\alpha_i)$ times the aggregate greenified content of inputs from $i-1$, which in turn is equal to the greenified content $\mu_{i-1,t}$ of each greenified variety in sector $i-1$ times the fraction $\chi_{i-1,t}$ of greenified varieties.

We can now solve for the price index in each sector. For the most upstream sector 1, we use (4) and (5) and solve for downstream prices by induction:

$$p_{1t} = (1+\tau)\, e^{-\chi_{1,t-1}Z} \text{ and } p_{it} = (1+\tau)\, e^{-\chi_{i,t-1}\mu_{i,t-1}Z}. \tag{7}$$

Therefore higher greenification in sector i or upstream of sector i reduces the price of sector i but only with a one-period delay. This is because the cost reduction achieved by greenification is only passed down to consumers once the monopoly position of the innovator expires.

Equilibrium Profits. The incentive to greenify depends on the profits that a newly greenified variety obtains, which we now derive. The producer of a newly greenified variety charges a price $1+\tau$ but faces marginal costs equal to $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$; therefore, she charges a mark-up θ_{it} given by

$$\theta_{it} = \frac{1+\tau}{e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}} = e^{Z\mu_{i,t-1}},$$

where the second equality uses Equations (6) and (7) to substitute for $p_{i-1,t}$. Higher greenification upstream (a higher $\mu_{i,t-1}$) enables the innovator to charge a higher mark-up but again only with a one-period delay. Once a clean input is produced competitively, it becomes cheaper than the dirty input, resulting in a lower marginal cost for the downstream clean producer. The profit share of revenues is $1-\theta_{it}^{-1}$, and the input cost share is θ_{it}^{-1}.

We next derive the equilibrium revenue r_{it} of a producer in sector i at time t. In the most downstream sector N, which produces the final consumption good, we know that

$$r_{Nt} = p_t c_t = 1.$$

From there we move upstream, as revenues trickle up from downstream to upstream sectors. Take as given the revenues $r_{i+1,t}$ of a producer in sector $i+1$ at time t. Then, sector i's good is only used as an input by the greenified varieties in sector $i+1$. For any variety in sector i, the revenue r_{it} includes both sales to previously greenified varieties and sales to newly greenified varieties. There is a mass $\chi_{i+1,t-1}$ of previously greenified varieties. These are produced competitively, so that a share $1-\alpha_{i+1}$ of their revenues goes to sector i. There is a mass $\chi_{i+1,t}-\chi_{i+1,t-1}$ of newly greenified varieties, where only a share $\theta_{i+1,t}^{-1} = e^{-Z\mu_{i+1,t-1}}$ of their revenues goes to the payment of inputs, out of which a share $1-\alpha_{i+1}$ goes to sector i. We then obtain:

$$\begin{aligned} r_{it} &= \underbrace{\chi_{i+1,t-1} r_{i+1,t} \left(1-\alpha_{i+1}\right)}_{\text{sales to previously greenified varieties}} + \underbrace{\left(\chi_{i+1,t}-\chi_{i+1,t-1}\right) r_{i+1,t} e^{-Z\mu_{i+1,t-1}} \left(1-\alpha_{i+1}\right)}_{\text{sales to newly greenified varieties}} \\ &= \widetilde{\chi}_{i+1,t} r_{i+1,t} \left(1-\alpha_{i+1}\right), \end{aligned}$$

where we define sector $i+1$'s revenue share spent on clean inputs as

$$\widetilde{\chi}_{i+1,t} \equiv \chi_{i+1,t-1} + \left(\chi_{i+1,t}-\chi_{i+1,t-1}\right) e^{-Z\mu_{i+1,t-1}}. \tag{8}$$

This in turn immediately yields the following expression for the equilibrium revenue accruing from downstream to good i production:[20]

$$r_{it} = \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{jt}(1-\alpha_j)\right). \tag{9}$$

Higher greenification downstream (higher $\widetilde{\chi}_{jt}$ for $j>i$) increases demand for upstream producers and therefore their revenues.

The corresponding profit from greenification for any variety producer in sector i, is then simply equal to the profit share times revenues:

$$\pi_{it} = (1-e^{-Z\mu_{i,t-1}}) r_{it}. \tag{10}$$

Greenification. A variety producer in sector i greenifies at time t if and only if π_{it} is bigger than the cost of greenification of that variety. It then immediately follows that the equilibrium share of greenifying varieties χ_{it} in sector i at time t satisfies

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i(\pi_{it})\right\}, \tag{11}$$

where the $\max$ operator reflects that there is no dis-greenification (note that this expression still applies if there is full greenification: $\chi_{it}=1$). Plugging in (10) and (9), we get:

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i\left(\left(1-e^{-Z\mu_{i,t-1}}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{jt}(1-\alpha_j)\right)\right)\right\}. \tag{12}$$

Greenification incentives in sector i depend on both upstream and downstream greenification: On the one hand, (past and contemporaneous) downstream greenification increases total revenue $r_{it} = \Pi_{j=i+1}^{N}\widetilde{\chi}_{jt}(1-\alpha_j)$ in sector i (a demand effect), and on the other hand, (past) upstream greenification increases $\mu_{i,t-1}$ and therefore the profit share $\left(1-e^{-Z\mu_{i,t-1}}\right)$ of a newly greenified

[20] We follow convention and set $\prod_{j=a}^{b} x_j = 1$ when $a>b$.

producer in sector i (an input cost effect).[21]

Equilibrium Equations. We can then characterize the equilibrium as follows. Given initial greenification shares $\{\chi_{i0}\}$ at $t = 0$, an equilibrium with a carbon tax τ is a sequence of greenification shares $\{\chi_{it}\}_{t>0}$ such that (12) is satisfied where the sequences $\{\mu_{it}\}$ and $\{\tilde{\chi}_{it}\}$ are defined iteratively through $\mu_{1t} = 1$, (6) and (8). Despite the complementarities in greenification, the market structure ensures that the equilibrium is unique. In Appendix B.1, we show:

Proposition 1. *Given initial condition $\{\chi_{i0}\}$, the economy with carbon taxes $\{\tau_t\}$ features a unique equilibrium path $\{\chi_{it}\}_{t>0}$.*

The initial level of greenification, χ_{i0}, determines the full vector of markups at time 1. This enables us to solve for χ_{i1} starting from the most downstream sector and moving up. In the following period, the share of competitive greenified products will have changed, so that markups in each sector will have increased, prompting the next round of greenification. The cycle repeats until we reach a steady-state.

Multiplicity of Steady-States. We next characterize the steady-states of the economy. In a steady-state, there are no newly greenified varieties, so $\widetilde{\chi}_i = \chi_i$ for all i. It follows that a steady-state with a carbon tax τ is a set of greenification shares $\{\chi_i\}$ such that:

$$\mu_1 = 1, \quad \mu_i = \alpha_i + \chi_{i-1}\mu_{i-1}(1-\alpha_i) \text{ and} \tag{13}$$

$$\chi_i \geq F_i\left(\left(1 - e^{-Z\mu_i}\right) \prod_{j=i+1}^{N} \chi_j\left(1-\alpha_j\right)\right). \tag{14}$$

Condition (14) is an inequality for the same reason as the "max" notation in (12): technically, since greenification cannot decrease, any $\{\chi_i^{ss}\}$ for which (14) holds as a strict inequality is also a steady-state. These are, however, not very interesting steady-states since, starting from $\chi_{i0} < \chi_i^{ss}$, there is no path that the economy can follow to reach them without direct government intervention. We therefore generally ignore them in our analysis, and we focus only on steady-states in which Condition (14) holds as an equality:

$$\chi_i = F_i\left(\left(1 - \left[e^{z}\left(1+\tau\right)\right]^{-\mu_i}\right) \prod_{j=i+1}^{N} \chi_j\left(1-\alpha_j\right)\right). \tag{15}$$

[21] Upstream greenification also decreases revenues as it reduces the sales to newly greenified varieties (see (8)). This effect is temporary and its impact on profits is generally dominated by the effect of upstream greenification on the profit share. $Z < \ln 2$ is a sufficient condition to get π_{it} always increasing in μ_{it}.

Equation (15) also clearly establishes that there is strategic complementarity in greenification in steady-state as its right-hand side is weakly increasing in all χ_j (for $j \neq i$) and strictly so if all $\chi_j \neq 0$. This strategic complementarity ensures that the economy generally features more than one steady-state. In Appendix B.2, we show:

Proposition 2. *For a given carbon tax τ, there is strategic complementarity in greenification in steady-state and there exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$. There is a unique steady-state when $N = 1$.*

Naturally, the same logic extends to the case where the government implements a cap-and-trade system instead of a carbon tax. In particular, multiple steady-states remain possible: for instance, a steady-state with a high level of greenification and a low price of carbon can co-exist with a steady-state with a low level of greenification and a high price of carbon. Both steady-states may achieve the same level of emissions (if the cap binds), but the low greenification steady-state does it with lower output (see Appendix B.3).[22]

3.3 Social Optimum

We now solve for the Social Planner problem and compare the optimum with the decentralized economy. Starting from initial conditions $\{\chi_{i0}\}$, the Social Planner seeks to maximize intertemporal utility (1). Greenification, once the fixed costs are paid, weakly pushes out the production possibility frontier. The social costs of greenification correspond to the linear disutility of extra labor, which is independent of the share of already greenified varieties. Therefore, all greenification happens immediately in the optimum. With constant technology levels after the initial greenification, labor allocation to clean and dirty production, and output are constant. Keeping in mind that $c_t = y_{Nt}$, that ξ is the social cost of pollution, and using $\sum_{t=0}^{\infty} \beta^t = 1/(1-\beta)$, the Social Planner solves:

$$\max_{\{\ell_{di}, \ell_{ci}, \chi_i\}} \frac{1}{1-\beta} \left(\ln y_N - (1+\xi) \sum_{i=1}^{N} \ell_{di} - \sum_{i=1}^{N} \ell_{ci} \right) - \sum_{i=1}^{N} \left(\mathcal{F}_i(\chi_i) - \mathcal{F}_i(\chi_{i,0}) \right). \tag{16}$$

We define $Z^* \equiv z + \ln(1+\xi)$ and assume that $Z^* > 0$, which ensures that the Social Planner uses the clean technology whenever it is available.

Thus, we can characterize the Social Planner's problem as (proof in Appendix B.4):

[22] A cap at zero, however, eliminates the multiplicity since it forces full greenification in all sectors, but it is generally not optimal to force emissions to zero.

Proposition 3. *If the solution χ_i is interior (or $\chi_i = 1$), it must satisfy the first-order conditions:*

$$\chi_i = F_i\left(\frac{\mu_i Z^*}{1-\beta}\prod_{j=i+1}^{N}\left[\chi_j\left(1-\alpha_j\right)\right]\right). \tag{17}$$

If the lower bound binds, i.e. χ_i=χ_{i0}, then the equality in (17) is replaced with $\geq$.

Proposition 3 closely follows the logic of Hulten's theorem: the argument of the CDF is the perpetual value of the revenue of a sector $r_i = \prod_{j=i+1}^{N}\left[\chi_j\left(1-\alpha_j\right)\right]$ times the productivity improvement from switching to the green technology $\ln\theta_i = \mu_i Z^*$. That is, the Planner is willing to commit resources to greenify a sector according to its importance—given by its Domar weight—and the degree to which the green technology is superior.

Steady-States versus the Social Optimum. We now compare the steady-state decentralized equilibrium with the social optimum. Obviously, for given degrees of greenification $\chi's$, the decentralized economy in steady-state allocates production resources (labor) efficiently provided that the carbon tax is at the Pigouvian level: $\tau = \xi$ (so that $Z = Z^*$).

More interestingly, we can analyze greenification in the two economies by comparing the steady-state equations (15) and the first-order conditions (17). There are two differences. First, the factor $\frac{1}{1-\beta}$ on the RHS of (17) is absent from the RHS of (15). This captures an intertemporal spillover effect whereby the social gain from greenification carries over to the whole future, whereas private producers benefit from greenification for one period only as patents expire after one period. This inefficiency could be corrected with a uniform subsidy to greenification equal to the discount factor β.

A second difference comes from the term $1-e^{-Z^*\mu_i}$ on the RHS of (15) (assuming a Pigouvian tax $\tau = \xi$) versus $\mu_i Z^*$ on the RHS of (17). This difference reflects the fact that the Social Planner maximizes total surplus, including consumer surplus, whereas producers only maximize their private rent from greenification.[23] The social surplus is larger than the private surplus, so that ceteris paribus, there is too little greenification in equilibrium (for $Z^* > 0$, we get that $1 - e^{-Z^*\mu_i} < \mu_i Z^*$). Yet, for a small Z^*, this difference is negligible: $1-e^{-Z^*\mu_i} = \mu_i Z^* + o\left(Z^*\right)$. Then, abstracting from the $\frac{1}{1-\beta}$ factor on the RHS of (17), the two Equations (15) and (17) become

[23] To see this, suppose there is just one sector, and that the carbon tax is set equal to the social cost of pollution. The model is then equivalent to having a (dirty) technology with productivity $\frac{1}{1+\xi}$ and a clean technology with productivity e^z. Under Cobb-Douglas preferences, the demand curve is $p = 1/y$. With competitive production, the consumer surplus of improving the productivity of a variety from $\frac{1}{1+\xi}$ to e^z is $\int_{\frac{1}{1+\xi}}^{e^z}\frac{1}{y}\,\mathrm{d}y = Z^*$, which differs from the private rent from such a productivity improvement.

nearly identical.

Is it the case, then, that a uniform subsidy β to greenification coupled with a Pigouvian carbon tax is enough to decentralize the optimum in steady-state when the overall advantage of the clean technology is small (Z^* close to 0)? The answer is no, which reveals the fundamental reason why industrial policy is warranted in our set-up: cross-sectoral strategic complementarities in greenification. That is, insufficient greenification in sectors downstream and/or upstream to sector i, reduces private incentives to greenify in sector i (see (17)), typically leading to multiple steady-state equilibria as shown above. Similarly, the first-order conditions (17) are generally not sufficient for the global optimum. Then, starting from initially low levels of greenification in all sectors, the economy may get stuck in a steady-state which also features low levels of greenification, even though the optimum might involve a high level of greenification in all sectors. The following example with Pigouvian taxation on emissions illustrates this point.[24]

Example 1. We consider a two-sector supply chain, so $N = 2$. We set $z = 0$ for simplicity and assume that ξ is small. In addition, we assume that greenification is uniformly subsidized at rate β , so that firms only face the greenification cost $(1-\beta)\,\phi$. Under these conditions, both the steady-state equilibrium equations and the first-order conditions for the social optimum can be written as:

$$\chi_1 = F_1\left(\frac{\xi\chi_2\,(1-\alpha_2)}{1-\beta}\right) \text{ and } \chi_2 = F_2\left(\frac{\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))}{1-\beta}\right). \tag{18}$$

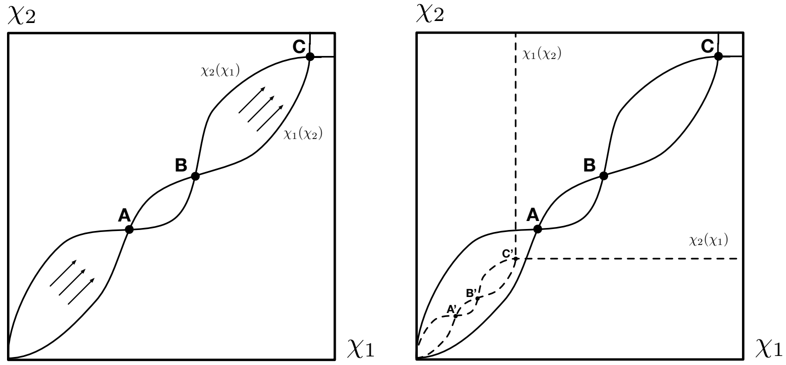
where F_1 and F_2 are chosen so that the two curves in Figure 2a intersect three times—at A, B, and C—all of which correspond to a steady-state of the decentralized economy. As Figure 2a shows, the two steady-states A and C are both stable, whereas B is unstable. For β sufficiently large, the social optimum corresponds to point C, whereas a decentralized economy starting from initially low or no greenification ends up being stuck at the low greenification steady-state A.

Without the uniform subsidy β, the decentralized steady-states satisfy

$$\chi_1 = F_1\left(\xi\chi_2\,(1-\alpha_2)\right) \text{and} \chi_2 = F_2\left(\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))\right).$$

These two equations correspond to the two dashed lines in Figure 2b, which also intersect three times at the steady-state equilibria A', B', and C'. Then an economy with low or no initial greenification will end up being stuck at the—even lower greenification—steady-state A'. The uniform subsidy β allows an economy with initially very low or no greenification to converge to A instead

[24]Of course, a sufficiently high carbon tax can remove the multiplicity of steady-states, but implementing such a tax above its Pigouvian value ξ is suboptimal.



(a) Decentralized economy with a subsidy β

(b) Decentralized economy without a subsidy β

Figure 2. Multiple steady-states: First-order conditions of the Social Planner's problem and decentralized economy with and without a subsidy β.

of A', but additional sector-specific subsidies are required on top of the uniform subsidy to make the economy converge to C.

Implementing the Social Optimum. Starting from the same initial conditions of low levels of greenification in all sectors, the Social Planner can achieve the optimum through a set of temporary subsidies to greenification. A subsidy $q_{i,t}$ to greenification in sector i reduces private greenification costs from $\phi_i(\nu)$ to $(1 - q_{it})\,\phi_i(\nu)$. Subsidies are financed lump-sum by consumers. As shown in the previous section, for any given policy and initial conditions, the equilibrium is unique, so that we establish (proof in Appendix B.5):

Proposition 4. *The optimal steady-state can be uniquely implemented through a carbon price together with a whole set of time-varying sector-specific greenification subsidies.*

Note that Proposition 4 is about decentralizing the optimal steady-state but not the full optimal allocation.[25] In the full optimum, users of intermediate inputs should buy newly greenified inputs at marginal costs, which requires the implementation of subsidies for the use of intermediates. However, with such instruments (a Pigouvian carbon tax, sector-specific subsidies for greenification, and subsidies to the use of intermediates), the equilibrium will generally not be unique and a Social Planner would have to use the type of instruments analyzed by Sturm (2023) to ensure uniqueness.[26]

[25]In the proof of Proposition 4, at $t = 1$, the optimal environmental tax and greenification levels are implemented, but newly greenified varieties are priced monopolistically and thus inefficiently under-supplied.

[26]The logic extends to the case where carbon pricing is enforced through a cap-and-trade system instead of a tax.

Small Subsidies can make a Big Difference. Are large subsidies—big-push policies—always necessary to move the economy away from a low greenification steady-state? Figure B.1 describes an example with three steady-state equilibria: no greenification, full greenification, and an interior, unstable steady-state, which is close to the no greenification steady-state. Starting from no greenification, small, targeted, and temporary sector-specific subsidies are enough to move the economy a little beyond the unstable steady-state from which point the economy converges on its own toward the high greenification steady-state. Provided that consumers are sufficiently patient, the full greenification steady-state dominates the other two and corresponds to the optimum. We develop a formal example along these lines in Appendix B.6.

3.4 Propagation and Appropriate Industrial Policy

Our analysis so far has considered a government that could intervene in several sectors at the same time, since implementing the optimum generically requires such a multi-faceted intervention. In practice, however, a government may be constrained to focus on one or a few key sectors at a time. What can be achieved in that case? To answer that question, we focus on marginal changes in greenification around a stable steady-state.[27]

Consider a stable steady-state equilibrium and assume that the government implements a (now permanent) marginal subsidy to greenification q_i in sector i. Then the new steady-state equilibrium satisfies:

$$\chi_i = F_i\left(\frac{\pi_i}{1-q_i}\right) \text{ with } \pi_i = \underbrace{\left(1-e^{-\mu_i Z}\right)}_{\text{input cost reduction}} \underbrace{\prod_{j=i+1}^{N} \chi_j\left(1-\alpha_j\right)}_{\text{demand from downstream}}, \tag{19}$$

and μ_i still given by (6). While greenification incentives propagate in both directions (downstream through the input cost channel and upstream through a demand effect), the effect is asymmetric: the same perturbation in the greenification rate χ_k in sector k tends to generate more greenification incentives in upstream sectors $i < k$ through the demand channel than in downstream sectors ($i > k$) through the input cost channel—in fact, the latter is close to zero when greenification rates are low. In other words, starting from a steady-state with low greenification across all sectors, the bang-for-the-buck is generically higher for policy that induces greenification first

As long as there are multiple steady-states, sector-specific subsidies to greenification remain necessary, on top of cap-and-trade, to ensure that the economy converges toward the optimal steady-state.

[27] Absent policy intervention, the economy is likely to be in a stable steady-state, which justifies our focus on such an environment. Alternatively, in Appendix B.7, we analyze how a targeted policy may or may not get the economy out of a no-greenification trap and consider the full transition.

in more downstream sectors.

To see this more formally, note that the elasticity of greenification in sector i to a subsidy in sector k can be decomposed into

$$\frac{\partial \ln \chi_i}{\partial \ln q_k} = \frac{\partial \ln \chi_i}{\partial \ln \pi_i} \frac{\partial \ln \pi_i}{\partial \ln \chi_k} \frac{\partial \ln \chi_k}{\partial \ln q_k}.$$

The first and third terms on the right depend on the cost distribution of sectors i and k respectively. In contrast, the middle term, representing the elasticity of greenification incentives in i (i.e. profits) with respect to greenification in k, depends only on network structure and is therefore the object of our attention. In Appendix B.8, we prove:

Proposition 5. *i) An increase in greenification downstream raises greenification incentives one-for-one:* $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = 1$ *if* $k > i$. *ii) An increase in greenification upstream raises incentives less-than-one-for-one*

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{\mu_i Z e^{-\mu_i Z}}{1 - e^{-\mu_i Z}} \frac{\mu_k \prod_{j=1}^{i-k} \chi_{i-j} \left(1 - \alpha_{i-j+1}\right)}{\mu_i} < 1 \text{ if } k < i. \tag{20}$$

iii) Incentives propagated from upstream rely on greenification along the chain: $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} \to 0$ *if* $\chi_j \to 0$ *for any* $j \in [k, i)$.

Part i) establishes that exogenously more greenification in downstream sectors generates proportional gains in greenification incentives in all upstream sectors. This is because greenification downstream increases the market for greenified varieties upstream one-for-one. In contrast, Part ii) establishes that exogenously more greenification upstream generates less-than-proportional gains in greenification incentives in downstream sectors, and the incentives get smaller and smaller the further downstream we go. This is because both $\frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1$ and $\frac{\mu_k \prod_{j=1}^{i-k} \chi_{i-j}(1-\alpha_{i-j+1})}{\mu_i} < 1$. The latter expression is the share of greenified content coming from sector k in the greenified content of sector i. It decreases as k is further upstream from i (i.e for smaller k) and the presence of the term $\prod_{j=1}^{i-k} \chi_{i-j}$ shows that the downstream propagation of incentives can be subject to bottlenecks and weak links. Intuitively, greenification in upstream sector k reduces the cost of greenified varieties in downstream sector $i > k$ but less than proportionately because at each stage of the production process, the cost of producing a given intermediate depends on the costs of the more upstream intermediates—in proportion to how green the supply chain from k to i is—as well as on labor costs.

The contrast between downstream and upstream propagation is particularly stark when at least one sector features low greenification, as the additional downstream incentives resulting from upstream greenification can become arbitrarily small. To see this, note that μ_i (the network-adjusted share of greenified content for producing a greenified variety in sector i) is bounded away from 0 so that for any sector k upstream to i, the additional incentive in sector i generated by greenification in sector k, $\frac{\partial \ln \pi_i}{\partial \ln \chi_k}$, goes to 0 whenever one of the χ_j tends to 0 for any $j \in [k, i-1]$. In other words, the downstream propagation of greenification incentives from k to i is only as strong as the weakest link between these two sectors. Therefore, starting from a steady-state with low greenification in at least some sectors ($\chi_i \approx 0$ for some i), the downstream propagation of incentives from targeting upstream sectors is close to zero, and industrial policy should first target downstream sectors.[28]

Our intuitive reasoning in this subsection is incomplete in two respects. First, it considers the change in incentives in sector i from perturbing χ_k, holding all other χ_j's constant. However, in equilibrium, χ_j would respond for all j, generating further changes in incentives. This can be solved easily and in Appendix B.9, we show that for a stable steady-state, the implementation of a small vector of subsidies $\boldsymbol{dq}$ leads to changes in greenification given by:

$$\boldsymbol{\chi}^{\odot -1} \odot \mathrm{d}\boldsymbol{\chi} = \left(I - \mathrm{Diag}(\boldsymbol{\epsilon_F}) \frac{\mathrm{d} \ln \boldsymbol{\pi}}{\mathrm{d} \ln \boldsymbol{\chi}^\top} \right)^{-1} \mathrm{Diag}(\boldsymbol{\epsilon_F})\, \mathrm{d}\boldsymbol{q}, \tag{21}$$

where $\odot$ denotes a componentwise operator, $\boldsymbol{\epsilon_F}$ is the vector of elasticities of the F distributions ($\epsilon_{F,i} \equiv \frac{\pi_i f_i(\pi_i)}{F_i(\pi_i)}$, this is also the elasticity of technology with respect to innovation costs). The elasticities matrix $\frac{d \ln \boldsymbol{\pi}}{d \ln \boldsymbol{\chi}^\top} \equiv \left(\frac{\partial \ln \pi_i}{\partial \ln \chi_k} \right)_{ik}$ has a null diagonal, upper triangle coefficients all equal to 1, and lower triangle coefficients all smaller than 1. This expression traces the direct and indirect effects of a subsidy vector on the steady-state across the network.

Second, it focuses on perturbations in a steady-state, while ignoring the sequence of changes in incentives along the transition. But these intuitions continue to hold when the path of χ_{jt}'s responds to the exogenous shock in χ_k as well.

3.5 Industrial Policy with Incomplete Carbon Prices

In our optimal policy analysis, we assumed that the planner could implement Pigouvian carbon prices. Unfortunately, global carbon prices are often far below the true social cost of carbon, and some countries, notably the US, have tried to use industrial policy as a substitute for a carbon price, rather than as a complement (as in our model). For this reason, we now study the role of industrial policy in the face of suboptimal carbon prices.

[28] This argument requires the mild condition that the relative marginal costs of greenification across sectors are bounded when the level of greenification is low.

A Second-Best Problem. We consider a version of the Planner's problem where the carbon price is restricted. That is, the Planner has a complete set of greenification subsidies, but due to some political restriction, the carbon price τ is set to an exogenous level below the true social cost ξ. We still assume that $Z = z + \ln(1+\tau) > 0$ so that greenified varieties are produced with the clean technology. Then, the policy problem requires selecting a level of greenification knowing that production decisions are made in a distorted equilibrium. To keep the analysis comparable to that of Section 3.3, we allow the Planner to remove the monopoly distortion, and as before, all greenification happens in the initial period. In Appendix B.10, we establish:

Proposition 6. *In the absence of a Pigouvian carbon price, optimal greenification satisfies*

$$\chi_i = F_i\left(\frac{\mu_i Z}{1-\beta}\prod_{j=i+1}^{N}\left[\chi_j\left(1-\alpha_j\right)\right] + \frac{\xi-\tau}{1-\beta}\Big(-\frac{\partial E}{\partial \chi_i}\Big)\right), \tag{22}$$

if the solution $\chi_i \in (\chi_{i,0}, 1]$. If the lower bound binds, $\chi_i = \chi_{i,0}$ and $\geq$ replaces $=$ in equation (22).

Proposition 6 shows how to adjust industrial policy in the face of suboptimal carbon prices. The first term mirrors that of Equation (17) in reflecting the direct, perpetual benefit of greenification through expanding production. The only difference is that equilibrium output is determined by the carbon price τ, rather than ξ (Z instead of Z^*). The second term reflects the wedge between the carbon price and its true social cost, and it pushes industrial policy to consider how greenification influences the equilibrium level of emissions. Now, greenification in a given sector should be higher in so far as it generates a larger reduction in equilibrium emissions (i.e. a more negative $\frac{\partial E}{\partial \chi_i}$), and the emphasis on emission reductions should scale with the carbon price wedge. Naturally, we recover the first-best whenever carbon prices are Pigouvian.

Equilibrium Emissions. Proposition 6 raises a natural question similar to that considered in Section 3.4: where along the supply chain does greenification most effectively reduce emissions? Interestingly, it is again downstream greenification that generates the greatest equilibrium response when the level of greenification elsewhere is low.[29]

We focus on a steady-state of the decentralized economy (or equivalently the solution to the second-best problem). Recall that sector i's spending share on labor in dirty production is equal to the share of non-greenified varieties $1-\chi_i$. Summing across sectors and taking into account

[29]Downstream's greater emission reductions hold in addition to downstream's greater propagation of greenification because in this section we hold greenification elsewhere constant.

that the overall price of labor in dirty production is $1+\tau$, we get that emissions are given by:

$$E = \frac{1}{1+\tau}\sum_{i=1}^{N} r_i(1-\chi_i).$$

Differentiating, we have that emission reductions follow

$$-\frac{\partial E}{\partial \chi_i} = \frac{\chi_i r_i - \sum_{j=1}^{i-1} r_j(1-\chi_j)}{(1+\tau)\,\chi_i}. \tag{23}$$

Therefore, the impact of greenification on emissions depends on the level of greenification both downstream and upstream. The first term in the square brackets represents the direct displacement of dirty production, which scales with a sector's market size r_i: greenifying a sector is less effective at reducing emissions when low downstream greenification causes the sector to be small. The second term reflects offsetting increases in emissions from increasing demand for dirty inputs in all upstream sectors. The total term in square brackets is weakly positive as a sector's clean revenues are always larger than its expenditures on dirty upstream goods.

While we have shown in the previous sections that there is strategic complementarity in the adoption of greenification, we can see here that there is also complementarity in emission reductions: greenification in each sector is more effective when the others are more greenified: formally, $-\frac{\partial^2 E}{\partial \chi_i \partial \chi_j} \geq 0$.[30] As before, this complementarity implies that, for low levels of greenification, targeting the downstream sector is more effective. To see this, consider the case where $\chi_j = \epsilon x_j$. Then for any $i < j$, we have that

$$\lim_{\epsilon \to 0} \frac{-\partial E/\partial \chi_i}{-\partial E/\partial \chi_j} \propto \lim_{\epsilon \to 0} \prod_{k=i+1}^{j} \chi_k(1-\alpha_k) = 0.$$

With low greenification, the marginal effect of greenification on emission reductions is larger in more downstream sectors, in which case policymakers should focus on downstream to reduce emissions. Intuitively, the marginal effect of greenification in a given sector goes to zero when downstream greenification is very low with a rate of reduction of the same order as the number of downstream sectors. In contrast, the marginal effect of greenifying a sector on emission reductions does not go to zero when upstream greenification goes to zero, because input shares are less than one. Thus, the effectiveness of greenification at reducing emissions is more "vulnera-

[30]This is trivial when $j > i$. For $j < i$, we differentiate (23) and obtain $-\frac{\partial^2 E}{\partial \chi_i \partial \chi_j} = \frac{\chi_j r_j - \sum_{l=1}^{j-1} r_l(1-\chi_l)}{(1+\tau)\chi_i \chi_j}$, which is weakly positive for the same reason as $-\frac{\partial E}{\partial \chi_i} \geq 0$.

ble” to low greenification downstream, rather than upstream.[31] In Appendix B.11, we extend the analysis beyond small greenification levels in the case of a two-layer supply chain.

4 The General Model

The vertical supply chain of Section 3 isolates the mechanism in its simplest form: each sector’s clean technology uses the output of the immediately upstream sector, so the supply chain must greenify together. Real production networks depart from this template in two ways. First, clean and dirty versions of the same sector typically draw on different sets of inputs: electric vehicles use batteries and electronics, while internal-combustion vehicles rely on refined petroleum. Second, a sector’s output generally flows to many downstream uses rather than to a single layer above. This section extends the model to accommodate both features while preserving the tractability of the vertical case.

Two sources of apparent complication can be handled by a straightforward reduction. The dirty-only part of the economy—sectors that neither greenify nor supply greenifiable sectors—can be collapsed into a composite dirty input without loss of generality, as Proposition 8 shows. Similarly, sector-level TFP heterogeneity can be absorbed into a transformation of the productivity vector $\mathbf{z}$. The one qualitatively new feature of the general model is the possibility that clean and dirty technologies within the same sector rely on different upstream inputs. This asymmetry, captured by two separate input-share matrices Σ^c and Σ^d, is the source of the one qualitatively new phenomenon of the general model: industrial policy that subsidizes the wrong upstream sector can push the economy away from the green steady state.

4.1 Setup

We extend the vertical chain in three dimensions. First, each sector’s clean and dirty recipes can draw on different input bundles. In the vertical chain, clean production used one specific upstream sector and dirty production used only labor; in the general model, both technologies combine labor with a bundle of intermediates from the rest of the economy, with the clean bundle and the dirty bundle allowed to differ. Second, the input network is no longer a single chain but an arbitrary acyclic graph: a sector can supply many downstream users, and a downstream sector

[31] Input shares also play a role in the sectoral heterogeneity of emission reductions. Low input shares downstream proportionally dampen emission reductions independent of the level of greenification elsewhere, but low input shares upstream—as well as in one’s own sector—enhance emission reductions by reducing the offsetting effect. Therefore, an economy characterized by low input shares throughout the supply chain favors downstream greenification as these low input shares penalize upstream sectors by reducing their market size but reward downstream sectors by reducing their total demand for dirty inputs.

can draw from many suppliers. Third, we allow sector-level heterogeneity in productivity (A_i), in the relative productivity of the clean over the dirty technology (z_i), in the carbon intensity of dirty production (e_i), and in the carbon tax rate (τ_i).

Formally, we consider a finite set of sectors indexed by $i \in \{1, \ldots, N\}$. Each good is produced with a unit mass of varieties $\nu \in [0,1]$ as in (2). As in the vertical chain model, each variety can always be produced with a dirty technology. If it is greenified, it can additionally be produced with a clean technology, which is a perfect substitute. Formally, a variety ν in sector i is produced according to:

$$y_{it}(\nu) = A_i \left[\left(\frac{\ell^d_{it}(\nu)}{\alpha^d_i}\right)^{\alpha^d_i} \prod_{j=1}^{N} \left(\frac{m^d_{ijt}(\nu)}{\sigma^d_{ij}}\right)^{\sigma^d_{ij}} + \gamma_{it}(\nu) \left(\frac{e^{z_i}\ell^c_{it}(\nu)}{\alpha^c_i}\right)^{\alpha^c_i} \prod_{j=1}^{N} \left(\frac{m^c_{ijt}(\nu)}{\sigma^c_{ij}}\right)^{\sigma^c_{ij}} \right]. \quad (24)$$

This equation generalizes (3). As before $\gamma_{it}(\nu) \in \{0,1\}$ indicates whether a variety has been greenified or not. The labor share parameters satisfy $\alpha^d_i, \alpha^c_i \in (0,1]$ and the input share parameters $\sigma^d_{ij}, \sigma^c_{ij} \in [0,1)$, and we impose constant returns to scale within each technology: $\alpha^d_i + \sum_{j=1}^N \sigma^d_{ij} = 1$ and $\alpha^c_i + \sum_{j=1}^N \sigma^c_{ij} = 1$.[32] We impose that the input network is acyclic: there exists an ordering of sectors such that whenever sector i uses sector j as an input in either technology, one has $j < i$ (i.e. $\sigma^d_{ij} = \sigma^c_{ij} = 0$ if $i \leq j$) and $\alpha^d_1 = \alpha^c_1 = 1$. Under that ordering, all input-share matrices are strictly lower triangular.[33]

We partition the network in two sets: Ω^d is a "dirty-only" subnetwork, it includes sectors that cannot be greenified either directly or indirectly, while Ω^c is the set of sectors that may be greenified or whose production involves at some step a greenifiable input. That is, if $i \in \Omega^d$, then $\gamma_{it}(\nu) = 0$ and $\sigma^d_{ij} = \sigma^c_{ij} = 0$ for all $j \in \Omega^c$; and if $i \in \Omega^c$ and $j \in \Omega^d$, then $\sigma^c_{ij} = 0$.

Greenification in greenifiable sectors is modeled as in Section 3.1: non-greenified varieties draw a one-time fixed cost from cdf F_i, and a producer that greenifies obtains one period of monopoly power before the clean technology becomes competitively available. The consumer problem remains identical to the baseline model, except that c_t is now a Cobb-Douglas aggregate of sector i consumption with shares b_i.

We use bold letters to denote (column) vectors: $\boldsymbol{e} \equiv (e_i)_{i=1}^N$ is the vector of emission rates (per unit of dirty labor), $\boldsymbol{\tau} \equiv (\tau_i)_{i=1}^N$ the vector of tax rates, $\boldsymbol{\chi_t} \equiv (\chi_{it})_{i=1}^N$ the vector of greenification shares. We denote by $\Sigma^d = (\sigma^d_{ij})_{i,j}$ and $\Sigma^c = (\sigma^c_{ij})_{i,j}$ the input-output matrices. Whenever we

[32]To alleviate notation we use the convention that $(m/\sigma)^\sigma = 1$ if $\sigma = 0$ in (24).

[33]This assumption plays a role in preserving equilibrium uniqueness. Liu and Tsyvinski (2024) show that the US input-output matrix is largely acyclical.

write expressions such as $\ln(\mathbf{1}+\boldsymbol{\tau})$, the log is component-wise and we use $x \odot y$ to denote element-wise multiplication.

In Appendix C.1, we solve for the equilibrium and show that it is still unique, for the same reason as before: upstream greenification only affects profits with a one period delay, ensuring that the equilibrium can be solved bottom-up.

Proposition 7. *Given initial condition $\boldsymbol{\chi_0}$, the economy with carbon taxes $\boldsymbol{\tau}$ features a unique equilibrium, in particular $\{\boldsymbol{\chi_t}\}_{t>0}$ is unique.*

The next proposition shows that two parts of this general environment can be abstracted from without loss of generality: the dirty-only subnetwork Ω^d can be summarized by an effective dirty input, and the sectoral productivity levels A_i can be normalized away by a change of units (proof in Appendix C.2).

Proposition 8. *This general network is isomorphic to a network where Ω^d sectors are eliminated and TFP parameters are normalized to 1. The isomorphic network relies on a transformation of the productivity shifters $\boldsymbol{z}$, emission rates $\boldsymbol{e}$, and tax rates $\boldsymbol{\tau}$. The transformation preserves prices, markups, revenues, profits, greenification incentives, and welfare (up to a constant).*

In other words, abstracting from a fully dirty supply chain as we did in the vertical model of Section 3 had no consequences. Intuitively, since sectors in Ω^d never greenify and only use dirty inputs from one another, their only impact for a sector in Ω^c is through the cost of a composite dirty input, which is why Ω^d can be collapsed. Likewise, the A_i's multiply dirty and clean production in the same way within a sector and therefore have no effect on greenification within that sector. Their effect on greenification in other sectors can then be captured through an adjustment of the vector $\mathbf{z}$. From now on, we work without loss of generality with the reduced network that contains only Ω^c sectors, and we normalize $A_i = 1$.

4.2 Equilibrium and Steady-State

As before, a first step to solve the equilibrium is to characterize when the green technology is used if it is available. From the production function (24), we directly get that the log cost ratio between the dirty and the clean technology in sector i is given by:

$$\ln\left(\frac{mc_{it}^d}{mc_{it}^c}\right) = \alpha_i^d \ln(1+\tau_i) + \alpha_i^c z_i + \sum_{j<i} (\sigma_{ij}^d - \sigma_{ij}^c) \ln p_{jt}. \tag{25}$$

The clean technology is used if that log cost ratio is weakly positive,[34] in which case the log cost ratio is also the log mark-up charged by innovators. To derive sufficient conditions for $\ln\left(\frac{mc^d_{it}}{mc^c_{it}}\right) \geq 0$, start with an economy with no greenification ($\boldsymbol{\chi}_t = \mathbf{0}$), and denote by $\mathbf{p}^d$ the price vector. Then, greenification weakly reduces marginal costs in all sectors if

$$\boldsymbol{\vartheta} \equiv \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) + \boldsymbol{\alpha}^c \odot \mathbf{z} + \left(\Sigma^d - \Sigma^c\right)\ln(\mathbf{p}^d) \geq 0. \tag{26}$$

With prices in the fully dirty economy given by $\ln(\mathbf{p}^d) = \left(I - \Sigma^d\right)^{-1}\left(\boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau})\right)$, we obtain

$$\boldsymbol{\vartheta} = \boldsymbol{\alpha}^c \odot \mathbf{z} + \left(I + \left(\Sigma^d - \Sigma^c\right)\left(I - \Sigma^d\right)^{-1}\right)\left(\boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau})\right).$$

Assume further that $\sigma^c_{ij} \geq \sigma^d_{ij}$ (i.e. $\Sigma^c \geq \Sigma^d$), that is the clean production process relies more heavily on *greenifiable* (Ω^c) sectors than the dirty production process. Then, from (25), an upstream price reduction further increases the relative cost of the dirty over the clean technologies.

Lemma 1. *Assume that $\boldsymbol{\vartheta} \geq 0$ and $\Sigma^c \geq \Sigma^d$, then clean technologies are used whenever they are available.*

When carbon taxes are homogeneous across sectors, the condition $\boldsymbol{\vartheta} \geq 0$ reduces to $Z \geq 0$, the same cost-advantage condition we imposed in the baseline vertical chain model. Appendix C.3 presents a formal proof of this lemma, fully characterizes the equilibrium (including when these assumptions are violated) and derives the expressions below. Note that the conditions $\boldsymbol{\vartheta} \geq 0$ and $\Sigma^c \geq \Sigma^d$ are only sufficient and not necessary conditions: contrary to the vertical supply chain case, clean technologies in a certain sector may be competitive for a given technology vector $\boldsymbol{\chi}$ but not for another one.[35]

From here onward, we assume that clean technologies are used whenever they are available. We can then write prices as:

$$\ln(\mathbf{p}_t) = \ln(\mathbf{p}^d) - \left(I - \Sigma_{t-1}\right)^{-1}\left(\boldsymbol{\chi}_{t-1} \odot \boldsymbol{\vartheta}\right), \tag{27}$$

where $\Sigma_{t-1} \equiv \text{Diag}\left(\mathbf{1} - \boldsymbol{\chi}_{t-1}\right)\Sigma^d + \text{Diag}\left(\boldsymbol{\chi}_{t-1}\right)\Sigma^c$ is the average input output matrix between clean and dirty technologies. $\left(I - \Sigma_{t-1}\right)^{-1} = I + \Sigma_{t-1} + \Sigma^2_{t-1} + \ldots$ is a Leontief inverse matrix: it captures how prices in a sector depend on greenification in that sector and its direct and indirect suppliers. Equation (27) shows that past own and upstream greenification reduces own prices.

[34] To avoid burdening language, we assume that producers use the clean technology when they are indifferent between the two.

[35] In particular, green technologies downstream may only become competitive after enough upstream greenification. That being said, in sectors where sunk costs have been spent, we should expect that green technologies are competitive and used, otherwise there would have been no justification for such an investment.

The log markups charged by owners of newly greenified varieties are then given by

$$\ln(\boldsymbol{\theta}_t) = \left[I + \left(\Sigma^c - \Sigma^d\right)\left(I - \Sigma_{t-1}\right)^{-1} \mathrm{Diag}(\boldsymbol{\chi}_{t-1})\right] \boldsymbol{\vartheta}. \tag{28}$$

If other sectors are dirty, greenification brings a cost advantage ϑ_i in sector i. Then, past upstream greenification (weakly) increases this cost advantage, and therefore mark-ups, through an input cost effect exactly as in the vertical chain model—but this effect now depends on how much more clean technologies rely on greenifiable inputs relative to dirty technologies (captured by the matrix $\Sigma^c - \Sigma^d$).

We derive revenues in a similar way as in the vertical model. The share of sector i revenues spent on sector j is given by

$$\tilde{\Sigma}_{ij,t} = (1 - \chi_{it})\sigma^d_{ij} + \tilde{\chi}_{it}\sigma^c_{ij}, \tag{29}$$

where $\tilde{\chi}_{it} = \chi_{i,t-1} + (\chi_{it} - \chi_{i,t-1})/\theta_{it}$ as before. A share $1 - \chi_{it}$ of revenues of sector i accrues to dirty varieties, of which their producers spend a share σ^d_{ij} on inputs from sector j. In addition, taking into account that producers of newly greenified varieties make profits, a share $\tilde{\chi}_{it}$ of sector i revenues is spent on the costs of clean varieties, and a share σ^c_{ij} of those is spent on sector j. Denoting by $\tilde{\Sigma}_t$ the sectoral share matrix, and taking into account consumer demand, we get that revenues are:

$$\mathbf{r}_t = \left(I - \tilde{\Sigma}_t^{\top}\right)^{-1} \mathbf{b}. \tag{30}$$

Revenues still depend on direct and indirect downstream greenification as $\left(I - \tilde{\Sigma}_t^{\top}\right)^{-1} = I + \tilde{\Sigma}_t^{\top} + \left(\tilde{\Sigma}_t^{\top}\right)^2 + \ldots$ and $\tilde{\Sigma}_t^{\top}$ is strictly upper triangular. Past downstream greenification still increases revenues but contemporaneous downstream greenification need not do so: greenification now reduces potential demand from the dirty producers and its increase in demand from clean producers is moderated by the mark-up they charge (i.e., we may have $\frac{\sigma^c_{ij}}{\theta_{it}} < \sigma^d_{ij}$). This effect disappears in steady-state (as $\chi_{it} = \chi_{i,t-1}$ then).

Profits are then given by $\boldsymbol{\pi}_t = \left(\mathbf{1} - \boldsymbol{\theta}_t^{\odot(-1)}\right) \odot \mathbf{r}_t$. Firms greenify up to the point where the cost of greenification equals the benefits, leading to the same law of motion for χ_{it} as before (11). We can then characterize steady-states as follows.[36]

[36] As in Section 3.2, we focus on steady-states where the first equality in Proposition 9 holds strictly. Steady-states for which it holds as a strict inequality cannot be reached from any initial condition strictly below them without direct government intervention.

Proposition 9. *A steady-state is a vector $\boldsymbol{\chi}$ such that*

$$\boldsymbol{\chi} = \mathbf{F}(\boldsymbol{\pi}) \text{ with } \boldsymbol{\pi} = \left(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)}\right) \odot \mathbf{r}.$$

Assuming that $\boldsymbol{\vartheta} \geq 0$ and $\Sigma^c \geq \Sigma^d$, revenues are characterized by $\mathbf{r} = \left(I - \Sigma^\top\right)^{-1} \mathbf{b}$, markups by $\ln \boldsymbol{\theta} = \left[I + \left(\Sigma^c - \Sigma^d\right)(I - \Sigma)^{-1} \operatorname{Diag}(\boldsymbol{\chi})\right] \boldsymbol{\vartheta}$, with $\Sigma = \operatorname{Diag}(\mathbf{1} - \boldsymbol{\chi}) \Sigma^d + \operatorname{Diag}(\boldsymbol{\chi}) \Sigma^c$. The economy features strategic complementarity in the sense that $\frac{\partial \pi_i}{\partial \chi_j} \geq 0$, and generally features multiple steady-states.

Therefore, the intuitions developed in the vertical supply chain model extend straightforwardly to this case: steady-state profits depend positively on upstream greenification through an input cost channel and on downstream greenification through a demand channel. The important assumption that we made then and that we have maintained here is that $\Sigma^c \geq \Sigma^d$: the clean production technology relies more on *greenifiable* inputs than the dirty technology. In the absence of input-output tables that are specific to each technology, we cannot test this assumption directly. However, we note that greenification of key emissions sources frequently involves either fuel switching from fossil to greenifiable carriers (electricity, hydrogen, synthetic- or biofuels, see, e.g., Gailani et al. 2024), or, for process emissions, often requires additional inputs (e.g., feed additives to reduce enteric fermentation in ruminants, which can be made from chemical synthesis or algae). While our assumption is unlikely to hold for every sector and every input in reality, our empirical analysis in Section 2also suggests that there is overall strategic complementarity in greenification, so that this assumption likely holds on average.

The Planner's problem also generalizes. In Appendix C.6 we show that the Planner's first-order condition for an interior χ_i takes the form $\chi_i = F_i\left(\frac{r_i \ln \theta_i}{1-\beta}\right)$, a direct generalization of equation (17). All of the qualitative conclusions from Section 3.3 carry over: a Pigouvian carbon tax is generally insufficient, and a temporary vector of sector-specific subsidies can uniquely implement the social optimum steady-state.

4.3 Multipliers

We now extend the analysis of Section 3.4 and trace out the effect of marginal subsidies to greenification on an economy in a stable steady-state.[37] The key object is the matrix of elasticities of profits with respect to greenified shares $\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi}^\top}$. As before we can decompose that elasticity into the elasticity of revenues (which captures the effect of downstream sectors) and that of profit

[37] In Appendix C.5 we derive a necessary condition for the stability of a steady-state.

margins (which captures the effect of upstream sectors):

$$\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi}^{\top}} = \frac{\partial \ln \mathbf{r}}{\partial \ln \boldsymbol{\chi}^{\top}} + \frac{\partial \ln(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})}{\partial \ln \boldsymbol{\chi}^{\top}}.$$

The downstream and upstream propagation channels from the vertical model carry over to arbitrary networks. Downstream greenification raises upstream revenues through a demand channel whose strength depends on how much more the clean technology relies on greenifiable inputs than the dirty one does, captured by the matrix $\Sigma^c - \Sigma^d$. Upstream greenification raises downstream profit margins through a cost channel governed by the same difference. Both effects vanish when clean and dirty technologies share the same input bundle: if greenification does not change a sector's suppliers, it does not spread through the network. Formally (proof in Appendix C.4):

Proposition 10. *Assume that $\boldsymbol{\vartheta} > 0$ and $\Sigma^c \geq \Sigma^d$. In steady-state, downstream greenification affects revenues according to the weakly positive matrix*

$$\frac{\partial \ln \mathbf{r}}{\partial \ln \boldsymbol{\chi}^{\top}} = \mathrm{Diag}(\mathbf{r})^{-1} \left(I - \Sigma^{\top}\right)^{-1} (\Sigma^c - \Sigma^d)^{\top} \mathrm{Diag}(\boldsymbol{r} \odot \boldsymbol{\chi}), \tag{31}$$

while upstream greenification affects profit margins according to the weakly positive matrix

$$\frac{\partial \ln(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})}{\partial \ln \boldsymbol{\chi}^{\top}} = \mathrm{Diag}\left(\boldsymbol{\theta} - \mathbf{1}\right)^{-1} (\Sigma^c - \Sigma^d) \left(I - \Sigma\right)^{-1} \mathrm{Diag}(\ln \boldsymbol{\theta} \odot \boldsymbol{\chi}). \tag{32}$$

This proposition describes in which sectors greenification has a large impact on other sectors incentives to greenify.[38] A sector j has a large impact on upstream greenification when $\frac{\partial \ln \mathbf{r}}{\partial \ln \chi_j}$ is large. To understand equation (31), note that a 1% increase in χ_j moves an amount $\chi_j r_j$ of revenues from the dirty to the clean production process. This affects the revenues of immediately upstream sectors in function of how much they sell more to the clean versus the dirty technology, hence the matrix $(\Sigma^c - \Sigma^d)^{\top}$. That shock then propagates to the more upstream sectors in function of how these sectors are related on average to the direct suppliers of sector j, hence the Leontief inverse matrix $\left(I - \Sigma^{\top}\right)^{-1} = I + \Sigma^{\top} + \left(\Sigma^{\top}\right)^2 + ...$ To obtain the proportional change in revenues upstream, we need to divide that shock by the revenues of the affected sectors (hence the matrix $\mathrm{Diag}(\mathbf{r})^{-1}$). Intuitively the greenification of sectors whose production draws heavily on greenifiable inputs, directly or through the supply chain generates greater spillover but only insofar as the clean technologies of these sectors rely more on greenifiable inputs than their dirty counterpart.

A sector j has a large impact on downstream greenification when $\frac{\partial \ln(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})}{\partial \ln \chi_j}$ is large. To

[38]To trace out the full effect of subsidies, the formula (21) still applies.

understand equation (32), note that a 1% increase in χ_j decreases the steady-state log price of sector j by $-\chi_j \ln \theta_j$, as the log markup equals the log difference between dirty and clean marginal costs. This shock is transmitted downstream to all sectors which use sector j directly or indirectly, hence the Leontief inverse matrix $(I - \Sigma)^{-1} = I + \Sigma + \Sigma^2 + ...$ This affects the marginal costs of dirty versus clean technologies in downstream sectors in proportion to how much these two technologies differently rely on upstream sectors (hence the matrix $\Sigma^c - \Sigma^d$). A 1% increase in the log mark-up in downstream sectors affects their log profit share by $\frac{1/\theta}{1-1/\theta} = \frac{1}{\theta-1}$ – hence the first diagonal matrix. Intuitively, the greenification of sectors that are more central suppliers generates greater spillovers but only insofar as they are ultimately used disproportionately by clean rather than dirty technologies. In addition, the greenification of sectors with large markups has a bigger effect on greenification incentives than sectors with low markups.

Whether these formulas deliver the vertical-model prediction—that downstream targeting dominates—depends on the network. With arbitrary Σ^c and Σ^d, the relative strength of the demand channel and the cost channel can go either way: it hinges on which sectors are central as markets versus as suppliers, and on how much more intensively clean technologies use greenifiable inputs than dirty ones do. Yet, the vertical intuition generalizes in one empirically plausible case, however: when dirty technologies make little direct use of greenifiable inputs ($\Sigma^d \approx 0$). In this case, the demand channel and the cost channel behave asymmetrically in the same way as in the vertical chain, and the argument for prioritizing downstream subsidies carries over.

To see this, set $\Sigma^d = 0$ in equation (31) to get:

$$\frac{\partial \ln \mathbf{r}}{\partial \ln \boldsymbol{\chi}^\top}|_{\Sigma^d=0} = \text{Diag}(\mathbf{r})^{-1} \sum_{k=1}^{\infty} \left((\Sigma^c)^\top \, \text{Diag} \left(\boldsymbol{\chi} \right) \right)^k \text{Diag}(\boldsymbol{r}).$$

The (i, j) entry of the downstream-to-upstream elasticity matrix $\partial \ln \mathbf{r} / \partial \ln \boldsymbol{\chi}^\top$ reduces to $\sigma^c_{ji} \chi_j r_j / r_i$ at first order, plus weakly positive higher-order terms. The first-order term has a direct economic interpretation: the marginal effect of further greenification in sector j on sector i's revenues equals the share of sector i's revenue derived from selling to sector j in steady-state. For any sector that does not sell directly to consumers ($b_i = 0$), summing across all downstream sectors yields $\sum_{j>i} \sigma^c_{ji} \chi_j r_j / r_i = 1$: every dollar of sales must be accounted for by some downstream use, regardless of the greenification level.

By contrast, the upstream-to-downstream elasticity in (32) behaves very differently: when

$\Sigma^d = 0$, the equation becomes:

$$\frac{\partial \ln(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})}{\partial \ln \boldsymbol{\chi}^\top}|_{\Sigma^d=0} = \text{Diag}\left(\boldsymbol{\theta} - \mathbf{1}\right)^{-1} \sum_{k=1}^{\infty} \left(\Sigma^c \text{Diag}(\boldsymbol{\chi})\right)^k \text{Diag}(\ln \boldsymbol{\theta}).$$

The summed elasticity $\sum_{j<i} \partial \ln(1 - \theta_i^{-1})/\partial \ln \chi_j$ tends to zero whenever $\chi_k \to 0$ for any $k \in [j, i)$. A single weak link anywhere between the subsidized sector and the downstream sector breaks the transmission.

The asymmetry of the vertical model therefore re-emerges in the general network: when green supply chains are in their infancy, the total input-cost channel coming from upstream greenification can be arbitrarily small, while the direct market-size effect coming from downstream greenification is still equal to one. Targeting downstream sectors again generates more incentives for greenification. In Appendix C.7 we extend the second-best analysis of Section 3.5 to the general model and find an analogous result: downstream targeting is more effective at reducing emissions when greenification levels are low.

4.4 Can Industrial Policy Backfire?

A common criticism of industrial policy is that a well-meaning government can subsidize the wrong sector and do more harm than good (Pack and Saggi, 2006). Indeed, figuring out which sector to subsidize may be difficult: the optimal greenification levels are hard to pin down without detailed knowledge of the distribution of innovation costs, and the preceding analysis on innovation incentive multipliers only gives a local answer and requires the knowledge of technology specific input-output matrices. Our general model delivers a sharp answer: industrial policy cannot backfire when greenification across sectors is strategically complementary, but it can when greenification is strategically substitutable. The first half is reassuring. As long as clean technologies rely on greenifiable inputs at least as intensively as dirty technologies do ($\Sigma^c \geq \Sigma^d$), a misguided subsidy can at worst waste resources on excess greenification during the transition; it cannot move the economy to a less green long-run outcome. Moreover, steady-state welfare weakly increases in greenification, so the policy cannot permanently reduce the utility flow. The second half is a genuine concern in cases we make concrete below, where two upstream sectors compete as inputs to a single downstream sector's clean and dirty technologies. In that case, subsidizing the wrong upstream sector can permanently lock the economy into the wrong steady-state.

Formally, we show (proof in Appendix C.8):

Proposition 11. *Assume that $\boldsymbol{\vartheta} \geq 0$ and $\Sigma^c \geq \Sigma^d$. Fix an initial condition $\boldsymbol{\chi}_0$ and a vector of carbon taxes $\boldsymbol{\tau}$. Let $\boldsymbol{\chi}^A$ denote the steady-state without greenification subsidies, and let $\boldsymbol{\chi}^B$ denote the steady-state when the government uses a sequence of sector-specific greenification subsidies $\mathbf{q}_t \geq 0$ with $\mathbf{q}_t$ constant (potentially null) for t large enough. Then $\boldsymbol{\chi}^B \geq \boldsymbol{\chi}^A$ component-wise, and the utility flow in steady-state is weakly larger with than without the subsidies.*

Intuitively, without greenification subsidies, the economy converges toward the closest from above steady-state (i.e. the steady-state immediately above $\boldsymbol{\chi}_0$). Since greenification is irreversible and since there is strategic complementarity in steady-state ($\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi}^\top} \geq 0$), a (weakly positive) set of greenification subsidies cannot lead to a worse steady-state.[39]

However, without strategic complementarity (i.e., $\Sigma^c \geq \Sigma^d$ is violated) and off a steady-state, a misdirected green industrial policy can bring permanent welfare losses if it "picks the wrong winner". To show this, we consider a simple example with 3 greenifiable sectors: Sectors 1 and 2 are upstream sectors and are used as input respectively by the clean production process and the dirty production process of the downstream sector 3.[40]

Formally, we set $\Sigma^c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1-\alpha & 0 & 0 \end{pmatrix}$ and $\Sigma^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \end{pmatrix}$ and for simplicity we keep z, τ, and ξ, homogeneous with $Z = z + \ln(1+\tau) > 0$. To avoid a dichotomy of cases, we assume $\alpha > 1/2$, which ensures that the green technology is always used when available in the downstream sector (and that using the green technology downstream does indeed reduce emissions even when taking into account indirect emissions).[41] This also ensures that the equilibrium equations of Section 4.2, directly apply to this case. From (30) and (28), we get that revenues and

[39] This is true even though greenification in sector i may (temporarily) decrease with greenification in sector j.

[40] For instance, primary steel production (3) can use coke (2) or hydrogen (1) as reductants, both of which can, in turn, be greenified through coal mine methane capture and the development of alternative coke ovens, and through electrolysis, respectively.

[41] To see this, note that $p_{it} = (1+\tau)\, e^{-Z\chi_{i,t-1}}$ for $i \in \{1,2\}$, and using ((25)) that

$$\ln\left(\frac{mc_{3t}^d}{mc_{3t}^c}\right) = \alpha Z + (1-\alpha)\ln\frac{p_{2t}}{p_{1t}} = \left(\alpha + (1-\alpha)\left(\chi_{1,t-1} - \chi_{2,t-1}\right)\right) Z, \tag{33}$$

which is positive regardless of $\chi_{1,t-1}$ and $\chi_{2,t-1}$ when $\alpha > 1/2$.

mark-ups are given by

$$\begin{pmatrix} r_{1t} \\ r_{2t} \\ r_{3t} \end{pmatrix} = \begin{pmatrix} \widetilde{\chi}_{3,t}(1-\alpha) \\ (1-\chi_{3,t})(1-\alpha) \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} \ln\theta_{1t} \\ \ln\theta_{2t} \\ \ln\theta_{3t} \end{pmatrix} = \begin{pmatrix} Z \\ Z \\ (\alpha + (1-\alpha)(\chi_{1,t-1} - \chi_{2,t-1}))\, Z \end{pmatrix}. \tag{34}$$

Revenues in the upstream sector 1 increase with downstream greenification (in sector 3), while revenues in the upstream sector 2 decrease with downstream greenification (as sector 2 is used by the dirty process in sector 3). Mark-ups in the downstream sector 3 increase with upstream greenification in sector 1 but decrease with upstream greenification in sector 2 (which benefits dirty production in sector 3). The economy converges toward a steady-state which satisfies:[42]

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} F_1\left(\left(1-e^{-Z}\right)\chi_3(1-\alpha)\right) \\ F_2\left(\left(1-e^{-Z}\right)(1-\chi_3)(1-\alpha)\right) \\ F_3\left(1-e^{-(\alpha+(1-\alpha)(\chi_1-\chi_2))Z}\right) \end{pmatrix}.$$

Greenifications in sectors 1 and 3 are strategic complements (χ_1 increases in χ_3, and vice versa), but greenifications in sectors 2 and 3 are strategic substitutes (χ_2 decreases in χ_3, and vice versa). Greenifying sector 2 (the input sector into the dirty downstream process) diverts effort away from greenifying the downstream sector 3 and (indirectly) sector 1 (the input sector into the clean downstream process) as it reduces the downstream dirty marginal cost.

The strategic substitutability between sectors 2 and 3 has three important implications: First, an industrial policy that favors greenifying 2 may backfire and halt the greenification that would have otherwise happened in 3 and 1 without government intervention—thereby reducing long-term welfare compared to no intervention (see Appendix C.9). Second, the laissez-faire equilibrium may involve too much greenification of 2 compared to the ex-ante optimum. But, given the history dependence in the dynamics of greenification shares over time, delaying greenification in 3 and 1 in turn can lead to irreversible long-term consequences (see Appendix C.10). Third, we note that strategic substitutability provides another argument for prioritizing greenification in the downstream sector: An increase in greenification in the downstream sector, incentivizes greenification in the "right" upstream sector (1). In contrast, and as just argued, a policy that targets the wrong upstream sector (2) discourages greenification downstream.[43]

[42]We ignore the possibility that the economy overshoots in which case the $=$ sign should be replaced with $\geq$.

[43]This example provides a cautionary word against innovations that make dirty production cleaner (here by making inputs cleaner) instead of switching away from it altogether – a distinct argument against innovation in intermediate technologies such as natural gas (instead of coal) is made by Acemoglu et al. (2023).

5 Numerical Example

We have shown theoretically that coordination issues along the supply chain can generate low greenification traps impeding the green transition. But, is this a real concern in practice? To answer this question, we present a quantitative illustration of our model to long-range and heavy-duty (LRHD) transportation, namely trucking, aviation, shipping, and rail, which account for around half of global transport-related CO_2 emissions. We consider "greenification" via a switch to hydrogen (H_2, either in fuel cells or direct combustion), a key technology to help decarbonize LRHD transport (IPCC 2022).[44] Indeed, hydrogen-based models are emerging or under development for trucks (e.g. Hyundai XCIENT, Nikola), trains (e.g. Alstom Coradia iLint, Siemens Mireo Plus H), and aircraft (e.g. Airbus ZEROe, Embraer Energia), though their market share remains close to zero. Importantly, hydrogen can itself be produced using either fossil fuels or (green) electricity. Currently, less than 1% of the world's hydrogen production is green (IEA 2024). Consequently, the greenification of the full value chain is precisely what is required in this context.

We calibrate the model to the US economy with a 2-sector vertical structure: gaseous hydrogen (GH_2) production and distribution as upstream sector 1, and LRHD transportation as downstream sector 2. Hydrogen is only used in the clean production process downstream. We assume that a model period corresponds to 25 years. We allow for heterogeneity across sectors in relative input efficiency z_i, emission rate e_i, and TFP A_i (as in Section 4.1). We relax the normalization $\kappa = 1$ in (1) to make units more easily interpretable. Appendix D presents the quantitative model structure in further detail and describes our calibration of initial wages and labor supplies to US data. All dollar values are in constant $2022 unless noted otherwise.

5.1 Calibration: LRHD Transportation

Our downstream sector differentiates eight varieties: trucks–class 7-8, trucks–class 4-6, air–narrow-body/short-medium range (NB/SMR), air–wide-body/long-range (WB/LR), rail–passenger, rail–freight, water–passenger, and water–freight. Among these varieties, the majority of both value-added and carbon emissions are due to trucking (71% and 62%, respectively) and aviation

[44] In contrast to light duty vehicles such as passenger cars, battery electrification is often less viable for LRHD applications due to the weight of the batteries and the high charging loads that would be required (e.g. Gross 2020; Tiwari et al. 2024). Even for rail, where electrification is advanced in some parts of the world (though not the US, where diesel fuels still account for over 95% of railway energy consumption, Davis and Boundy 2020), the high capital cost of catenary systems can favor hydrogen as a way of decarbonizing diesel locomotive areas already under current conditions (Ahluwalia et al. 2021). Of course, in reality, there is also technology crossover in light- and heavy-duty applications, as there are both hydrogen-powered passenger vehicles and electric trucks on the market.

(21% and 24%, respectively).[45] Our approach relies on engineering and industry estimates to quantify production and cost parameters. We briefly summarize the calibration here and provide further details in Appendix D.

First, we quantify the cost share of delivered gaseous hydrogen as $1 - \alpha_2 = 0.18$ based on the value-added-share weighted average across 26 estimates of future levelized costs (LCOs) of H_2 LRHD transport.[46] Second, we use pairs of LCO estimates using fossil fuels or hydrogen to calculate the value of z_2 implied by the relevant marginal cost ratio in the model, yielding a weighted average of $z_2 = -2.58$ (across 21 estimates). Third, we set A_2 to match calibration year (2022) data on prices and effective taxes, and calculate the baseline emissions intensity based on aggregate emissions, output, and prices. Finally, we include three types of costs in the fixed costs $\phi_2(.)$: technology development costs (e.g. of hydrogen aircraft), certain infrastructure adjustment costs (e.g. of refueling stations to accommodate hydrogen storage and dispensation), and the initial *excess costs* of current vs. future hydrogen-fueled technologies due to anticipated learning-by-doing and other technological advancements.

Figure 3 showcases our estimates of fixed costs across varieties, normalized as the percent increase in per-unit costs relative to current fossil fuel-based costs (over the model period).[47] Encouragingly, the estimates appear consistent with the empirical observation that H_2 development so far has focused more on lower-cost varieties such as passenger rail, trucks, and narrow-body aircraft compared to higher-cost varieties such as wide-body aircraft.

5.2 Calibration: Hydrogen Production and Distribution

Our upstream sector encompasses the production of (gaseous) hydrogen and its distribution across six US regions.[48] For H_2 production, we consider electrolysis from dedicated renewables[49] as "green" and steam-methane reforming with natural gas as the polluting benchmark technol-

[45]Based on 2022 data from the Bureau of Transportation Statistics (BTS 2024) and Environmental Protection Agency (EPA 2024).

[46]Given that we define Sector 1 output as *delivered* GH_2—rather than *dispensed* H_2—we adjust any LCO estimates that use a "dispensed" H_2 cost measure accordingly.

[47]Our calibration focuses on the US economy. In reality, the market for new technologies like hydrogen aircraft may include other countries. We thus adjust the fixed LRHD technology development costs downward (by 30%) to reflect that US revenues need not cover the full fixed costs of technology development. Our results are robust to relaxing this assumption as described in Appendix D.

[48]The motivation for modeling hydrogen production and distribution as having some substitutability is that hydrogen can be produced locally (such as near refueling stations) with lower efficiency, or produced at scale and in preferable locations (with, e.g., higher solar potential) but then requiring more distribution.

[49]While technically possible, grid-based electrolysis is generally estimated to be more expensive than with dedicated (or "islanded") renewable electricity (European Hydrogen Observatory 2023).

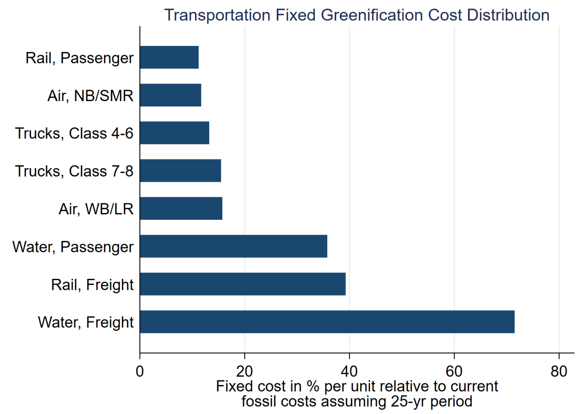


Figure 3. Distribution of fixed greenification costs in the downstream sector

ogy. While green H_2 may become cost-competitive in the future due to learning-by-doing and technological advancements (e.g. BloombergNEF 2023), at present it is associated with substantially higher production costs. We again consider these *excess initial costs* as a non-recurring cost that must be incurred for greenification. For distribution, we consider tube-trailer trucks as the polluting benchmark and hydrogen pipelines as the "green" option that can be accessed after incurring a fixed cost (of legal and technological development and construction).[50]

For the production function parameters, $\alpha_1 = 1$ is given and we set $z_1 = 1.608$ based on a weighted average across paired estimates of clean and dirty upstream costs from BloombergNEF and the Argonne National Laboratory's Hydrogen Delivery Scenario Analysis Model (HDSAM v4.5). Productivity A_1 matches 2022 data on prices, wages, and effective taxes. The emissions rate is based on IEA (2023) and the HDSAM. Finally, fixed costs $\phi_1(.)$ for hydrogen production capture *excess initial costs* (from BloombergNEF) and we estimate fixed costs for hydrogen pipelines in each region with the HDSAM (Brown et al. 2022). Appendix D provides further details, including how we re-scale fixed costs to account for demand outside our model, and presents the resulting distribution of fixed costs across upstream varieties.

5.3 Results

This section highlights three main results. First, steady-state multiplicity is empirically relevant. Figure 4 panel (a) presents an empirical counterpart similar to Figure 2b for a uniform carbon tax

[50]Pipelines are estimated to reduce H_2 distribution emissions by 85-95% (Demir et al. 2018; diLullo et al. 2022; Frank et al. 2021). We disregard the possibility of liquefied H_2 distribution via truck due to its limited scalability (e.g. Steer 2023). For aviation, we include liquefaction in the downstream costs of switching to and operating H_2 aircraft.

corresponding to the current effective average rate in the US (\$13/tCO_2, OECD 2023). Already at this low tax, there are multiple steady-states, including the current stable steady-state of ($\chi_{1,0} = 0, \chi_{2,0} = 0$), an unstable intermediate steady-state, and a high steady-state with greenification rates around 90% in both sectors.

Second, it is difficult to escape the low initial greenification steady-state using a carbon price alone. We consider the introduction of a carbon price of \$300/tCO_2, which is between recent estimates of the social cost of carbon from the US EPA (\$213/tCO_2 in \$2022,[51] EPA 2022) and those of some European countries (e.g. €300 ~ \$340/tCO_2 in Germany, Matthey et al. 2024; CHF430 ~ \$526/tCO_2 in Switzerland). Figure 4 panel (b) shows that even a US carbon price of \$300/tCO_2 is not sufficient to eliminate (0,0) as a steady-state. The welfare stakes of this multiplicity are large: Assuming an annual utility discount factor of $\beta_{yr} = 0.97$, we estimate a welfare difference between the low (0,0) and the stable high steady-state (0.89, 0.99) of around \$8.4 trillion (initial period consumption equivalent variation).[52] Escaping this multiplicity with carbon prices alone would require a tax of around \$440/tCO_2 to ensure a high greenification steady-state.

Third, we confirm that a temporary downstream subsidy may be sufficient to induce greenification and is preferable to an upstream subsidy. We specifically model the economy's transition path from the current (0,0) steady-state assuming that a temporary greenification subsidy is provided either downstream or upstream in addition to a Pigouvian carbon price of \$300/tCO_2. Figure 4 panel (c) shows that a two-period 50% downstream greenification subsidy succeeds in triggering greenification in both sectors, but that this is not the case for a comparable upstream subsidy.[53] In terms of welfare, the stakes are large: Over the next 50 years alone, adding the downstream greenification subsidy decreases US emissions by almost 16 billion metric tons of CO_2 (undiscounted total)—equivalent to about six months of current global energy-related CO_2 emissions. The welfare gain associated with the downstream (vs. upstream) subsidy is estimated to be \$3.2 trillion. Finally, we also find that the downstream subsidy + carbon price policy package achieves welfare gains of around +\$324 billion compared to a "high carbon price only" policy that seeks to induce full greenification without a clean technology subsidy (with a \$440/tCO_2 tax).

[51] We convert the benchmark \$190/tCO_2 in \$2020 figure from EPA (2022) into \$2022 based on the relevant US GDP deflator values from the St. Louis Fed's FRED database.

[52] We calculate the counterfactual steady-state welfare assuming that the government provides sufficient subsidies to reach the high steady-state greenification shares in the first period, so that the economy is in the steady-state from the second period as in the proof of Proposition 4. Both steady-states are modeled with a \$300/tCO_2 carbon price.

[53] Note that i) the 50% downstream subsidy is sub-optimal, and ii) a uniform subsidy that solely corrects for innovators' myopia is insufficient to induce even moderate levels of greenification.

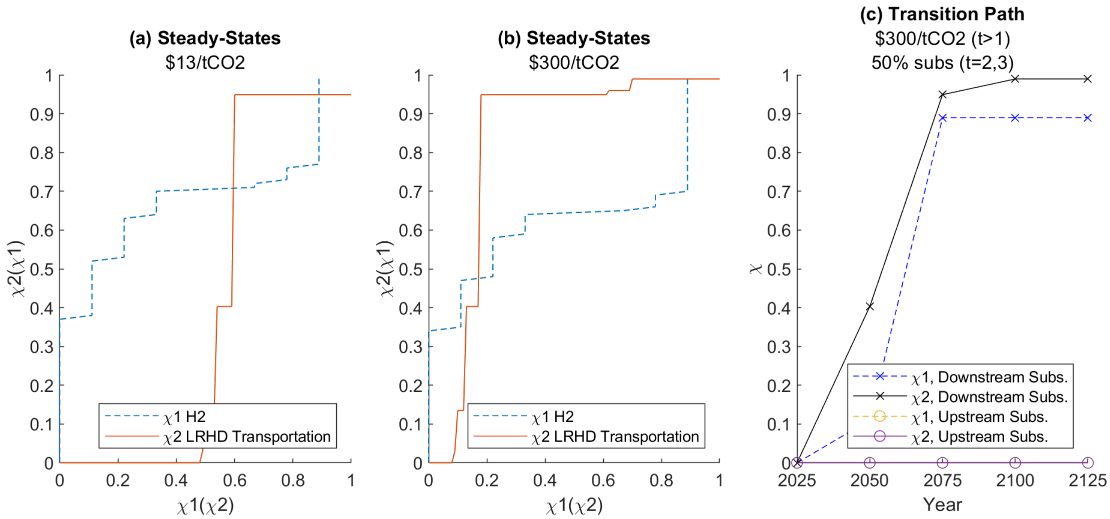


Figure 4. Panels (a) and (b) show steady-state greenification shares consistent with the current (\$13/tCO_2, (a)) and Pigouvian (\$300/tCO_2, (b)) carbon price, respectively. Panel (c) shows transition dynamics with a Pigouvian carbon price and temporary 50% greenification subsidy applied downstream (x-marker) or upstream (circle marker).

6 Conclusion

In this paper we analyzed a model of green technological transition along a supply chain. We then provided a quantitative application of our model to decarbonization of long-range and heavy-duty transportation via hydrogen, and showed that even with a uniform carbon price set equal to the social cost of carbon, the US economy could remain stuck in the "wrong" steady-state with CO_2 emissions far above the social optimum, whereas adding a temporary downstream greenification subsidy can induce high levels of greenification and yield large welfare gains.

Our analysis could be extended in several directions. A first extension would be to measure technology-specific input-output tables and calibrate our general network model in order to trace out the green transition of the overall economy across sectors. A second extension would be to look at coordination of greenification incentives not only across sectors and layers within a country but also along international value chains. A third extension would be to explore the extent to which allowing for vertical integration between different layers in the production chain affects our main conclusions. A fourth extension would be to use our framework to compute the overall elasticities of substitution between clean and dirty inputs to produce final goods once the supply chains involved in these technologies are fully taken into account. We know from previous work (e.g. Acemoglu et al. 2012, 2023; Donald 2023) that these elasticities play a major role in the design of optimal policies, yet rigorous methodologies to compute them remain to be found. Finally, our framework could also be used to explore quantitatively other technological transitions. These and other extensions are left for future research.

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Online Appendix

Transition to Green Technology along the Supply Chain

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Contents

A Reduced Form Evidence for the Role of the Supply Chain

This appendix describes the details and robustness checks of the empirical exercises in Section 2. Our EPA data attributes emissions to sectors using the FLOWSA model, and we clean this data in two steps. First, because the FLOWSA attribution system is model-based, rather than direct measurement, some sectors see implausibly high swings in emissions across years. To address this, we remove any sector that has a yearly change in emissions level of greater than 50%. Such sectors make up 12.3% of total emissions over this period.[A.1] After mapping from the EPA sectors (in NAICS) to BLS sectors, we are left with 70 sectors. In Figure A.4, we take an alternative approach, where instead of excluding EPA sectors with jumps, we winsorize the change in log emissions intensity at the BLS sector level to the 90th percentile. The results are largely the same.

Our sector-level emissions data are available from 2012 to 2022, but the BLS input-output data extend back to 1997. Therefore, all of our empirical exercises consider five year bins from at most 1997 to 2022. Our patent data has coverage over this entire time period. When we make use of cube root emissions weighting, we use emissions from the first year of a period, and every emissions variable uses CO2 equivalence, calculated using EPA equivalencies for other greenhouse gases.

The input share distance metrics used for Figures 1, A.1, A.2, A.3, and A.4, are computed as

$$TV(n)_{it} = \left(\frac{1}{2}\sum_j |\sigma_{ijt} - \sigma_{ijt-1}|^n\right)^{\frac{1}{n}},$$

where σ_{ijt} is the share of revenue sector i spends on sector j at time t. This distance metric always lies between zero and the sum of the expenditure shares and gives a measure of the degree to which a sector changed its input shares. As we describe in the main text, we consider $n = 1$ as our baseline and $n = 2$ for robustness.

For Tables 1, A.2, A.3, A.4, and A.5 we compute the upstream and downstream level of emis-

[A.1] Because BLS sectors are generally more aggregated than EPA sectors, removing an EPA sector does not necessarily remove the associated BLS sector. Our approach implicitly assumes that after removing the EPA sectors with implausible emissions changes, the value-added-weighted emissions intensity reduction of the remaining sectors provides an accurate estimate of BLS-sector-level change.

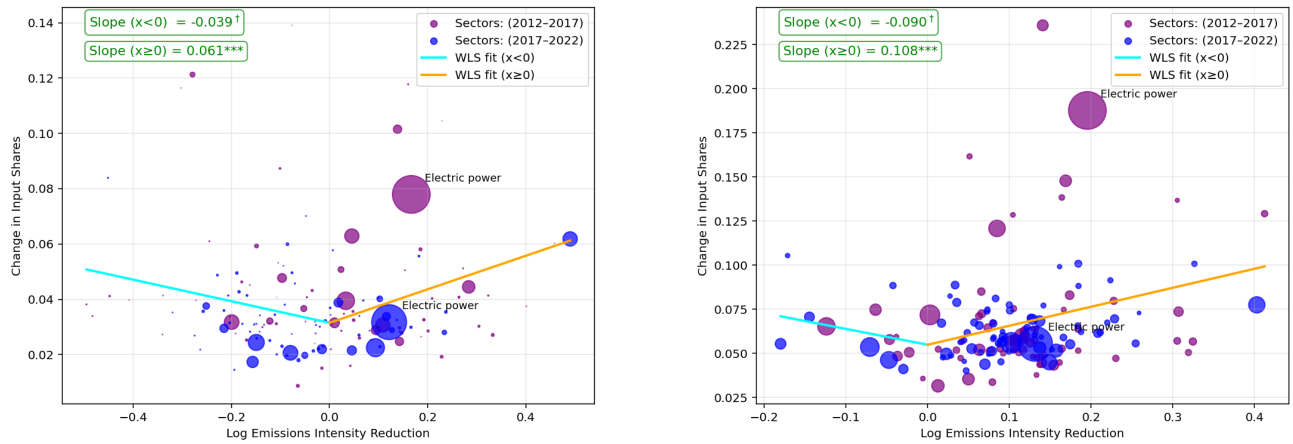
Table A.1. Summary Statistics of Regression Variables

	Mean	Standard Deviation	Interquartile Range	Obs
Log Emissions Intensity Reduction	-0.032	0.146	0.246	144
Network Log Emissions Intensity Reduction	0.269	0.135	0.202	144
Green Patent Share	0.059	0.067	0.069	273
Network Green Patent Share	0.298	0.166	0.190	273
Green Citation Share	0.062	0.076	0.067	273
Network Green Citation Share	0.306	0.180	0.221	273

Table A.2. Directional Supply Chain Incentives for Greenification

	Dependent Variable:						
	Log Emissions Intensity Reduction			Green Patent Share		Green Citation Share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Upstream Log Emissions Intensity Reduction	-0.022			0.473***		0.530***	
	(0.298)			(0.108)		(0.124)	
Downstream Log Emissions Intensity Reduction	0.240*			0.033		-0.008	
	(0.127)			(0.063)		(0.068)	
Upstream Green Patents		-0.327			0.539***		
		(0.346)			(0.178)		
Downstream Green Patents		0.242**			0.084**		
		(0.103)			(0.040)		
Upstream Green Citations			-0.149				0.517**
			(0.332)				(0.218)
Downstream Green Citations			0.219**				0.061†
			(0.098)				(0.040)
R^2	0.071	0.082	0.077	0.095	0.135	0.082	0.094
Obs	144	144	144	75	273	75	273

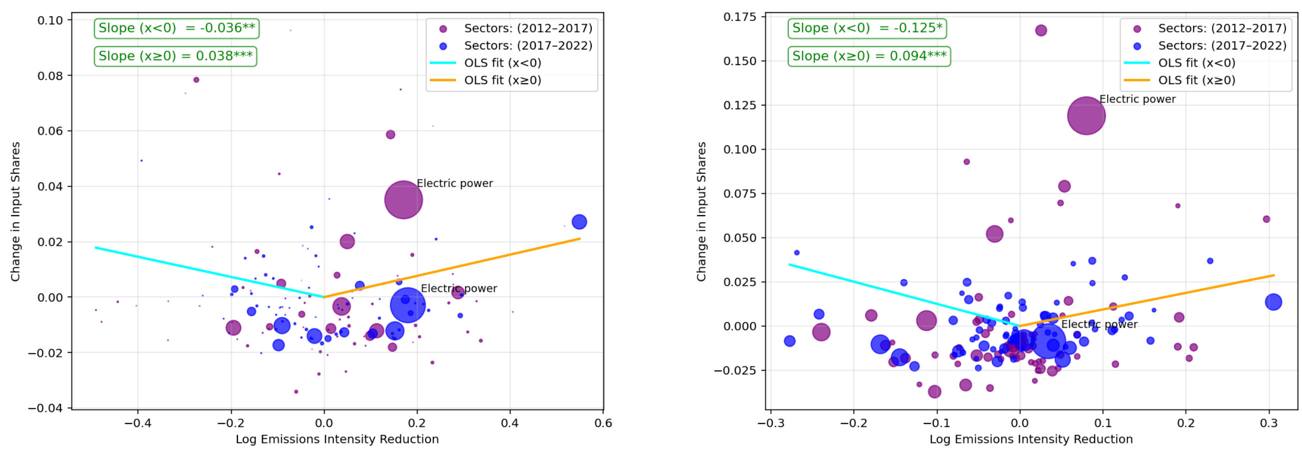
Notes: Standard errors clustered by sector are reported in parentheses. Columns 4-7 only consider sectors with at least one green patent in a period. All variables are winsorized to the 90th percentile, and all specifications include time fixed effects. † $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.



(a) IO Table (WLS)

(b) Leontief Inverse (WLS)

Figure A.1. Reductions in CO2e emissions are associated with changes in the supply chain for weighted sectors

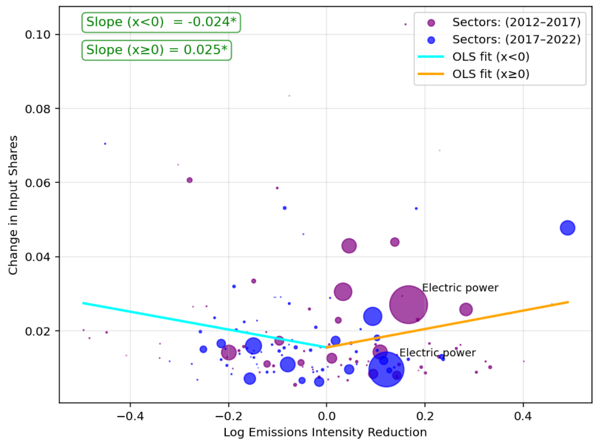


(a) IO Table (Time Fixed Effects)

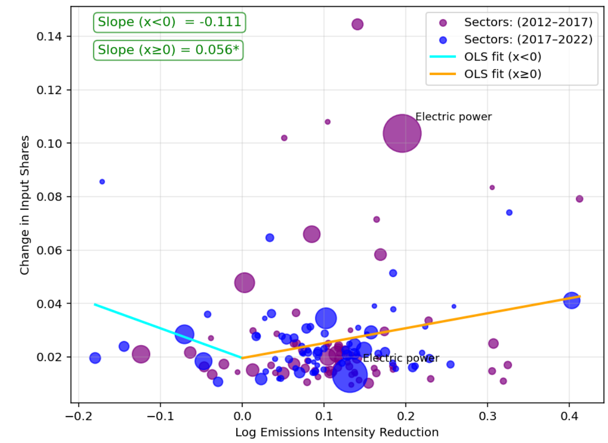
(b) Leontief Inverse (Time Fixed Effects)

Figure A.2. Reductions in CO2e emissions are associated with changes in the supply chain within time periods

sions reduction or green patenting as $\left((I-\Sigma)^{-1}-I\right)\boldsymbol{z}$ and $\left((I-\Sigma)^{-1}-I\right)^{\top}\boldsymbol{z}$, respectively, where Σ is the production network at the end of a period (only including the non-service sectors

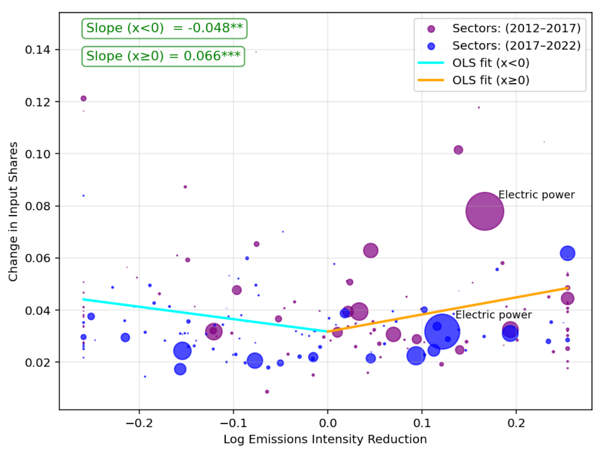


(a) IO Table

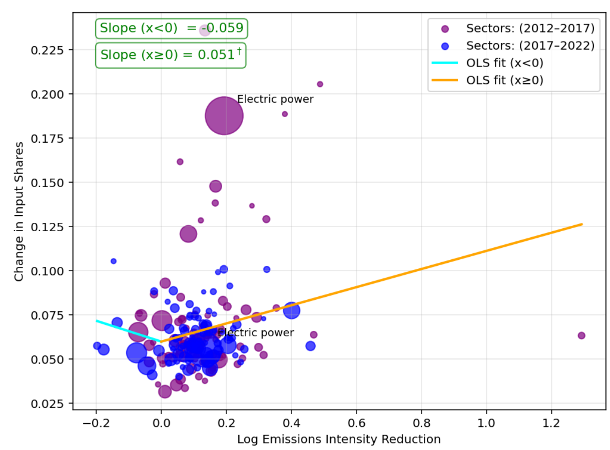


(b) Leontief Inverse

Figure A.3. Reductions in CO2e emissions are associated with relatively large changes in the supply chain



(a) Winsorized IO Table



(b) Winsorized Leontief Inverse

Figure A.4. Reductions in CO2e emissions are associated with changes in the supply chain for the complete sample

described in the main text, removing the diagonal, and renormalizing to sum to one minus the value-added and import share) and z is the sector-level outcome in question. This allows for the

Table A.3. Lagged Supply Chain Incentives for Greenification

	Dependent Variable:						
	Log Emissions Intensity Reduction			Green Patent Share		Green Citation Share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Network Log Emissions Intensity Reduction (lag)	0.008			0.057		0.057	
	(0.094)			(0.064)		(0.063)	
Network Green Patents (lag)		$0.101^{\dagger}$			0.097^{**}		
		(0.065)			(0.041)		
Network Green Citations (lag)			$0.091^{\dagger}$				0.078^{*}
			(0.057)				(0.042)
R^2	0.000	0.058	0.059	0.015	0.061	0.017	0.037
Obs	72	144	144	34	227	34	227

Notes: Standard errors clustered by sector are reported in parentheses. Columns 4-7 only consider sectors with at least one green patent in a period. All variables are winsorized to the 90th percentile, and all specifications include time fixed effects. † $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

inclusion of higher-order network effects in addition to immediate neighbors. Tables 1, A.3, A.4, and A.5 use the sum of upstream and downstream, while Table A.2 considers each individually. When z is reductions in log emissions intensity, we only consider upstream and downstream reductions (setting increases in emissions intensity to zero) because we are explicitly testing for the network effects of increases in greenification. For green patenting, we take the standard practice of considering both patent counts and patent citations. For citations, we follow Arora et al. (2023) and normalize by application year and 3-digit CPC average.[A.2] We also add one citation to each patent so as to not exclude patents with zero citations. Green patenting is irrelevant in some sectors and we exclude sectors with 0 green patents from our green patenting share regressions. For our weighted specifications, regressions with change in log emissions intensity as the outcome again use the cube root of emission levels, while those with green patenting (citations) shares as the outcome use the cube root of total green patents (citations).

Table A.4 considers the weighted relationship, finding similar but slightly weaker effects. Tables A.2 and A.3 show that the network effects still hold when splitting the total network effect into downstream and upstream effects, or considering lags (though insignificantly for emissions reduction). In our specification with split downstream and upstream effects, we typically find downstream effects for emission reductions and upstream effects for patenting, but we interpret this as stemming from noise in a relatively small sample, rather than a genuine asymmetry.Our lagged results are consistent with our model's implication that there is strategic complementarity

[A.2]Patents with multiple CPC codes are normalized by the average of averages across their CPC codes.

Table A.4. Supply Chain Incentives for Greenification (WLS)

	Dependent Variable: Emissions Reduction			Green Patent Share		Green Citation Share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Network Log Emissions Intensity Reduction	0.210			0.084		0.056	
	(0.163)			(0.065)		(0.064)	
Network Green Patents		$0.212^{\dagger}$			0.108^{**}		
		(0.142)			(0.045)		
Network Green Citations			$0.202^{\dagger}$				0.085^{**}
			(0.135)				(0.042)
R^2	0.073	0.077	0.078	0.027	0.096	0.010	0.057
Obs	144	144	144	75	273	75	273

Notes: Standard errors clustered by sector are reported in parentheses. Columns 1-3 are weighted by the cube root of emissions, and columns 4-5 and 6-7 are weighted by the cube root of clean patent counts and citations, respectively. All variables are winsorized to the 90th percentile, and all specifications include time fixed effects. † $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

for greenification with respect to both χ_t and χ_{t-1} as well as the fact that green patenting simply takes time to affect the technologies used in production.

Finally, all of our patenting regressions so far have made two exclusions (in addition to the EPA exclusions discussed above): eliminating patents related to internal combustion engine vehicles (Y02T10/10-40) and the aerospace product and parts manufacturing sector. The first exclusion is meant to eliminate "grey" patents that improve the efficiency of dirty technologies. Our model explicitly considers a switch to green production technologies, so we view the improvement of dirty technology as a distinct (and potentially counter-productive) phenomenon. The second exclusion comes from related reasoning: the aerospace manufacturing sector is a massive outlier in green patenting as we have defined it, but most of these patents pertain to general efficiency improvement for aircraft (e.g. Y02T50/60 efficient propulsion technologies). Again, we do not view our theory as pertaining to such improvement in fossil fuel technologies, so we exclude this outlier sector. Table A.5 considers our baseline specification with the full sample, not excluding any EPA sectors with jumps, patents from the aviation sector, or patents related to ICE vehicles. The results are largely unchanged, though the estimates for sector-level citations become somewhat attenuated due to the presence of grey patents.

Table A.5. Supply Chain Incentives for Greenification (No Exclusions)

	Dependent Variable:						
	Log Emissions Intensity Reduction			Green Patent Share		Green Citation Share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Network Log Emissions Intensity Reduction	0.054†			0.027		0.021	
	(0.034)			(0.023)		(0.026)	
Network Green Patents		0.193*			0.079**		
		(0.103)			(0.040)		
Network Green Citations			0.201**				0.054
			(0.095)				(0.041)
R^2	0.068	0.072	0.077	0.026	0.054	0.011	0.027
Obs	184	184	184	91	279	91	279

Notes: Standard errors clustered by sector are reported in parentheses. Columns 4-7 only consider sectors with at least one green patent in a period. All variables are winsorized to the 90th percentile, and all specifications include time fixed effects. † $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B Theory Derivations in the Supply Chain Model

B.1 Proof of Proposition 1

We proceed by induction. Suppose we know the sequence of greenification shares $\{\chi_{i,t-1}\}$ at time $t-1$. Then, we can compute the sequence $\{\mu_{it}\}$ recursively from upstream moving downstream using the equations $\mu_{1t} = 1$ and (6). Next, given $\chi_{N,t-1}$ and the μ's at time $t-1$, one can compute χ_{Nt} using that $r_{Nt} = 1$ as:

$$\chi_{Nt} = \max\left\{\chi_{N,t-1}, F_N\left(\left(1 - e^{-\mu_{N,t-1}Z}\right)\right)\right\}.$$

For given downstream greenification shares χ_{jt} for $j \in \{i+1, N\}$, we can compute the upstream equilibrium greenification share χ_{it} using (8) and (12). By induction, the equilibrium sequence of shares at date t, $\{\chi_{it}\}_{i=1}^{N}$, is uniquely determined. $\square$

B.2 Proof of Proposition 2

When $N = 1$, the steady-state equilibrium greenification share χ_1 satisfies the equation $\chi_1 = F_1\left(1 - e^{-Z}\right)$, which has a unique solution.

Assume now that $N \geq 2$. As μ_i is increasing in upstream χ_j (strictly so unless one sector is not greenified at all) and as all downstream χ_j appear in a multiplicative way in the argument of F, we directly get that the right-hand side of (15) is increasing in χ_j for $j \neq i$ and strictly so unless one of the χ_k is null.

To establish that there are multiple steady-states, we simply need to build an example. We do so by assuming that the distribution of greenification costs is a mass point at some value ϕ. We further assume that all the α_i 's are equal to the same α (except for $i = 1$). We derive conditions under which there is a steady-state where all sectors fully greenify and another one where no sector greenifies.

Consider first the case where no firm greenifies in any sector. In this case, the profit from greenification in all sectors $i < N$ is zero due to zero demand. The rent from greenifying in the most downstream sector N is $\pi_N = 1 - e^{-\alpha Z}$. Thus there will be no greenification in sector N whenever $1 - e^{-\alpha Z} < \phi$.

Next, suppose that all firms greenify in all sectors. In this case, the profit from greenification in sector i is $\pi_i = (1-\alpha)^{N-i}\left[1 - e^{-Z}\right]$. For all firms to have an incentive to greenify, we need that $(1-\alpha)^{N-1}\left[1 - e^{-Z}\right] > \phi$.

Hence, both the full and no greenification steady-states exist if ϕ satisfies

$$1 - e^{-\alpha Z} < \phi < (1-\alpha)^{N-1}\left[1 - e^{-Z}\right].$$

This is possible as soon as there exist values of z, τ, and α such that

$$1 - e^{-\alpha Z} < (1-\alpha)^{N-1}\left[1 - e^{-Z}\right].$$

For instance, for small Z ($e^Z \approx 1 + Z$) this inequality boils down to $\alpha < (1-\alpha)^{N-1}$, which is satisfied for α sufficiently small. This completes the proof. $\square$

B.3 Example with Multiple Steady-States in the Cap-and-Trade Case

In this section, we show that in the presence of a cap-and-trade system with a cap $\bar{\ell}_d$, there exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$.

Revenues of the dirty production process get allocated to the payment of labor in these processes and emission permits. Given a price on emissions τ_t, and using that revenues of each sector are given by (9), we get that

$$(1+\tau_t)\,\ell_{dt} = \sum_{i=1}^{N} (1-\chi_{it}) \prod_{j=i+1}^{N} \widetilde{\chi}_{jt}\,(1-\alpha_j)\,, \tag{B.1}$$

under the maintained assumption that $(1+\tau_t)\,e^{z} > 1$. If the cap does not bind, then $\tau_t = 0$, and if it binds, $\ell_{dt} = \bar{\ell}_d$ and the previous equation uniquely determines the price of emissions for given technology levels (noting that $\widetilde{\chi}_{jt}$ decreases in τ_t).

With $N = 1$, a steady-state is then uniquely characterized by

$$\chi_1 = F_1\left(1 - \frac{e^{-z}}{1+\tau}\right) \text{ and } (1+\tau_t)\,\ell_{dt} = 1 - \chi_1.$$

To show that there can be multiple steady-states for $N \geq 2$, we build an example with two sectors. We consider parameter values for which the cap always binds. A steady-state is a pair $\{\chi_1, \chi_2\}$ and a price on emissions τ, which satisfy (13), (15), and (B.1) such that

$$(1+\tau)\,\bar{\ell}_d = (1-\chi_1)\,\chi_2\,(1-\alpha_2) + (1-\chi_2) \tag{B.2}$$

$$\chi_1 = F_1\left[\left(1 - \frac{e^{-z}}{1+\tau}\right)\chi_2\,(1-\alpha_2)\right] \tag{B.3}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}}{1+\tau}\right]^{\mu_2}\right) \text{ with } \mu_2 = \alpha_2 + \chi_1\,(1-\alpha_2)\,. \tag{B.4}$$

We construct non-knife edge examples where one steady-state features $\chi_1 = 0$, $\chi_2 > 0$ and the other one features $\chi_1^\dagger = 1$, $\chi_2^\dagger > \chi_2$.

In the first steady-state, $\mu_2 = \alpha_2$ and given (B.2), $1+\tau = (1-\alpha_2\chi_2)\,/\bar{\ell}_d$, hence (B.3) and (B.4) give

$$0 = F_1\left[\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2\,(1-\alpha_2)\right] \tag{B.5}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2}\right). \tag{B.6}$$

In the second steady-state, $\mu_2^\dagger = 1$ and given (B.2), $1+\tau = \left(1-\chi_2^\dagger\right)/\bar{\ell}_d$, hence (B.3) and (B.4) give

$$1 = F_1\left[\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger\,(1-\alpha_2)\right] \tag{B.7}$$

$$\chi_2^\dagger = F_2\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right). \tag{B.8}$$

The right-hand sides of (B.6) and (B.8) are decreasing in χ_2 and $\chi_2^\dagger$ respectively. For a sufficiently low cap $\bar{\ell}_d$, we have $1 - \left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2} < 1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}$. Then, (B.6) and (B.8) imply that $\chi_2^\dagger > \chi_2$, and one can build F_2 such that there is a large gap between $\chi_2^\dagger$ and χ_2. Again, for a sufficiently low cap, we get $\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2 < \left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger$. Building F_1 such that all the mass of the distribution is between these two values, we can satisfy both (B.5) and (B.7). This shows that multiple steady-states are possible.

B.4 Proof of Proposition 3

The maximization problem can be decomposed in three steps. First, for a given level of greenification and a given scale of production y_N, the Planner chooses a labor allocation that minimizes the social costs. Second, for a given level of greenification, the Planner chooses the optimal scale of production. Third, the Planner chooses the optimal level of greenification. We follow these three steps.

Step 1: The social costs of production are equal to 1 per unit of clean labor and $1+\xi$ per unit of dirty labor. With wages normalized to 1, $\tau = \xi$, and a constant returns to scale production technology, the social costs per unit of production are then exactly equal to the marginal cost of producing the final good. Since the economy behaves competitively in steady-state, the marginal cost of producing the final good (which we denote $\hat{p}$ to highlight that it is a shadow price) simply corresponds to the steady-state sectoral price of the final good for $\tau = \xi$. Using (7), we get:

$$\hat{p} = \hat{p}_N = (1+\xi)\, e^{-\chi_N \mu_N Z^*}.$$

Step 2: Given the level of greenification initially chosen, the Planner maximizes the constant utility flow which is given by

$$\ln c - \ell_c - (1+\xi)\,\ell_d = \ln c - \hat{p}c,$$

so that we get $\hat{p} = 1/c$, which is exactly what we get in a steady-state of the decentralized economy for given greenification levels (provided that $\tau = \xi$). Therefore, the labor allocation is identical to that obtained in the decentralized economy with $\tau = \xi$ for the right level of greenification. We can then rewrite the post-greenification utility flow as

$$\begin{aligned} \ln c - \ell_c - (1+\xi)\,\ell_d &= -\ln \hat{p} - 1 \\ &= \chi_N \mu_N Z^* - \ln(1+\xi) - 1 \end{aligned}$$

Step 3: Eliminating the constants, we then get that the Planner solves the maximization problem

$$\max_{\{1 \geq \chi_i \geq \chi_{i0}\}} Z^* \chi_N \mu_N - (1-\beta) \sum_i \mathcal{F}_i(\chi_i)\,. \tag{B.9}$$

Using the definition of μ_i (13), we get $\chi_N \mu_N = \sum_{i=1}^{N} \chi_i \alpha_i \prod_{j=i+1}^{N} [\chi_j (1-\alpha_j)]$, so that the Planner's problem can finally be rewritten as:

$$\max_{\{1\geq\chi_i\geq\chi_{i0}\}} Z^* \sum_{i=1}^{N} \chi_i \alpha_i \prod_{j=i+1}^{N} \left[\chi_j \left(1-\alpha_j\right)\right] - \left(1-\beta\right) \sum_i \mathcal{F}_i \left(\chi_i\right). \tag{B.10}$$

We define $\omega_i \equiv \prod_{j=i+1}^{N} \chi_j \left(1-\alpha_j\right)$ and note by ι_i^u the Lagrange multiplier on the upper bound constraint $\chi_i \leq 1$ and by ι_i^l the Lagrange multiplier on the lower bound constraint $\chi_i \geq \chi_{i0}$. We then obtain the first order conditions:

$$\frac{Z^*}{1-\beta} \sum_{j=1}^{i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} - \iota_i^u + \iota_i^l = \mathcal{F}_i' \left(\chi_i\right). \tag{B.11}$$

Note that for $i > 1$,

$$\chi_i \omega_i \mu_i = \chi_i \omega_i \left(\alpha_i + \left(1-\alpha_i\right) \chi_{i-1} \mu_{i-1}\right) = \chi_i \omega_i \alpha_i + \chi_{i-1} \mu_{i-1} \omega_{i-1} = \sum_{j=1}^{i} \omega_j \alpha_j \chi_j.$$

We can then rewrite (B.11)

$$\frac{\omega_i \mu_i Z^*}{1-\beta} - \iota_i^u + \iota_i^l = \mathcal{F}_i' \left(\chi_i\right). \tag{B.12}$$

For an interior solution, $\iota_i^u = \iota_i^l = 0$ and $\mathcal{F}_i' \left(\chi_i\right) = F_i^{-1} \left(\chi_i\right)$, so that (B.12) directly delivers (17). Because a cdf is constant and equal to 1 for a fixed cost above the upper-bound of the distribution, then (17) also applies when $\chi_i = 1$. When the lower bound binds $\chi_i = \chi_{i,0}$ and we get:

$$\chi_{i,0} \geq F_i \left(\frac{\mu_i Z^*}{1-\beta} \prod_{j=i+1}^{N} \left[\chi_j \left(1-\alpha_j\right)\right] \right).$$

This establishes the Proposition. □

B.5 Proof of Proposition 4

Consider any initial allocation $\{\chi_{i,0}\}$ and assume that the Social Planner imposes a Pigouvian tax $\tau_t = \xi$ and a set of sector specific subsidies $\{q_{i,t}\}$. Then, the equilibrium level of greenification at time t is given by

$$\chi_{i,t} = F_i \left(\frac{\left(1-e^{-\mu_{i,t-1} Z}\right)}{1-q_{i,t}} \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{j,t}(1-\alpha_j)\right) \right).$$

For sector N at time 1, we can always set $q_{N,1}$ such that

$$\chi_N^{SP} = F_N \left(\frac{\left(1-e^{-\mu_{N,0} Z}\right)}{1-q_{N,1}} \right),$$

where χ_N^{SP} is the optimal level of greenification in sector N and $\mu_{N,0}$ is predetermined. Assume now that the Social Planner uses a set of sector specific subsidies $\{q_{j,1}\}$ for $j > i$, in order to implement the optimal level of greenification χ_j^{SP} for $j > i$ at $t = 1$. Then for sector i, the Social Planner can choose $q_{i,1}$ such that

$$\chi_i^{SP} = F_i \left(\frac{\left(1 - e^{-\mu_{i,0} Z}\right)}{1 - q_{i,1}} \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{j,1}(1 - \alpha_j)\right) \right),$$

since $\mu_{i,0}$ is again pre-determined and $\widetilde{\chi}_{j,1} = \chi_{j,0} + \left(\chi_j^{SP} - \chi_{j,0}\right) e^{-\mu_{j,0} Z}$ is also given. Then, by induction, the Social Planner can implement the socially optimal levels of greenification χ_i^{SP} in all sectors from the most downstream to the most upstream at time $t = 1$.

At time $t = 2$, there is no more incentive to greenify when $q_{i,2} = 0$, because if $\chi_{j,2} = \chi_j^{SP}$, we get that

$$\chi_i^{SP} = F_i \left(\frac{\mu_i^{SP} Z}{1 - \beta} \prod_{j=i+1}^{N} \left(\chi_j^{SP}(1 - \alpha_j\right) \right) > F_i \left(\left(1 - e^{-\mu_{i,1} Z}\right) \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{j,2}(1 - \alpha_j)\right) \right),$$

where the equality stems from the fact that χ_i^{SP} is the optimum, but the inequality uses that $\mu_{i,1} = \mu_i^{SP}$, $\widetilde{\chi}_{j,2} = \chi_j^{SP}$ if there is no further greenification and that $\frac{\mu_i^{SP} Z}{1-\beta} > \left(1 - e^{-\mu_{i,1} Z}\right)$. The overall inequality implies that there is no further incentive to greenify in the decentralized economy. Since there is no greenification, there are no profits either, and the social optimum is implemented from $t = 2$ onward. This completes the proof. □

B.6 Example: Small Interventions can have Large Effects

Figure B.1 illustrates an example where small interventions are enough to push the economy out of a low-greenification steady-state. In that example, there are three steady-state equilibria: no greenification, full greenification, and an interior, unstable steady-state, which is close to the no greenification steady-state. Starting from no greenification, small sector-specific subsidies are enough to move the economy a little beyond the unstable steady-state from which point the economy converges on its own toward the high greenification steady-state. Provided that consumers are sufficiently patient, the full greenification steady-state dominates the other two and corresponds to the optimum.

We develop a formal example along these lines.We assume that the distributions of fixed costs F_1 and F_2 have mass points at ϕ_1 and ϕ_2, respectively. We first derive conditions under which there are three steady-states characterized by no greenification, full greenification, and an interior level of greenification. We then derive conditions under which moving from the no greenification

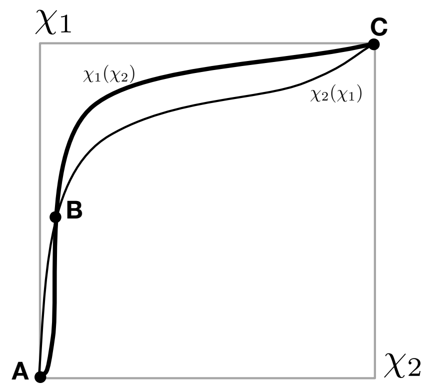


Figure B.1. Small subsidies are sufficient to escape no-greenification steady-state

to the interior level of greenification only requires a small intervention.

First, assume that the economy features no greenification, so there is no incentive to greenify downstream (sector 2) provided that $1 - e^{-\alpha_2 Z} < \phi_2$. In that case, there is no market for sector 1 and no greenification upstream either as long as $\phi_1 > 0$. Because these are strict inequalities, there are still no incentives to greenify for χ's slightly different from 0, so this steady-state is stable.

Second, full greenification is also a steady-state provided that $\left(1 - e^{-Z}\right)(1 - \alpha_2) > \phi_1$, which ensures full greenification upstream, and $\left(1 - e^{-Z}\right) > \phi_2$, which ensures full greenification downstream. For the same reason as before, this steady-state is also stable.

Then the no-greenification and full-greenification steady-states coexist provided that

$$1 - e^{-\alpha_2 Z} < \phi_2 < 1 - e^{-Z} \text{ and } 0 < \phi_1 < \left(1 - e^{-Z}\right)(1 - \alpha_2).$$

Third, an interior steady-state equilibrium (χ_1^*, χ_2^*) must satisfy

$$\left(1 - e^{-Z}\right) \chi_2^* (1 - \alpha_2) = \phi_1 \tag{B.13}$$

$$1 - e^{-\left(\alpha_2 + \chi_1^*(1-\alpha_2)\right)Z} = \phi_2. \tag{B.14}$$

Given that $1 - e^{-\alpha_2 Z} < \phi_2 < 1 - e^{-Z}$, there always exists a $\chi_1^* \in (0, 1)$ which satisfies the second equation. Similarly, given that $\left(1 - e^{-Z}\right)(1 - \alpha_2) > \phi_1 > 0$, there also always exists a $\chi_2^* \in (0, 1)$ that satisfies the first equation. Since the left hand side of (B.13) is increasing in χ_2 and the left-hand side of (B.14) is increasing in χ_1, while the right-hand sides are fixed at ϕ_1 and ϕ_2, this interior steady-state is necessarily unstable. Therefore, a small increase in χ_1 and/or χ_2 starting from (χ_1^*, χ_2^*) will lead to further greenification.

Next, we derive a set of subsidies sufficient to ensure that the economy moves from the no greenification to the interior steady-state at time 1. For greenification in sector 1 to be interior, we need a subsidy $q_{1,1}$ which satisfies

$$\frac{1-e^{-Z}}{1-q_{1,1}}\widetilde{\chi}_{2,1}\left(1-\alpha_2\right)=\phi_1 \text{ with } \widetilde{\chi}_{2,1}=\chi_2^* e^{-\alpha_2 Z},$$

which, using (B.13), yields $q_{1,1}=1-e^{-\alpha_2 Z}$. Similarly, for greenification in sector 2 to be interior, we need a subsidy $q_{2,1}$ such that

$$\frac{1-e^{-\alpha_2 Z}}{1-q_{2,1}}=\phi_2 \Longrightarrow q_{2,1}=1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}.$$

Overall, the total amount of subsidies to move the economy to the interior unstable steady-state, is given by

$$Q_1=\chi_1^* q_{1,1}\phi_1+\chi_2^* q_{2,1}\phi_2=\chi_1^*\left(1-e^{-\alpha_2 Z}\right)\phi_1+\chi_2^*\left(1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}\right)\phi_2$$

If ϕ_2 is above but close to $1-e^{-\alpha_2 Z}$, then $q_{2,1}$ is small. In addition, in that case, (B.14) implies that χ_1^* is small too, which ensures that the total amount spent Q_1 is small. This establishes the result described above: if greenification costs are just a little too high downstream, then a small intervention is enough to push the economy toward the interior steady-state and eventually the full greenification one as well.[B.1]

B.7 Escaping the No Greenification Trap

If an economy is trapped in a no-greenification steady-state, which sector should a government prioritize? To answer this question, we consider a special case of a supply chain with $N \geq 3$ sectors. We suppose that the basic parameters and the F_i's are such that no greenification in all sectors is a steady-state. We assume that initially the economy is stuck in this no greenification steady-state and that the government can directly greenify a positive mass of varieties in one sector only.

Then, provided that greenification costs are not too high, greenification starts propagating upstream immediately if the government targets the most downstream sector N as the demand channel operates contemporaneously. If instead the government targets sector $N-1$, greenification starts propagating with a one-period delay: through the input cost channel it propagates

[B.1] This logic does not extend to the case where greenification costs are just a little too high upstream: If ϕ_1 is positive but close to 0, then χ_2^* is close to 0 and subsidies spent for sector 2 are small. However, there is no guarantee that subsidies spent for greenification in sector 1 are small without additional assumptions.

next period to the most downstream sector and then through the demand channel it propagates immediately from that sector to all upstream sectors (again for sufficiently low greenification costs). However, should the government target sectors that are more upstream than $N-1$, then the input cost channel never reaches the most downstream sector so that greenification never propagates. This analysis reinforces our general point that, if limited, an industrial policy should favor downstream sectors in priority.

Proof. **Part (i):** Greenification starts propagating immediately if the government intervenes in the most downstream sector N.

Suppose the government greenifies sector N at the beginning of time t, so that $\chi_{N,t} > 0$. Then, greenification in sector $N-1$ at t is given by

$$\chi_{N-1,t} = F_{N-1}\left(\left(1-e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_N Z}\left(1-\alpha_N\right)\right).$$

With $\chi_{N,t} > 0$, we get $\chi_{N-1,t} > 0$ (as long as greenification costs at $N-1$ are not too high: $F_{N-1}\left(\left(1-e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_N Z}\left(1-\alpha_N\right)\right) > 0$). In contrast at $t-1$, pre-intervention, we had $\chi_{N,t-1} = \chi_{N-1,t-1} = 0$.

Now suppose greenification did propagate to sectors $j > i$ (i.e. $\chi_{j,t} > 0$). Then it also propagates to sector i since:

$$\chi_{it} = F_i\left(\left(1-e^{-\alpha_i Z}\right)\prod_{j=i+1}^{N}\chi_{j,t}e^{-\alpha_j Z}\left(1-\alpha_j\right)\right) \text{ for } i < N,$$

which is positive if greenification costs are not too high. Hence greenification propagates all the way from the most downstream sector N to the most upstream sector 1 at time t.

At time $t+1$, we have

$$\chi_{i,t+1} = F_i\left(\left(1-e^{-\mu_{i,t}Z}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,t+1}\left(1-\alpha_j\right)\right)\right),$$

with $\widetilde{\chi}_{j,t+1}$ given by (8) and μ_{it} by (6). Then, $\mu_{it} \geq \alpha_i$ and $\widetilde{\chi}_{j,t+1} \geq \chi_{j,t} > \chi_{j,t}e^{-\alpha_j Z}$, which implies that further greenification occurs in all sectors at time $t+1$ (as long as F_i has positive mass around the relevant range), and this continues in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (ii): Greenification starts propagating with a one-period delay if the government intervenes in sector $N-1$.

Suppose now that the government starts greenifying in sector $N-1$. Then we get that

$\mu_{N,t-1} = \alpha_N$ (i.e. the pre-intervention value) so that χ_{Nt} must satisfy

$$\chi_{Nt} = F_N \left(1 - e^{-\alpha_N Z}\right) = \chi_{N,t-1} = 0.$$

In other words, greenifying first in sector $N-1$ at time t does not immediately propagate to the most downstream sector N. Consider now sector $i < N-1$. Since, $\chi_{N,t} = 0$, $r_{it} = 0$ and there is no greenification in any sector besides $N-1$.

Consider now time $t+1$. In sector N, we get $\mu_{N,t} = \alpha_N + \chi_{N-1,t}\alpha_{N-1}(1-\alpha_N) > \alpha_N$ and greenification incentives in sector N obey

$$\chi_{N,t+1} = F_N \left(1 - e^{-\mu_{N,t} Z}\right).$$

Since $\mu_{N,t} > \alpha_N$, then provided that greenification costs are not too high, we can have $F_N \left(1 - e^{-\mu_{N,t} Z}\right) > F_N \left(1 - e^{-\alpha_N Z}\right) = 0$, such that $\chi_{N,t+1} > 0$, in which case greenification propagates to sector N.

Moving back to sector $N-1$, we have

$$\chi_{N-1,t+1} = \max\left\{\chi_{N-1,t}, F_{N-1}\left(\left(1 - e^{-\alpha_{N-1} Z}\right)\widetilde{\chi}_{N,t+1}(1-\alpha_N)\right)\right\},$$

with $\widetilde{\chi}_{N,t+1} = \chi_{N,t+1}\left(e^z(1+\tau_t)\right)^{-\mu_{N,t}}$. Now moving to sector $N-2$, we get that

$$\chi_{N-2,t+1} = F_{N-2}\left(\left(1 - e^{-\alpha_{N-2} Z}\right)\widetilde{\chi}_{N-1,t+1}(1-\alpha_{N-1})\widetilde{\chi}_{N,t+1}(1-\alpha_N)\right),$$

with $\widetilde{\chi}_{N-1,t+1} = \chi_{N-1,t} + (\chi_{N-1,t+1} - \chi_{N-1,t})\, e^{-\mu_{N-1,t} Z} \geq \chi_{N-1.t}$ and $\widetilde{\chi}_{N,t+1} > 0$ if $\chi_{N,t+1} > 0$. This in turn implies that greenification also propagates to sector $N-2$, provided that the distribution of fixed costs F_i has positive mass in the relevant range. The logic extends to all sectors $j < N-1$ so that greenification propagates to all sectors at time $t+1$. And greenification intensifies in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (iii): Greenification never propagates if the government intervenes in a sector which is more upstream than sector $N-1$.

Now suppose that the government starts greenifying in a sector j more upstream than $N-1$, i.e. $j < N-1$, at time t. Consider first sector N at time t. Given that greenification incentives in that sector only depend upon $\mu_{N,t-1}$, they are the same as pre-intervention so that $\chi_{N,t} = 0$. As a result $r_{it} = 0$ for any $i < N$ and greenification does not propagate: $\chi_{k,t} = 0$ for all $k \neq j$. At time $t+1$, since $\chi_{N-1,t} = 0$, $\mu_{N,t} = \mu_{N,t-1}$, and greenification incentives in sector N are the same as in period t, i.e. $\chi_{N,t+1} = 0$. It follows that we also have $\chi_{k,t+1} = 0$ for all $k \neq j$. Since the same reasoning carries over to all future periods, greenification never propagates. This establishes the proposition. □

B.8 Proof of Proposition 5

Part i) is trivial. To establish Part ii), we first derive $\frac{\partial \mu_i}{\partial \chi_k}$. We immediately note that $\frac{\partial \mu_i}{\partial \chi_k} = 0$ for $k > i$. Further, we have that $\frac{\partial \mu_i}{\partial \chi_{i-1}} = (1-\alpha_i)\,\mu_{i-1}$, while for $k < i-1$, we get $\frac{\partial \mu_i}{\partial \chi_k} = \chi_{i-1}(1-\alpha_i)\frac{\partial \mu_{i-1}}{\partial \chi_k}$. Iterating on j such that $i-j$ goes down to $k+1$, we get

$$\frac{\partial \mu_i}{\partial \chi_k} = \begin{cases} (1-\alpha_{k+1})\left(\prod_{j=0}^{i-k-2}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k & k < i \\ 0 & \text{otherwise} \end{cases}. \tag{B.15}$$

For $k > i$, it is then immediate that $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = 1$. While for $k < i$, we get

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} \frac{\partial \ln \mu_i}{\partial \ln \chi_k},$$

which combined with (B.15) gives (20).

We note that $\frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1 \iff 1+\mu_i Z < e^{\mu_i Z}$, which is true for $Z > 0$. Next, for $k = i-1$, we get using (6) that

$$\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} = \frac{(1-\alpha_i)\chi_{i-1}\mu_{i-1}}{\mu_i} = \frac{\mu_i - \alpha_i}{\mu_i} < 1.$$

In addition, for any $k < i-1$, then, using again (6), we get

$$\begin{aligned}\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} &= \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)(\mu_{k+1}-\alpha_{k+1})}{\mu_i} \\ &< \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_{k+1}}{\mu_i}.\end{aligned}$$

Then, we get that $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i}$ is increasing in k for $k \leq i-1$, and by induction, $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} < 1$ for all $k \leq i-1$. As both fractions on the right-hand side of (20) are smaller than 1, $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} < 1$, establishing Part ii). Part iii) is then trivial.

B.9 Full Effect of Subsidies

In this Appendix, we trace out the effect of a set of marginal subsidies on steady-state greenification level. With subsidies, an interior steady-state is characterized by (19). For a stable steady-state, the economy moves toward the new "perturbed steady-state". Differentiating (19) and introducing $\epsilon_{F,i} \equiv \frac{\pi_i f_i(\pi_i)}{F_i(\pi_i)}$, we get:

$$\frac{d\chi_i}{\chi_i} = \epsilon_{F,i}\frac{d\pi_i}{\pi_i} + \epsilon_{F,i}\frac{dq_i}{1-q_i}.$$

Introducing the elasticities $\frac{\partial \ln \pi_i}{\partial \ln \chi_j}$, we obtain:

$$\frac{d\chi_i}{\chi_i} = \epsilon_{F,i} \sum_j \frac{\partial \ln \pi_i}{\partial \ln \chi_j} \frac{d\chi_j}{\chi_j} + \epsilon_{F,i} \frac{dq_i}{1 - q_i}.$$

Rearranging and stacking these equations, we get:

$$\left(I - \text{Diag}(\boldsymbol{\epsilon_F}) \frac{d \ln \boldsymbol{\pi}}{d \ln \boldsymbol{\chi}^\top} \right) \boldsymbol{\chi}^{\odot -1} \odot d\boldsymbol{\chi} = \text{Diag}(\boldsymbol{\epsilon_F})(\mathbf{1} - \mathbf{q})^{\odot -1} \odot d\boldsymbol{q}.$$

For a stable steady-state, the matrix $\left(I - \text{Diag}(\boldsymbol{\epsilon_F}) \frac{d \ln \boldsymbol{\pi}}{d \ln \boldsymbol{\chi}^\top} \right)$ is invertible. Assuming that there is initially no subsidy, we obtain (21).

B.10 Proof of Proposition 6

As discussed in the main text, all greenification still occurs in the first period, which enables us to drop time subscripts from the problem. To consider the equilibrium relationship between the allocation of labor and the level of greenification, we first define the factor demand correspondences $\{\ell_{di}(\{\chi_j, p_N\}), \ell_{ci}(\{\chi_j, p_N\})\}$ which reflect profit maximization by producers:

$$\left\{\ell_{di}\big(\{\chi_j, p_N\}\big), \ell_{ci}\big(\{\chi_j, p_N\}\big)\right\} \equiv \text{argmax}\Big[p_N y_N - \sum_{i=1}^{N} \Big((1+\tau)\ell_{di} + \ell_{ci}\Big)\Big]. \tag{B.16}$$

Since the economy is production efficient, it is as if there is a single, price-taking firm solving the above profit maximization problem, the solution of which defines a ray of possible factor demands. This firm must pay the numeraire wage for labor as well as the carbon price of τ for dirty labor. The final good price p_N is then set by household consumption maximization and solves $p_N y_N = 1$. Therefore, we can define the equilibrium labor allocation $\{\ell_{di}(\{\chi_j\}), \ell_{ci}(\{\chi_j\})\}$ as the solution to the fixed point problem $\{\ell_{di}, \ell_{ci}\} = \{\ell_{di}(\{\chi_j, y_N^{-1}\}), \ell_{ci}(\{\chi_j, y_N^{-1}\})\}$, where the dependency of y_N on $\{\ell_{di}, \ell_{ci}\}$ pins down the scale of factor demand.

The Planner then maximizes household utility with industrial policy that anticipates this equilibrium relationship. They solve

$$\max_{\{\chi_j\}} \frac{1}{1-\beta}\Big(\ln(y_N) - (1+\xi)\sum_{i=1}^{N} \ell_{di} - \sum_{i=1}^{N} \ell_{ci}\Big) - \sum_{i=1}^{N} \mathcal{F}_i(\chi_i). \tag{B.17}$$

We can now establish Proposition 6.

The arguments of Appendix B.4 give us a mapping from technology and the labor allocation to output via Equation (B.10). This mapping implies that the Planner's problem can be rewritten as

$$\max_{\{1\geq\chi_j\geq\chi_{j0}\}} Z\sum_{i=1}^{N}\omega_i\chi_i\alpha_i-(\xi-\tau)\sum_{i=1}^{N}\ell_{di}-(1-\beta)\sum_{i=1}^{N}\mathcal{F}_i(\chi_i),$$

where the additional labor term comes from the fact that total payments to factors differs from labor disutility due to the pollution: $\ell_c+(1+\xi)\ell_d=1+(\xi-\tau)\ell_d$. This gives us the first-order conditions

$$\frac{Z}{1-\beta}\sum_{j=1}^{i}\frac{\omega_j\alpha_j\chi_j}{\chi_i}+\frac{\xi-\tau}{1-\beta}\Big(-\frac{\partial\ell_d}{\partial\chi_i}\Big)-\iota_i^u+\iota_i^l=\mathcal{F}_i'\left(\chi_i\right).$$

Noting that $\sum_{j=1}^{i}\frac{\omega_j\alpha_j\chi_j}{\chi_i}=\mu_i\omega_i$ gives us Equation (22). □

B.11 Two Sector Example

The relative effectiveness of downstream greenification is well-illustrated in the special case of a two-layer supply chain, with only an upstream and a downstream sector. In that case, Equation (23) implies

$$-\frac{\partial E}{\partial\chi_2}=\frac{1-(1-\alpha_2)(1-\chi_1)}{1+\tau}\text{ and }-\frac{\partial E}{\partial\chi_1}=\frac{\chi_2(1-\alpha_2)}{1+\tau}.$$

Downstream greenification always reduces emissions, but upstream greenification only reduces emissions as long as downstream is already greenified ($\chi_2>0$). While greenification in each sector is more effective when the other one is more greenified, the upstream sector is more "vulnerable" to low greenification of its counterpart. If nobody buys electric vehicles (EVs), greenifying the production of EV batteries has little effect on emissions.

We can better understand this asymmetry by sketching the line where the two sectors are equally effective at reducing emissions:

$$\chi_2=\frac{\alpha_2}{1-\alpha_2}+\chi_1.$$

When χ_2 is greater than this line, upstream greenification is more effective at reducing emissions, while downstream is more effective otherwise. The slope is always one, but the intercept is decreasing in the downstream input share, as a larger input share strengthens the offsetting effect for downstream, while raising the market size for upstream. In the extreme case of an input share equal to one, the situation is symmetric and greenifying the lagging sector is more effective. But as the input share shrinks below one, the space where upstream greenification is better at reducing emissions shrinks. Once the input share reaches one half, downstream greenification is always more effective (for any χ).

C Theory Derivations in the General Model

C.1 Equilibrium in the Very General Model

We solve for the equilibrium in the general model presented in Section 4.1 and establish that it is unique.

Marginal Costs. The production technology (24) implies that the marginal costs of producing one unit of a variety in sector i at time t with the dirty technology (mc^d_{it}), and, whenever available, with the clean technology (mc^c_{it}) are given respectively by:

$$mc^d_{it} = \frac{1}{A_i}(1+\tau_i)^{\alpha^d_i} \prod_{j<i} p_{jt}^{\sigma^d_{ij}} \text{ and } mc^c_{it} = \frac{1}{A_i} e^{-\alpha^c_i z_i} \prod_{j<i} p_{jt}^{\sigma^c_{ij}}. \tag{C.1}$$

If a variety is not greenified ($\gamma_{it}(\nu) = 0$), its marginal cost equals mc^d_{it}. If it is greenified ($\gamma_{it}(\nu) = 1$), its marginal cost is $\min\{mc^d_{it}, mc^c_{it}\}$. We summarize the clean-vs-dirty cost advantage by the (potential) markup factor

$$\theta_{it} \equiv \max\left(\frac{mc^d_{it}}{mc^c_{it}}, 1\right). \tag{C.2}$$

Thus $\theta_{it} > 1$ when the clean technology is (strictly) cheaper, and $\theta_{it} = 1$ otherwise. Using (C.1), the log difference between the dirty and clean technologies' marginal costs is given by:

$$\ln\left(\frac{mc^d_{it}}{mc^c_{it}}\right) = \alpha^d_i \ln(1+\tau_i) + \alpha^c_i z_i + \sum_{j<i} (\sigma^d_{ij} - \sigma^c_{ij}) \ln p_{jt}. \tag{C.3}$$

Technology Use. Within a sector, the marginal costs (mc^d_{it}, mc^c_{it}) are common across varieties, so that at a given date t, either all greenified varieties in sector i use the clean technology, or none does. We denote by $\hat{\chi}_{it}$, the share of varieties that are greenified by the end of t (including newly greenified at t) and that actually use the clean technology:

$$\hat{\chi}_{it} \equiv \mathbf{1}\{mc^c_{it} \leq mc^d_{it}\}\chi_{it}. \tag{C.4}$$

Among these varieties, a mass $\bar{\chi}_{it}$ is competitively supplied at time t at the clean marginal cost mc^c_{it} and uses input shares σ^c_{ij}. These varieties correspond to those greenified by the end of $t-1$, so that:

$$\bar{\chi}_{it} \equiv \mathbf{1}\{mc^c_{it} \leq mc^d_{it}\}\chi_{i,t-1}. \tag{C.5}$$

The remaining mass, $\hat{\chi}_{it} - \bar{\chi}_{it}$, is newly greenified. These varieties use the clean technology, but are sold at price mc^d_{it}; their input expenditures are therefore scaled down by $1/\theta_{it}$. As in the vertical chain model, we then define the *effective* clean share for input spending

$$\tilde{\chi}_{it} \equiv \bar{\chi}_{it} + (\hat{\chi}_{it} - \bar{\chi}_{it})\frac{1}{\theta_{it}}. \tag{C.6}$$

The remaining mass of varieties $1 - \hat{\chi}_{it}$ use the dirty technology and spend input shares σ^d_{ij}. We can then write sectoral prices as

$$p_{it} = (mc^c_{it})^{\bar{\chi}_{it}} \left(mc^d_{it}\right)^{1-\bar{\chi}_{it}}. \tag{C.7}$$

Revenues and Profits. We can then write the value share of sector i's revenue spent on input j at time t as:

$$\tilde{\Sigma}_{ij,t} = (1 - \hat{\chi}_{it})\, \sigma^d_{ij} + \tilde{\chi}_{it}\, \sigma^c_{ij}. \tag{C.8}$$

Let $\tilde{\Sigma}_t = (\tilde{\Sigma}_{ij,t})_{i,j}$. Summing across all purchasing sectors i gives total intermediate demand revenue for sector j equal to $\sum_i \tilde{\Sigma}_{ij,t} r_{it}$, i.e. $(\tilde{\Sigma}^\top_t \mathbf{r}_t)_j$. Adding direct final demand b_j yields $\mathbf{r}_t = \mathbf{b} + \tilde{\Sigma}^\top_t \mathbf{r}_t$. For an acyclic network, $I - \tilde{\Sigma}^\top_t$ is invertible and its inverse is weakly positive. We then obtain that revenues satisfy:

$$\mathbf{r}_t = \left(I - \tilde{\Sigma}^\top_t\right)^{-1} \mathbf{b}. \tag{C.9}$$

With the mark-up θ_{it} defined as in (C.2) to include the case where the green technology is not used, we obtain that the profit vector of newly greenified varieties can be written as:

$$\boldsymbol{\pi}_t = \left(\mathbf{1} - \boldsymbol{\theta}_t^{\odot(-1)}\right) \odot \mathbf{r}_t. \tag{C.10}$$

Labor Allocation. Let ℓ^c_{it} and ℓ^d_{it} denote the total amounts of clean and dirty labor used in sector i at time t, aggregating across the unit mass of varieties. Following the same logic as with sectoral sales above, we get that expenditures on clean labor correspond to a share $\alpha^c_i \tilde{\chi}_{it}$ of revenues and expenditures on dirty labor and taxes correspond to a share $\alpha^d_i(1 - \hat{\chi}_{it})$. As a result, sector-$i$ labor demands satisfy

$$\ell^c_{it} = \frac{\alpha^c_i \tilde{\chi}_{it} r_{it}}{w_t}, \text{ and } \ell^d_{it} = \frac{\alpha^d_i (1 - \hat{\chi}_{it}) r_{it}}{(1 + \tau_i) w_t}. \tag{C.11}$$

Emissions at t are $E_t = \sum_i e_i \ell^d_{it}$.

Equilibrium Uniqueness. To establish uniqueness, we fix a date t and a state vector $\boldsymbol{\chi}_{t-1}$. We show that this establishes a unique price $\mathbf{p}_t$ vector and hence a unique markup vector $\boldsymbol{\theta}_t$. We then show we obtain unique revenues $\mathbf{r}_t$, profits $\boldsymbol{\pi}_t$, and current greenification shares $\boldsymbol{\chi}_t$.

Step 1 (prices and markups). We solve for prices by forward induction on $i = 1, \ldots, N$. Given upstream prices $(p_{jt})_{j<i}$, the marginal costs mc^d_{it} and mc^c_{it} are pinned down by (C.1). This de-

termines θ_{it} via (C.2) and therefore $\bar{\chi}_{it}$. We then obtain the price index p_{it} through (C.7). By acyclicity there is no feedback to already-solved upstream prices. As marginal costs and prices in sector 1 are pinned down by the wage normalization, the induction yields a unique price vector $\mathbf{p}_t$, hence a unique markup vector $\boldsymbol{\theta}_t$.

Step 2 (revenues and innovation). Given $\boldsymbol{\theta}_t$ and the downstream rows of $\tilde{\Sigma}_t$, the revenue system can be solved by backward substitution because $\tilde{\Sigma}_t^\top$ is strictly upper triangular:

$$r_{it} = b_i + \sum_{k>i} \tilde{\Sigma}_{ki,t} r_{kt}.$$

We therefore compute (r_{it}, χ_{it}) recursively from the most downstream sector to the most upstream sector. Start with $i = N$, for which $r_{Nt} = b_N$. Given r_{it} and θ_{it}, we obtain the profits of a newly greenified variety π_{it}. For each greenifiable sector $i \in \Omega^c$, we then obtain:

$$\chi_{it} = \max\left(\chi_{i,t-1}, F_i\left(\pi_{it}\right)\right),$$

while for $i \in \Omega^d$ we have $\chi_{it} \equiv 0$. This pins down χ_{it} uniquely. With χ_{it} in hand, the row i of the input-share matrix $\tilde{\Sigma}_t$ is uniquely determined by (C.8) (and $\tilde{\chi}_{it}$), which then enters the revenue recursion of upstream sectors $j < i$. Proceeding backward from $i = N$ down to $i = 1$ yields a unique sequence $(r_{it}, \chi_{it})_{i=1}^N$, hence a unique equilibrium.

C.2 Proof of Proposition 8

In this Appendix, we prove Proposition 8. We proceed in two steps, first we show that we can collapse the dirty-only subnetwork Ω^d, and then we show that we can eliminate the TFP vector $\mathbf{A}$. Before doing so, we note one caveat on the first step: With homogeneous emission rates and taxes, the isomorphism preserves the relationship between a common tax rate and a common emission rate (i.e $\tau = \tau^{\text{CO}_2} e$ both before and after the isomorphism). The isomorphism also preserves Pigouvian taxation with heterogeneous emission rates (i.e. $\tau_i = \xi^{CO_2} e_i$ both before and after the transformation). If taxes are small and proportional to the emission rate (i.e. $\tau_i = \tau^{\text{CO}_2} e_i$ with τ^{CO_2} small), then the isomorphism also preserves that relationship at first order ($\tau_i = \tau^{\text{CO}_2} e_i + o\left(\tau^{\text{CO}_2}\right)$). However, the relationship is not preserved in general. Since our focus is on innovation incentives (more than on emissions per se), and since the Pigouvian case and the case of τ^{CO_2} small are the most relevant ones, we will nevertheless focus on the collapsed economy.

C.2.1 Step 1: Collapsing the Dirty-Only Subnetwork Ω^d

We proceed in four steps: first, we solve for prices in the dirty-only subnetwork; second, we present a transformation that preserves prices, markups, revenues, profits, and innovation incentives in Ω^c; third, we show that this transformation also preserves welfare; and, fourth, we discuss when a potential relationship between τ_i and e_i is preserved or not.

Prices in the Dirty-Only Subnetwork Let $\Sigma^{d,dd}$ be the restriction of Σ^d to intermediate inputs inside Ω^d. Define $\mathbf{p}^{\Omega^d} \equiv (p_k)_{k\in\Omega^d}$, $\mathbf{A}^d \equiv (A_k)_{k\in\Omega^d}$, $\boldsymbol{\alpha}^{d,d} \equiv (\alpha_k^d)_{k\in\Omega^d}$, and $\boldsymbol{\tau}^d \equiv (\tau_k)_{k\in\Omega^d}$. Define

$$\ln \bar{\mathbf{A}}^d = \left(I - \Sigma^{d,dd}\right)^{-1} \ln \mathbf{A}^d. \tag{C.12}$$

Note that $I - \Sigma^{d,dd}$ is invertible since (by acyclicity) $\Sigma^{d,dd}$ is strictly lower triangular. Define the matrix of total (tax-inclusive) dirty-labor cost shares

$$B^d \equiv \left(I - \Sigma^{d,dd}\right)^{-1} \operatorname{Diag}\left(\boldsymbol{\alpha}^{d,d}\right). \tag{C.13}$$

CRS within Ω^d implies that $(I - \Sigma^{d,dd})\mathbf{1} = \boldsymbol{\alpha}^{d,d}$, so premultiplying by $(I - \Sigma^{d,dd})^{-1}$ yields $B^d\mathbf{1} = \mathbf{1}$. Then we get that prices in the dirty subnetwork are given by a simple Cobb-Douglas price index:

Lemma 2. *Competitive equilibrium prices in Ω^d obey:*

$$p_k = \frac{1}{\bar{A}_k^d} \prod_{\ell\in\Omega^d} (1+\tau_\ell)^{B_{k\ell}^d}. \tag{C.14}$$

Proof. Goods in the dirty subnetwork are produced competitively with inputs from the dirty subnetwork only, so that for any $k \in \Omega^d$, we have:

$$p_{kt} = mc_{kt}^d = \frac{1}{A_k}(1+\tau_k)^{\alpha_k^d} \prod_{\ell\in\Omega^d} p_{\ell t}^{\sigma_{k\ell}^d}.$$

Taking logs and stacking these equations across $k \in \Omega^d$ gives

$$\ln \mathbf{p}^{\Omega^d} = -\ln \mathbf{A}^d + \boldsymbol{\alpha}^{d,d} \odot \ln\left(\mathbf{1} + \boldsymbol{\tau}^d\right) + \Sigma^{d,dd} \ln \mathbf{p}^d. \tag{C.15}$$

With $I - \Sigma^{d,dd}$ invertible, we directly obtain:

$$\ln \mathbf{p}^{\Omega^d} = \left(I - \Sigma^{d,dd}\right)^{-1} \left[-\ln \mathbf{A}^d + \operatorname{Diag}(\boldsymbol{\alpha}^{d,d}) \ln(\mathbf{1} + \boldsymbol{\tau}^d)\right].$$

With $\bar{\mathbf{A}}^d$ defined by (C.12), and B^d by (C.13), exponentiating the previous equation componentwise gives (C.14). □

Collapsing Ω^d inside Downstream Sectors

Lemma 3. *For each sector $i \in \Omega^c$, define the reduced-form parameters*

$$\alpha_i^{d,\text{red}} \equiv \alpha_i^d + \sum_{k \in \Omega^d} \sigma_{ik}^d, \tag{C.16}$$

$$A_i^{\text{red}} \equiv A_i \prod_{k \in \Omega^d} \left(\bar{A}_k^d\right)^{\sigma_{ik}^d}, \tag{C.17}$$

$$\ln(1+\tau_i^{\text{red}}) \equiv \frac{1}{\alpha_i^{d,\text{red}}} \left[\alpha_i^d \ln(1+\tau_i) + \sum_{k \in \Omega^d} \sigma_{ik}^d \sum_{\ell \in \Omega^d} B_{k\ell}^d \ln(1+\tau_\ell)\right], \tag{C.18}$$

$$z_i^{\text{red}} \equiv z_i - \frac{1}{\alpha_i^c} \sum_{k \in \Omega^d} \sigma_{ik}^d \ln \bar{A}_k^d. \tag{C.19}$$

Then, holding fixed all remaining input shares among sectors in Ω^c, one can drop Ω^d and work with an equivalent economy in which, for every $i \in \Omega^c$,

$$mc_{it}^d = \frac{1}{A_i^{\text{red}}} (1+\tau_i^{\text{red}})^{\alpha_i^{d,\text{red}}} \prod_{j \in \Omega^c} p_{jt}^{\sigma_{ij}^d}, \tag{C.20}$$

$$mc_{it}^c = \frac{1}{A_i^{\text{red}}} e^{-\alpha_i^c z_i^{\text{red}}} \prod_{j \in \Omega^c} p_{jt}^{\sigma_{ij}^c}. \tag{C.21}$$

This mapping preserves (mc_{it}^d, mc_{it}^c) for all $i \in \Omega^c$ and hence preserves prices, markups, revenues, profits, and innovation incentives in Ω^c.

Proof. Fix $i \in \Omega^c$. The dirty marginal cost in the original economy is

$$mc_{it}^d = \frac{1}{A_i} (1+\tau_i)^{\alpha_i^d} \left(\prod_{k \in \Omega^d} p_k^{\sigma_{ik}^d}\right) \left(\prod_{j \in \Omega^c} p_{jt}^{\sigma_{ij}^d}\right).$$

Substituting the dirty-only price factorization (C.14) gives

$$\prod_{k \in \Omega^d} p_k^{\sigma_{ik}^d} = \left(\prod_{k \in \Omega^d} \left(\bar{A}_k^d\right)^{\sigma_{ik}^d}\right)^{-1} \prod_{k \in \Omega^d} \prod_{\ell \in \Omega^d} (1+\tau_\ell)^{\sigma_{ik}^d B_{k\ell}^d}.$$

Define A_i^{red} by (C.17) to collect all purely technological terms, and define τ_i^{red} by (C.18) so that

$$(1+\tau_i^{\text{red}})^{\alpha_i^{d,\text{red}}} = (1+\tau_i)^{\alpha_i^d} \prod_{k \in \Omega^d} \prod_{\ell \in \Omega^d} (1+\tau_\ell)^{\sigma_{ik}^d B_{k\ell}^d}.$$

Then the original mc_{it}^d becomes exactly (C.20).

Since the clean technology never uses Ω^d inputs, in the original economy

$$mc_{it}^c = \frac{1}{A_i} e^{-\alpha_i^c z_i} \prod_{j \in \Omega^c} p_{jt}^{\sigma_{ij}^c}.$$

To keep this expression unchanged when replacing A_i by A_i^{red}, choose z_i^{red} so that $\frac{1}{A_i^{\text{red}}}e^{-\alpha_i^c z_i^{\text{red}}} = \frac{1}{A_i}e^{-\alpha_i^c z_i}$, which is equivalent to (C.19). This yields (C.21). The mapping therefore preserves $(\mathbf{mc}_t^d, \mathbf{mc}_t^c)$. This then preserves $\bar{\boldsymbol{\chi}}_t$, equilibrium prices $\mathbf{p}_t$, and mark-ups $\boldsymbol{\theta}_t$. $\hat{\boldsymbol{\chi}}_t$, $\tilde{\boldsymbol{\chi}}_t$, and revenues $\boldsymbol{r}_t$ and therefore profits are preserved as a function of $\boldsymbol{\chi}_t$. Since the system has a unique solution, $\boldsymbol{\chi}_t$, revenues and profits end up preserved. $\square$

Welfare

Lemma 4. *Fix t and any downstream sector $i \in \Omega^c$, then: i) The collapse of the dirty network preserves ℓ_{it}^c. ii) The tax-inclusive spending on dirty labor embodied in the entire dirty supply chain of sector-i output (directly in i and indirectly through Ω^d inputs) equals $\alpha_i^{d,\text{red}}(1-\hat{\chi}_{it})\, r_{it}$ both before and after the collapse. iii) The social costs of labor associated with producing a dirty unit of sector-i output (directly in i and indirectly through Ω^d inputs) are preserved by the collapse provided that the effective emission rate for the composite dirty input in sector i, e_i^{red}, is defined by*

$$1 + \xi^{CO_2} e_i^{\text{red}} \equiv \frac{1+\tau_i^{\text{red}}}{\alpha_i^{d,\text{red}}}\left[\alpha_i^d \frac{1+\xi^{CO_2}e_i}{1+\tau_i} + \sum_{k\in\Omega^d}\sigma_{ik}^d \sum_{\ell\in\Omega^d} B_{k\ell}^d \frac{1+\xi^{CO_2}e_\ell}{1+\tau_\ell}\right]. \tag{C.22}$$

iv) Consumption and welfare are preserved by the transformation.

Proof. In both economies, clean labor hired in sector i is given by $\ell_{it}^c = \alpha_i^c \tilde{\chi}_{it} r_{it}$, since $\tilde{\chi}_{it}$ and r_{it} are preserved then so is clean labor, which establishes statement i).

In the dirty technology of sector i, a share α_i^d of revenues is direct spending on taxed dirty labor in i, and a share $\sum_{k\in\Omega^d}\sigma_{ik}^d$ is spending on Ω^d intermediates. But every dollar spent on an Ω^d good ultimately pays for taxed dirty labor somewhere in Ω^d as Ω^d uses only dirty inputs. Hence the total tax-inclusive dirty-labor spending embodied in the dirty supply chain is exactly $(\alpha_i^d + \sum_{k\in\Omega^d}\sigma_{ik}^d(1-\hat{\chi}_{it})\, r_{it} = \alpha_i^{d,\text{red}}(1-\hat{\chi}_{it})\, r_{it}$, and the same is true in the collapsed representation by construction. This establishes statement ii).

Next, consider any $k \in \Omega^d$. By Lemma 2 and the definition of B^d, the exponent $B_{k\ell}^d$ is the share of expenditures on sector k paid (tax-inclusive) to dirty labor in sector $\ell \in \Omega^d$. One dollar of expenditure on good k therefore corresponds to $B_{k\ell}^d/(1+\tau_\ell)$ units of dirty labor in ℓ. The corresponding social costs (lack of leisure + emission costs) is then given by $\left(1+\xi^{CO_2}e_\ell\right) B_{k\ell}^d/(1+\tau_\ell)$. Summing over ℓ shows that the social costs per dollar of k equal $\sum_{\ell\in\Omega^d} B_{k\ell}^d \left(1+\xi^{CO_2}e_\ell\right)/(1+\tau_\ell)$.

Now consider dirty production in sector $i \in \Omega^c$. For each dollar spent on a sector i variety that uses the dirty technology, a share α_i^d is spent directly on dirty labor in i (+ the associated taxes), implying direct social costs of $\alpha_i^d\left(1+\xi^{CO_2}e_i\right)/(1+\tau_i)$. A share σ_{ik}^d is spent on each Ω^d input k, implying indirect social costs $\sigma_{ik}^d \sum_\ell B_{k\ell}^d \left(1+\xi^{CO_2} e_\ell\right)/(1+\tau_\ell)$. Adding the two contributions yields total social costs per expenditure equal to the bracketed term in (C.22). In the collapsed economy, a share $\alpha_i^{d,\text{red}}$ is spent on dirty labor in sector i, so that employment per expenditure is given by $\alpha_i^{d,\text{red}}/\left(1+\tau_i^{\text{red}}\right)$, corresponding to social costs $\left(1+\xi^{CO_2} e_i^{\text{red}}\right) \alpha_i^{d,\text{red}}/\left(1+\tau_i^{\text{red}}\right)$. Since expenditures are equal in the two economies, then the social costs are also the same provided that e_i^{red} satisfies (C.22). This establishes statement iii).

Since prices and revenues are preserved, consumption c_t is preserved. Further, clean labor is preserved, the social costs of dirty labor are preserved, and investments in greenification are preserved. This ensures that welfare overall is preserved. □

Relationship between τ_i and e_i. Because the tax rate and the emission rate τ_i^{red} and e_i^{red} are re-defined separately through (C.18) and (C.22), there is no guarantee that the isomorphism preserves a potential relationship between τ_i and e_i in general. However, such a relationship is preserved in important cases. We establish the following result:

Corollary 1. *i) The isomorphism preserves Pigouvian taxation: if $\tau_i = \xi^{CO_2} e_i$ then $\tau_i^{\text{red}} = \xi^{CO_2} e_i^{\text{red}}$. ii) The isomorphism preserves homogeneous tax rates and emission rates: if $\tau_i = \tau$ and $e_i = e$ for all i, then $\tau_i^{\text{red}} = \tau$ and $e_i^{\text{red}} = e$; it also preserves dirty labor used and emissions generated directly and indirectly through Ω_d in the dirty production of a variety in a given sector, iii) The isomorphism preserves a constant tax per unit of CO_2 at first order in τ^{CO_2}, ξ^{CO_2}, that is if $\tau_i = \tau^{\text{CO}_2} e_i$ and $\tau^{\text{CO}_2}, \xi^{CO_2}$ are small, then $\tau_i^{\text{red}} = \tau^{\text{CO}_2} e_i^{\text{red}} + o\left(\tau^{\text{CO}_2}\right)$; it also preserves dirty labor used and emissions generated directly and indirectly through Ω_d in the dirty production of a variety in a given sector at first order in $\tau^{\text{CO}_2}, \xi^{CO_2}$.*

Proof. Statement (i) is directly derived from plugging in $\tau_i = \xi^{CO_2} e_i$ in (C.22).

For statement (ii), (C.18) directly implies that if all τ_i's are equal to τ, then so is τ_i^{red}. Then if all e_i are equal to e, then $(C.22)$ implies that $e_i^{\text{red}} = e$ as well. With the direct and (through Ω^d) indirect social costs of dirty labor equalized and with identical social costs per unit of dirty labor, then we get that direct and indirect through Ω^d dirty production and emissions are equalized.

Assume now that taxes are proportional to emissions, $\tau_i = \tau^{\text{CO}_2} e_i$ and that $\tau^{\text{CO}_2}$is small, then (C.18) implies

$$\tau_i^{\text{red}} = \tau^{\text{CO}_2} \frac{\alpha_i^d e_i + \sum_{k\in\Omega^d} \sigma_{ik}^d \sum_{\ell\in\Omega^d} B_{k\ell}^d e_l}{\alpha_i^{d,\text{red}}} + o\left(\tau^{\text{CO}_2}\right).$$

With ξ^{CO_2} also small, then ($C.22$) implies:

$$\alpha_i^{d,\text{red}}\left(1+\xi^{CO_2}e_i^{\text{red}}\right)$$
$$=\left(1+\tau^{\text{CO}_2}\frac{\alpha_i^d e_i+\sum_{k\in\Omega^d}\sigma_{ik}^d\sum_{\ell\in\Omega^d}B_{k\ell}^d e_l}{\alpha_i^{d,\text{red}}}\right)\left[\alpha_i^{d,\text{red}}+\left(\xi^{CO_2}-\tau^{\text{CO}_2}\right)\left(\alpha_i^d e_i+\sum_{k\in\Omega^d}\sigma_{ik}^d\sum_{\ell\in\Omega^d}B_{k\ell}^d e_l\right)\right]$$
$$+o\left(\tau^{\text{CO}_2},\xi^{CO_2}\right),$$

which is equivalent to:

$$e_i^{\text{red}}=\frac{\alpha_i^d e_i+\sum_{k\in\Omega^d}\sigma_{ik}^d\sum_{\ell\in\Omega^d}B_{k\ell}^d e_l}{\alpha_i^{d,\text{red}}}+o\left(\tau^{\text{CO}_2},\xi^{CO_2}\right).$$

From there, we directly get that $\tau_i^{\text{red}}=\tau^{\text{CO}_2}e_i^{\text{red}}+o\left(\tau^{\text{CO}_2},\xi^{CO_2}\right)$.

To produce one unit of revenues in sector i, the amount of labor that needs to be hired directly or indirectly through Ω^d is equal to $\frac{\alpha_i^d}{1+\tau_i}+\sum_{k\in\Omega^d}\sigma_{ik}^d\sum_{\ell\in\Omega^d}\frac{B_{k\ell}^d}{1+\tau_\ell}$ (i.e. the social costs per unit of revenues if $\xi^{CO_2}=0$). After the collapse of the dirty sector, it is equal to $\frac{\alpha_i^{d,\text{red}}}{1+\tau_i^{d,\text{red}}}$, the two are equal at first order. Since social costs are also equal, then emissions are equal. □

C.2.2 Step 2: Eliminating Sectoral TFP Levels A_i

From now on, we drop Ω^d and work only with the collapsed Ω^c economy. To avoid heavy notation, we also drop the superscript "red" and simply rename the transformed parameters as $(A_i,\alpha_i^d,\tau_i,z_i,e_i)$. We now show that we can also eliminate *all* remaining A_i by a relabeling of units and a redefinition of the z_i's. We proceed in 3 steps: First, we solve for prices, then we introduce the transformation and show that it preserves a re-scaled version of prices; and, third, we show that the transformation preserves revenues, profits, labor allocation, greenification incentives, and welfare up to a constant.

Price System with A's. Using (C.1) and (C.7), we can write

$$\ln p_{it}=-\ln A_i+(1-\bar{\chi}_{it})\left(\alpha_i^d\ln(1+\tau_i)+\sum_{j<i}\sigma_{ij}^d\ln p_{jt}\right)+\bar{\chi}_{it}\left(-\alpha_i^c z_i+\sum_{j<i}\sigma_{ij}^c\ln p_{jt}\right). \tag{C.23}$$

Define $\bar{\Sigma}_{t-1}$, the effective input-share matrix for prices,

$$\bar{\Sigma}_{t-1}\equiv\text{Diag}(\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\Sigma^d+\text{Diag}(\bar{\boldsymbol{\chi}}_t)\Sigma^c, \tag{C.24}$$

which is a (row-specific) weighted average between the dirty and clean input-share matrices (we use a $t-1$ index as it depends on $\boldsymbol{\chi}_{t-1}$, making future notation simpler). Stacking and re-arranging

(C.23) gives:

$$\left(I-\bar{\Sigma}_{t-1}\right)\ln \mathbf{p}_t = -\ln \mathbf{A} + (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z}. \tag{C.25}$$

Relabeling Units. Define $\bar{\mathbf{A}}$ as $\ln \bar{\mathbf{A}} \equiv \left(I-\Sigma^d\right)^{-1}\ln \mathbf{A}$, noting that $I-\Sigma^d$ is invertible. Then, we re-define units such that $y_i^{\text{red}} = y_i/\bar{A}_i$. By definition, the price vector of re-scaled goods, $\mathbf{p}_t^{\text{red}}$, is given by:

$$\ln \mathbf{p}_t^{\text{red}} = \ln \mathbf{p}_t + \ln \bar{\mathbf{A}}.$$

We then define the transformed clean-productivity vector $\mathbf{z}^{\text{red}}$ by

$$z_i^{\text{red}} \equiv z_i + \frac{1}{\alpha_i^c}\sum_{j<i}(\sigma_{ij}^c - \sigma_{ij}^d)\ln \bar{A}_j. \tag{C.26}$$

We establish:

Lemma 5. *For a given t, and given technologies $\boldsymbol{\chi}_{t-1}$, the pair $(\mathbf{p}_t^{\text{red}}, \bar{\boldsymbol{\chi}}_t)$ solves the same log-price system as a model with $A_i \equiv 1$ for all i and with $\mathbf{z}$ replaced by $\mathbf{z}^{\text{red}}$. The mark-up vector $\boldsymbol{\theta}_t$ is the same in the two economies.*

Proof. We first take as given the vector $\bar{\boldsymbol{\chi}}_t$. Start from (C.25), substitute $\ln \mathbf{p}_t = \ln \mathbf{p}_t^{\text{red}} - \ln \bar{\mathbf{A}}$, and re-arrange to obtain:

$$\begin{aligned}
& \left(I-\bar{\Sigma}_{t-1}\right)\ln \mathbf{p}_t^{\text{red}} \\
= \;& -\ln \mathbf{A} + \left(I-\bar{\Sigma}_{t-1}\right)\ln \bar{\mathbf{A}} + (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z} \\
= \;& -\ln \mathbf{A} + \left(I - \text{Diag}(\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\Sigma^d - \text{Diag}(\bar{\boldsymbol{\chi}}_t)\Sigma^c\right)\ln \bar{\mathbf{A}} + (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z} \\
= \;& \text{Diag}(\bar{\boldsymbol{\chi}}_t)\left(\Sigma^d - \Sigma^c\right)\ln \bar{\mathbf{A}} + (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z} \\
= \;& (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \left(\boldsymbol{\alpha}^c \odot \mathbf{z} + (\Sigma^c - \Sigma^d)\ln \bar{\mathbf{A}}\right),
\end{aligned}$$

where we used the definition of $\bar{\Sigma}_{t-1}$and of $\ln \bar{\mathbf{A}}$. By definition (C.26), the bracketed term equals $\boldsymbol{\alpha}^c \odot \mathbf{z}^{\text{red}}$ componentwise, so

$$\left(I-\bar{\Sigma}_{t-1}\right)\ln \mathbf{p}_t^{\text{red}} = (\mathbf{1}-\bar{\boldsymbol{\chi}}_t)\odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z}^{\text{red}}.$$

This is exactly the $A_i \equiv 1$ version of (C.25) with $\mathbf{z}$ replaced by $\mathbf{z}^{\text{red}}$ (for a given $\bar{\boldsymbol{\chi}}_t$).

Using (C.1), we get that the log price difference in marginal costs is given by

$$\begin{aligned}
\ln \mathbf{mc}_t^d - \ln \mathbf{mc}_t^c &= \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) + \boldsymbol{\alpha}^c \odot \mathbf{z} + (\Sigma^d - \Sigma^c)\ln \mathbf{p}_t \\
&= \boldsymbol{\alpha}^d \odot \ln(\mathbf{1}+\boldsymbol{\tau}) + \boldsymbol{\alpha}^c \odot \mathbf{z}^{\text{red}} + (\Sigma^d - \Sigma^c)\ln \mathbf{p}_t^{\text{red}}
\end{aligned}$$

Therefore, the system with $A_i = 1$ and $\mathbf{z}$ replaced by $\mathbf{z}^{\text{red}}$ also gives the same relative marginal costs. This ensures that for given $\boldsymbol{\chi}_{t-1}$, the ranking between mc^c_{it} and mc^d_{it} is the same in the two settings ensuring that $\bar{\boldsymbol{\chi}}_t$ is also the same. We then get that $(\mathbf{p}^{\text{red}}_t, \bar{\boldsymbol{\chi}}_t)$ solves the log-price system when $A_i = 1$ and $\mathbf{z}$ is replaced by $\mathbf{z}^{\text{red}}$, and with relative mark-ups constant, we immediately get that $\boldsymbol{\theta}_t$ is the same in both settings. □

Invariance of Equilibrium Objects.

Lemma 6. *The transformation in the previous paragraph has no consequences on revenues, profits, greenification incentives, and technology path $\boldsymbol{\chi}_t$. Equilibrium allocations of clean and dirty labor are also unchanged, and welfare only differs by an additive constant $\sum_i b_i \ln \bar{A}_i$, which is immaterial for any decision.*

Proof. Given that relative marginal costs and mark-ups are constant, then for a given $\boldsymbol{\chi}_{t-1}$, $\bar{\boldsymbol{\chi}}_t$, $\hat{\boldsymbol{\chi}}_t$, and $\tilde{\boldsymbol{\chi}}_t$ are the same in the original and the rescaled economy. As a result, the matrix $\tilde{\Sigma}_t$ is unchanged, so that revenues remain the same. Profits are therefore also unchanged, which ensures that greenification incentives and the full technology path remain the same. Equilibrium allocations of clean and dirty labor follow (C.11) and are therefore identical. Using the redefined units, we have

$$\ln c^{\text{red}}_t = \sum_i b_i \ln c^{\text{red}}_{it} = \sum_i b_i \ln c_{it} - \sum_i b_i \ln \bar{A}_i,$$

so that the relabeling shifts utility by a constant. □

C.3 Equilibrium in the Reduced Model

We now characterize the equilibrium in the reduced model where Ω_d sectors have been collapsed and the TFP vector $\mathbf{A}$ has been normalized to 1.

Prices. First, note that (25) in the text corresponds to (C.3). Then use (C.25) with $\mathbf{A} = \mathbf{1}$ to obtain the simplified price system:

$$\left(I - \overline{\Sigma}_{t-1}\right) \ln \mathbf{p}_t = (\mathbf{1} - \bar{\boldsymbol{\chi}}_t) \odot \boldsymbol{\alpha}^d \odot \ln(\mathbf{1} + \boldsymbol{\tau}) - \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\alpha}^c \odot \mathbf{z}. \tag{C.27}$$

Setting $\bar{\boldsymbol{\chi}}_t = \mathbf{0}$, we obtain the price vector $\mathbf{p}^d$ when no varieties have been greenified:

$$\left(I - \Sigma^d\right) \ln \mathbf{p}^d = \boldsymbol{\alpha}^d \odot \ln(\mathbf{1} + \boldsymbol{\tau}), \tag{C.28}$$

so that $\ln \mathbf{p}^d = \left(I - \Sigma^d\right)^{-1} \boldsymbol{\alpha}^d \odot \ln(\mathbf{1} + \boldsymbol{\tau})$. Subtracting (C.28) from (C.27) gives:

$$\left(I - \overline{\Sigma}_{t-1}\right) \left(\ln \mathbf{p}_t - \ln \mathbf{p}^d\right) = -\text{Diag}(\bar{\boldsymbol{\chi}}_t) \left[\boldsymbol{\alpha}^d \odot \ln(\mathbf{1} + \boldsymbol{\tau}) + \boldsymbol{\alpha}^c \odot \mathbf{z} + \left(\Sigma^d - \Sigma^c\right) \ln \mathbf{p}^d\right]. \tag{C.29}$$

Using the definition (26), we then obtain:

$$\ln \mathbf{p}_t = \ln \mathbf{p}^d - \left(I - \overline{\Sigma}_{t-1}\right)^{-1} \bar{\boldsymbol{\chi}}_t \odot \boldsymbol{\vartheta}. \tag{C.30}$$

Proof of Lemma 1. Assume that $\boldsymbol{\vartheta} \geq \mathbf{0}$. Note that $\left(I - \overline{\Sigma}_{t-1}\right)^{-1} = I + \overline{\Sigma}_{t-1} + \overline{\Sigma}_{t-1}^2 + \ldots$ is weakly positive as $\overline{\Sigma}_{t-1} \geq 0$ and strictly lower triangular. Using (C.30), we get: $\ln \mathbf{p}_t \leq \ln \mathbf{p}^d$. Using (25) and (26), we get that the log difference in marginal costs between dirty and clean technologies is given by:

$$\ln \mathbf{mc}_t^d - \ln \mathbf{mc}_t^c = \boldsymbol{\vartheta} + \left(\Sigma^d - \Sigma^c\right)\left(\ln \mathbf{p}_t - \ln \mathbf{p}^d\right). \tag{C.31}$$

With $\boldsymbol{\vartheta} \geq \mathbf{0}$, $\Sigma^d \leq \Sigma^c$, and now $\ln \mathbf{p}_t \leq \ln \mathbf{p}^d$, we get that $\mathbf{mc}_t^d \geq \mathbf{mc}_t^c$: the clean technology is used whenever it is available.

Prices and Mark-Ups when Clean Technologies are Always Used. From now on, we assume that clean technologies are used whenever they are available (for instance because the conditions of Lemma 1 are met). We then immediately get that $\bar{\boldsymbol{\chi}}_t = \boldsymbol{\chi}_{t-1}$ so that $\overline{\Sigma}_{t-1} = \Sigma_{t-1}$ with Σ_{t-1} defined in the text. We then immediately get that (C.30) leads to (27), and an increase in past greenification reduces prices. In addition, we get that $\ln \boldsymbol{\theta}_t = \ln \mathbf{mc}_t^d - \ln \mathbf{mc}_t^c$, plugging (27) into (C.31), we obtain (28).

Revenues and Profits. Still assuming that clean technologies are used whenever they are available, $\hat{\boldsymbol{\chi}}_t = \boldsymbol{\chi}_t$, so that $\tilde{\Sigma}_t$ as defined in (29) corresponds to (C.8). Revenues and profits are then directly given by (30) and the profit equation in the text, which we had derived before (C.9) and (C.10).

C.4 Steady-State Multipliers in the Reduced Model

In this Appendix we prove Proposition 10. To derive (31), we start from the steady-state relationship

$$\mathbf{r} = \mathbf{b} + \Sigma^\top \mathbf{r},$$

knowing that in a steady-state where clean technologies are used,

$$\Sigma = \mathrm{Diag}\left(\mathbf{1} - \boldsymbol{\chi}\right)\Sigma^d + \mathrm{Diag}\left(\boldsymbol{\chi}\right)\Sigma^c. \tag{C.32}$$

Differentiating with respect to $\boldsymbol{\chi}$, we obtain:

$$d\mathbf{r} = \Sigma^\top d\mathbf{r} + \left(\Sigma^c - \Sigma^d\right)^\top \mathrm{Diag}\left(\boldsymbol{r}\right) d\boldsymbol{\chi},$$

so that

$$d\mathbf{r} = \left(1 - \Sigma^{\top}\right)^{-1} \left(\Sigma^{c} - \Sigma^{d}\right)^{\top} \mathrm{Diag}\left(\boldsymbol{r}\right) d\boldsymbol{\chi}.$$

Going from simple differential to log differentials gives (31).

To derive (32), we first note that in a steady-state where green technologies are always in use, (C.31) can be rewritten as

$$\ln \boldsymbol{\theta} = \boldsymbol{\vartheta} + \left(\Sigma^{d} - \Sigma^{c}\right) \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right). \tag{C.33}$$

Differentiating this expression, we get

$$d \ln \boldsymbol{\theta} = (\Sigma^{d} - \Sigma^{c}) d \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right). \tag{C.34}$$

Next, in a steady-state where green technologies are always in use, (C.29) can be rewritten as

$$\left(I - \Sigma\right) \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right) = -\boldsymbol{\chi} \odot \boldsymbol{\vartheta}. \tag{C.35}$$

Differentiating this expression, we obtain:

$$\begin{aligned} \left(I - \Sigma\right) d \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right) &= \mathrm{Diag}\left(d\boldsymbol{\chi}\right) \left(\left(\Sigma^{c} - \Sigma^{d}\right) \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right) - \boldsymbol{\vartheta}\right) \\ &= -\ln \boldsymbol{\theta} \odot d\boldsymbol{\chi} \end{aligned}$$

where the second line uses (C.33). We then get

$$d \left(\ln \mathbf{p} - \ln \mathbf{p}^{d}\right) = -\left(I - \Sigma\right)^{-1} \left(\ln \boldsymbol{\theta} \odot d\boldsymbol{\chi}\right) \tag{C.36}$$

Plugging that expression into (C.34), we get

$$d \ln \boldsymbol{\theta} = -(\Sigma^{d} - \Sigma^{c}) \left(I - \Sigma\right)^{-1} \left(\ln \boldsymbol{\theta} \odot d\boldsymbol{\chi}\right). \tag{C.37}$$

Further, note that $d \ln \left(1 - \theta_i^{-1}\right) = \frac{\theta_i^{-1}}{1 - \theta_i^{-1}} d \ln \theta_i$ so that (C.37) implies (32).

C.5 Stability of Steady-States

To assess the stability of a steady-state, we will consider the following dynamic system representation of our model:

$$\ln \boldsymbol{\chi}_t = \mathcal{H}\big(\ln \boldsymbol{\chi}_{t-1}\big),$$

where a steady-state is a fixed point of $\mathcal{H}$. Approximating around a steady-state $\boldsymbol{\chi}$, we have that

$$\ln \boldsymbol{\chi}_t - \ln \boldsymbol{\chi} \approx \frac{\partial \mathcal{H}\big(\ln \boldsymbol{\chi}\big)}{\partial \ln \boldsymbol{\chi}^{\top}} \big(\ln \boldsymbol{\chi}_{t-1} - \ln \boldsymbol{\chi}\big).$$

We then have the standard result that the steady-state is locally stable when the spectral radius of the Jacobian is below one. To characterize the Jacobian, we can differentiate the steady-state condition $\boldsymbol{\chi} = \mathbf{F}(\boldsymbol{\pi})$ to derive

$$\begin{aligned}
\frac{\partial \ln \boldsymbol{\chi_t}}{\partial \ln \boldsymbol{\chi_{t-1}}^\top} &= \mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\Big(\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi_{t-1}}^\top} + \frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi_t}^\top}\frac{\partial \ln \boldsymbol{\chi_t}}{\partial \ln \boldsymbol{\chi_{t-1}}^\top}\Big) \\
&= \Big(\mathbf{I} - \mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi_t}^\top}\Big)^{-1}\mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\frac{\partial \ln \boldsymbol{\pi}}{\partial \ln \boldsymbol{\chi_{t-1}}^\top} \\
&= \Big(\mathbf{I} - \mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\frac{\partial \ln \mathbf{r}}{\partial \ln \boldsymbol{\chi_t}^\top}\Big)^{-1}\mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\Big(\frac{\partial \ln(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})}{\partial \ln \boldsymbol{\chi_{t-1}}^\top} + \frac{\partial \ln \mathbf{r}}{\partial \ln \boldsymbol{\chi_{t-1}}^\top}\Big) \\
&= \Big(\mathbf{I} - \mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\mathrm{Diag}(\mathbf{r})^{-1}\left(I - \Sigma^\top\right)^{-1}\left(\mathrm{Diag}(\boldsymbol{\theta})^{-1}\Sigma^c\right)^\top \mathrm{Diag}(\boldsymbol{r} \odot \boldsymbol{\chi})\Big)^{-1} \\
&\quad \mathrm{Diag}(\boldsymbol{\epsilon}_\mathbf{F})\Big(\mathrm{Diag}(\boldsymbol{\theta} - 1)^{-1}(\Sigma^c - \Sigma^d)\left(I - \Sigma\right)^{-1}\mathrm{Diag}(\ln \boldsymbol{\theta}) \\
&\quad\quad + \mathrm{Diag}(\mathbf{r})^{-1}\left(I - \Sigma^\top\right)^{-1}\left(\mathrm{Diag}(\mathbf{1} - \boldsymbol{\theta}^{\odot(-1)})\Sigma^c - \Sigma^d\right)^\top \mathrm{Diag}(\boldsymbol{r})\Big)\mathrm{Diag}(\boldsymbol{\chi}),
\end{aligned}$$

which plugs in the multiplier formulas from Proposition 10 while also splitting the elasticity of revenue into effects from current and lagged greenification. Therefore, for a steady-state to be locally stable, all of the eigenvalues of this matrix need to be inside of the unit circle.

C.6 Social Planner Problem in the General Model

In this Appendix, we solve for the Planner's problem in the general model. We follow the same steps as in Section B.4. Given greenification levels, the social cost of producing one unit of consumption bundle (denoted $\hat{p}^C$) is equal to the price of the consumption bundle in a steady-state:

$$\ln p^C = \ln\left(\mathbf{p}\right)^\top \mathbf{b},$$

The vector of prices is given by (C.30) – knowing that in steady-state, the time indices drop and that we set $\tau_i = \xi^{CO_2} e_i$. Maximizing the consumption level again leads to $p^C c = 1$. Therefore, dropping constants, we can rewrite the maximization problem as

$$\max_{\{1 \geq \chi_i \geq \chi_{i0}\}} -\ln\left(\mathbf{p}\right)^\top \mathbf{b} - (1-\beta)\sum_i \mathcal{F}_i\left(\chi_i\right). \tag{C.38}$$

Naturally, a Planner would never want to pay to further greenify a sector unless the clean technology is used in the optimum (and brings net welfare gains). Therefore, we get either $\bar{\chi}_i = \chi_i$ or $\chi_i = \chi_{i0}$. For simplicity, we focus here on an interior equilibrium so that $\bar{\boldsymbol{\chi}} = \boldsymbol{\chi} > \boldsymbol{\chi}_0$, and we obtain the first order conditions

$$\mathcal{F}_i'(\chi_i) = -\frac{1}{1-\beta}\frac{\partial \ln(\mathbf{p})}{\partial \chi_i}^\top \mathbf{b}.$$

Using (C.36), we get

$$\frac{\partial \ln(\mathbf{p})}{\partial \chi_i} = -(I-\Sigma)^{-1}(\ln \boldsymbol{\theta} \odot \boldsymbol{\delta}_i), \tag{C.39}$$

where $\boldsymbol{\delta}_i$ is the vector with 0s in all entries except for a 1 in entry i. Using that $\mathcal{F}_i'(\chi_i)$ is a scalar, we then get:

$$\mathcal{F}_i'(\chi_i) = \frac{1}{1-\beta}(\boldsymbol{\delta}_i \odot \ln \boldsymbol{\theta})^\top \left(I-\Sigma^\top\right)^{-1}\mathbf{b}.$$

The shadow revenues $\mathbf{r}$ are still given by $\mathbf{r} = \left(I-\Sigma^\top\right)^{-1}\mathbf{b}$, so that

$$\mathcal{F}_i'(\chi_i) = \frac{1}{1-\beta}(\boldsymbol{\delta}_i \odot \ln \boldsymbol{\theta})^\top \mathbf{r} = \frac{r_i \ln \theta_i}{1-\beta}.$$

We then directly obtain:

$$\chi_i = F_i\left(\frac{r_i \ln \theta_i}{1-\beta}\right). \tag{C.40}$$

C.7 Incomplete Carbon Prices in a General Supply Chain

We have argued that downstream greenification is more effective at reducing emissions, so we will again examine the generality of this argument by considering a general supply chain network like the one described in Section 4. Note that the optimal policy results of Proposition 6 carry over to this more general environment as the deviation from Hulten's theorem is the same. The substantive difference will be in the equilibrium emission reductions: $-\frac{\partial E}{\partial \chi_i}$.

Total emissions in this more general setting are given by

$$E = \frac{1}{1+\tau}(\mathbf{e} \odot \boldsymbol{\alpha}^d \odot \boldsymbol{r})^\top(\mathbf{1}-\boldsymbol{\chi}).$$

That is, emissions $\times(1+\tau)$ equal equilibrium revenue for each sector multiplied by its dirty share. Using Equation (31), we have that

$$-\frac{\partial E}{\partial \boldsymbol{\chi}} = \frac{1}{1+\tau}\mathrm{Diag}(\mathbf{e} \odot \boldsymbol{\alpha}^d \odot \boldsymbol{r})\Big[\mathbf{1} - \left(\Sigma^c - \Sigma^d\right)(I-\Sigma)^{-1}(\mathbf{1}-\boldsymbol{\chi})\Big]. \tag{C.41}$$

Therefore, emissions reductions in the general supply chain take a very similar form to Equation (23), with terms that include both a direct reduction in emissions as well as an offsetting increase in emissions due to demand for the inputs of other sectors.

Downstream greenification will be more effective in reducing emissions under the same conditions where it is more effective in propagating greenification incentives: when greenifiable inputs are used primarily by green technology and greenification levels are low across the board. Under these conditions, the effectiveness of upstream greenification for reducing emissions is more "vulnerable" to low levels of greenification. In particular, let $n(j)$ denote the minimum number of steps through the network needed before the goods from sector j are sold directly to the consumer. That is, $n(j) = 0$ for any sector that supplies to the consumer ($b_j > 0$), $n(j) = 1$ for any sector that supplies to these one-step sectors (but not the consumer), and so on. In this more general setting, n provides us with a measure of a sector's upstreamness. If we again consider $\Sigma^d = 0$, $\chi_j = \epsilon x_j$ and take ϵ to zero, then the term in square brackets will converge to $\boldsymbol{\alpha}^c$. Moreover, the revenue of a sector will be $\mathcal{O}(\epsilon^{n(j)})$ because revenue will have to pass through $n(j)$-many steps before it reaches sector j. Thus, as in the vertical network case, we have that more upstream sectors are less effective at reducing emissions for low levels of greenification.

C.8 Proof of Proposition 11

Consider an economy with a set of greenification subsidies $\boldsymbol{q}_t$ which is constant at $\boldsymbol{q} \in [0,1)^N$ for t large enough. Because subsidies can be temporarily higher, we must now consider the full set of steady-states, which is characterized by any vector $\boldsymbol{\chi} \in [0,1]^N$ such that

$$\boldsymbol{\chi} \geq \mathbf{F}\left((\mathbf{1}-\boldsymbol{q})^{\odot(-1)} \odot \boldsymbol{\pi}(\boldsymbol{\chi})\right)$$

and $\boldsymbol{\pi}(\boldsymbol{\chi})$ still defined as in Proposition 9.

For a vector $\mathbf{x}$ (and a given $\boldsymbol{q}$), we can define the closest from above steady-state as the steady-state $\boldsymbol{\chi}^u \geq \mathbf{x}$ componentwise, such that for any other steady-state $\boldsymbol{\chi} \geq \mathbf{x}$ componentwise, then $\boldsymbol{\chi} \geq \boldsymbol{\chi}^u$. Note that the steady-state $\boldsymbol{\chi}^u$ exists because $\boldsymbol{\pi}(\boldsymbol{\chi})$ is weakly increasing in $\boldsymbol{\chi}$: If $\boldsymbol{\chi}^1$ and $\boldsymbol{\chi}^2$ are two steady-states with $\boldsymbol{\chi}^1, \boldsymbol{\chi}^2 \geq \mathbf{x}$, then we have that $\min(\boldsymbol{\chi}^1, \boldsymbol{\chi}^2) \geq \mathbf{x}$ componentwise (where the min operator operates component by component), $\chi_i^1 \geq F_i\left((1-q_i)^{-1}\pi_i(\boldsymbol{\chi}^{\mathbf{1}})\right) \geq F_i\left((1-q_i)^{-1}\pi_i(\min(\boldsymbol{\chi}^1, \boldsymbol{\chi}^2)\right)$ and similarly $\chi_i^2 \geq F_i\left((1-q_i)^{-1}\pi_i(\min(\boldsymbol{\chi}^1, \boldsymbol{\chi}^2)\right)$, so that $\min(\boldsymbol{\chi}^1, \boldsymbol{\chi}^2)$ is also a steady-state.

For a current state $\mathbf{x}$ and a constant subsidy vector $\boldsymbol{q}$, let $\mathcal{L}_{\boldsymbol{q}}(\mathbf{x})$ denote the limit of the dynamics starting from $\boldsymbol{\chi}_0 = \mathbf{x}$ when the subsidy vector is kept forever equal to $\boldsymbol{q}$. Under a constant $\boldsymbol{q}$,

$$\boldsymbol{\chi}_t = \max\left\{\boldsymbol{\chi}_{t-1}, \mathbf{F}\left((\mathbf{1}-\boldsymbol{q})^{\odot(-1)} \odot \boldsymbol{\pi}_t\right)\right\},$$

where we still have $\boldsymbol{\pi}_t = \left(\mathbf{1} - \boldsymbol{\theta}_t^{\odot(-1)}\right) \odot \mathbf{r}_t$ and $\boldsymbol{\theta}_t$ defined as in (28) and $\mathbf{r}_t$ defined as in (30). Recall that we have assumed that $\boldsymbol{\vartheta} \geq 0$ and $\Sigma^c \geq \Sigma^d$ so that green technologies are always in use when available. We first establish:

Lemma 7. *Under the assumptions of Proposition 11, for every constant subsidy vector $\boldsymbol{q} \in [0,1)^N$ and every $\mathbf{x} \in [0,1]^N$, the constant-$\boldsymbol{q}$ path starting from $\mathbf{x}$ converges to the closest to $\mathbf{x}$ from above steady-state. In addition, $\mathcal{L}_{\boldsymbol{q}}(\mathbf{x})$ is weakly increasing in $\mathbf{x}$ and in $\boldsymbol{q}$.*

Proof. Fix $\boldsymbol{q}$ and $\mathbf{x}$, and let $\{\boldsymbol{\chi}_t\}_{t\geq 0}$ be the constant-$\boldsymbol{q}$ path with $\boldsymbol{\chi}_0 = \mathbf{x}$. Take any $\boldsymbol{q}$-steady-state $\mathbf{s} \geq \mathbf{x}$. We first show that $\boldsymbol{\chi}_t \leq \mathbf{s}$ for all t. Suppose $\boldsymbol{\chi}_{t-1} \leq \mathbf{s}$. Denote $\Sigma(\boldsymbol{\chi}) \equiv \mathrm{Diag}(\mathbf{1} - \boldsymbol{\chi})\Sigma^d + \mathrm{Diag}(\boldsymbol{\chi})\Sigma^c$, then as $\Sigma^c \geq \Sigma^d$, we get $\Sigma(\boldsymbol{\chi}_{t-1}) \leq \Sigma(\mathbf{s})$, and the Neumann series implies $\big(I - \Sigma(\boldsymbol{\chi}_{t-1})\big)^{-1} \leq (I - \Sigma(\mathbf{s}))^{-1}$. Because $\boldsymbol{\chi}_{t-1} \leq \mathbf{s}$ also implies $\mathrm{Diag}(\boldsymbol{\chi}_{t-1}) \leq \mathrm{Diag}(\mathbf{s})$, and because $\Sigma^c - \Sigma^d \geq 0$ and $\boldsymbol{\vartheta} \geq 0$, we obtain using (28) that $\boldsymbol{\theta}_t \leq \boldsymbol{\theta}(\mathbf{s})$.

Next, we prove by backward induction on sectors that $\chi_{it} \leq s_i$ and $r_{it} \leq r_i(\mathbf{s})$. For the most downstream sector, $r_{Nt} = b_N = r_N(\mathbf{s})$, so $\pi_{Nt} \leq \pi_N(\mathbf{s})$ and therefore

$$\chi_{Nt} = \max\left\{\chi_{N,t-1}, F_N\left(\frac{\pi_{Nt}}{1-q_N}\right)\right\} \leq \max\left\{s_N, F_N\left(\frac{\pi_N(\mathbf{s})}{1-q_N}\right)\right\} \leq s_N,$$

because $\mathbf{s}$ is a $\boldsymbol{q}$-steady-state. Now suppose $\chi_{kt} \leq s_k$ and $r_{kt} \leq r_k(\mathbf{s})$ have already been shown for all $k > i$. Since $\tilde{\chi}_{kt} \leq \chi_{kt} \leq s_k$, we have

$$\tilde{\Sigma}_{ki,t} = (1-\chi_{kt})\sigma^d_{ki} + \tilde{\chi}_{kt}\sigma^c_{ki} \leq (1-s_k)\sigma^d_{ki} + s_k\sigma^c_{ki} = \Sigma_{ki}(\mathbf{s}),$$

where the inequality uses $\Sigma^c \geq \Sigma^d$. Using the recursive revenue equation,

$$r_{it} = b_i + \sum_{k>i} \tilde{\Sigma}_{ki,t} r_{kt} \leq b_i + \sum_{k>i} \Sigma_{ki}(\mathbf{s}) r_k(\mathbf{s}) = r_i(\mathbf{s}).$$

With $\boldsymbol{\theta}_t \leq \boldsymbol{\theta}(\mathbf{s})$, we obtain $\pi_{it} \leq \pi_i(\mathbf{s})$, so that

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i\left(\frac{\pi_{it}}{1-q_i}\right)\right\} \leq \max\left\{s_i, F_i\left(\frac{\pi_i(\mathbf{s})}{1-q_i}\right)\right\} = s_i.$$

This proves $\boldsymbol{\chi}_t \leq \mathbf{s}$ for all t.

The path $\{\boldsymbol{\chi}_t\}$ converges to a steady-state $\boldsymbol{\chi}$, so that we must also have $\boldsymbol{\chi} \leq \mathbf{s}$. Therefore, $\boldsymbol{\chi}_t$ converges toward the steady-state which is the closest to $\mathbf{x}$ from above.

If $\mathbf{x} \leq \mathbf{y}$, then the set of $\boldsymbol{q}$-steady-states above $\mathbf{y}$ is a subset of the set of $\boldsymbol{q}$-steady-states above $\mathbf{x}$, so their least elements satisfy $\mathcal{L}_{\boldsymbol{q}}(\mathbf{x}) \leq \mathcal{L}_{\boldsymbol{q}}(\mathbf{y})$.

Finally, if $\boldsymbol{q} \leq \boldsymbol{q}'$ (again componentwise), then every $\boldsymbol{q}'$-steady-state is also a $\boldsymbol{q}$-steady-state, because $(\mathbf{1} - \boldsymbol{q}')^{\odot(-1)} \geq (\mathbf{1} - \boldsymbol{q})^{\odot(-1)}$ and each F_i is increasing. Hence the set of $\boldsymbol{q}'$-weak steady-states above $\mathbf{x}$ is a subset of the set of $\boldsymbol{q}$-weak steady-states above $\mathbf{x}$, which implies $\mathcal{L}_{\boldsymbol{q}}(\mathbf{x}) \leq \mathcal{L}_{\boldsymbol{q}'}(\mathbf{x})$. □

We can now establish Proposition 11. Let $\boldsymbol{\chi}_T$ denote the state at the date from which the subsidy sequence becomes constant, so that $\boldsymbol{q}_t = \boldsymbol{q}$ for all $t > T$. Since greenification is irreversible, $\boldsymbol{\chi}_T \geq \boldsymbol{\chi}_0$. From date $T+1$ onward, the economy follows the constant-$\boldsymbol{q}$ dynamics, so the long-run outcome with the policy sequence $\{\boldsymbol{q}_t\}_{t\geq 1}$ is

$$\boldsymbol{\chi}^B = \mathcal{L}_{\boldsymbol{q}}(\boldsymbol{\chi}_T),$$

whereas the laissez-faire long-run outcome from the original initial condition is

$$\boldsymbol{\chi}^A = \mathcal{L}_{\mathbf{0}}(\boldsymbol{\chi}_0).$$

Lemma 7 therefore gives

$$\boldsymbol{\chi}^B = \mathcal{L}_{\boldsymbol{q}}(\boldsymbol{\chi}_T) \geq \mathcal{L}_{\boldsymbol{q}}(\boldsymbol{\chi}_0) \geq \mathcal{L}_{\mathbf{0}}(\boldsymbol{\chi}_0) = \boldsymbol{\chi}^A,$$

as stipulated in Proposition 11. In steady-state there is no further greenification (so that no subsidies are paid), and the utility flow is weakly increasing in $\boldsymbol{\chi}$: therefore industrial policy cannot decrease the long-run utility flow.

C.9 Backfiring Industrial Policy

Here we build an example where an industrial policy initially focused on the upstream sector 2 backfires: given initial conditions and the greenification cost functions, the laissez-faire equilibrium is associated with full greenification in sectors 3 and 1, whereas a (misguided) industrial policy focusing on sector 2 in the initial period reduces long-run welfare by preventing full greenification in sectors 3 and 1. Key to our example is the fact that greenification in sector 2 reduces the cost-advantage of greenified varieties in sector 3, which reduces the incentives to greenify both sector 3 and its upstream sector, 1.

Before developing the example, we note that in Section C.6, we derived that in a steady-state when $\tau_i = \xi_i$, the utility flow is given by $-\ln(\mathbf{p})^\top.\mathbf{b}$ up to a constant. In our example, we can use (27) in steady-state (since green technologies are still used whenever they are available) and we obtain:

$$-\ln(\mathbf{p})^\top.\mathbf{b} = -\ln p_3 = (\chi_3\alpha + \chi_3(1-\alpha)\chi_1 + (1-\chi_3)(1-\alpha)\chi_2)Z - \ln(1+\xi). \quad \text{(C.42)}$$

Example 2. We assume that the economy initially features no greenification in all three sectors ($\chi_{i,0} = 0$ for $i \in \{1,2,3\}$). We consider two potential policies: In the first one, the government does not subsidize greenification. In the second one, the government subsidizes greenification in sector 2 at time $t=1$, such that $\chi_{2,1} = 1$. In both cases, the government implements a Pigouvian carbon tax: $\tau = \xi$.

Using (34), we can write profits as

$$\begin{pmatrix} \pi_{1t} \\ \pi_{2t} \\ \pi_{3t} \end{pmatrix} = \begin{pmatrix} \left(1-e^{-Z}\right)\widetilde{\chi}_{3,t}\left(1-\alpha\right) \\ \left(1-e^{-Z}\right)\left(1-\chi_{3,t}\right)\left(1-\alpha\right) \\ 1-e^{-(\alpha+(1-\alpha)(\chi_{1,t-1}-\chi_{2,t-1}))Z} \end{pmatrix}. \tag{C.43}$$

Consider first the case with no government intervention. Then using (C.43) greenification at time 1 in sector 3 is given by $\chi_{3,1} = F_3\left(1-e^{-Z\alpha}\right)$. In addition, since $\tilde{\chi}_{it} = \chi_{i,t-1} + (\chi_{it} - \chi_{i,t-1})/\theta_{it}$, we have$\widetilde{\chi}_{3,1} = \chi_{3,1}e^{-Z\alpha}$. Therefore, $\chi_{1,1}$ satisfies

$$\chi_{1,1} = F_1\left(\left(1-e^{-Z}\right)\chi_{3,1}e^{-Z\alpha}\left(1-\alpha\right)\right).$$

We assume that this is positive, i.e. the smallest fixed cost of greenification in sector 1 lies below $\left(1-e^{-Z}\right)\chi_{3,1}e^{-Z\alpha}\left(1-\alpha\right)$. For sector 2, we then get

$$\chi_{2,1} = F_2\left(\left(1-e^{-Z}\right)\left(1-\chi_{3,1}\right)\left(1-\alpha\right)\right),$$

which we assume to be equal to zero. This in turn will be the case if the smallest fixed cost of greenification in sector 2 is greater than $\left(1-e^{-Z}\right)\left(1-\alpha\right)$.

We now consider period $t=2$. Since $\chi_{3,t}$ is non-decreasing, the incentives to greenify sector 2 are weakly smaller, so that we still have $\chi_{2,2} = 0$. Using (C.43), we get that greenification in sector 3 is given by $\chi_{3,2} = F_3\left(1-e^{-(\alpha+(1-\alpha)\chi_{1,1})Z}\right)$, which we take to be equal to one. That is, all greenification costs in sector 3 are below $1-e^{-(\alpha+(1-\alpha)\chi_{1,1})Z}$. We now get $\widetilde{\chi}_{3,2} = \chi_{3,1} + \left(1-\chi_{3,1}\right)e^{-(\alpha+(1-\alpha)\chi_{1,1})Z}$ so that, using (C.43), $\chi_{1,2}$ satisfies

$$\chi_{1,2} = F_1\left(\left(1-e^{-Z}\right)\left(\chi_{3,1} + \left(1-\chi_{3,1}\right)e^{-(\alpha+(1-\alpha)\chi_{1,1})Z}\right)\left(1-\alpha\right)\right).$$

Again, we take this to be equal to one: namely, all greenification costs in sector 1 are below the term at which F_1 is evaluated in the expression above. With full greenification in sectors 1 and 3, the economy has reached a steady-state with $\chi_2^* = 0$ and $\chi_1^* = \chi_3^* = 1$. Not greenifying sector 2 in this context comes at no cost, since the input from that sector is used only for dirty production in sector 3 which disappears from $t=3$ onward. Using (C.42), the corresponding utility flow is (up to a constant) given by:

$$-\ln p_3 = z.$$

Now, suppose instead that starting from no greenification in all sectors, the government decides to fully greenify the upstream sector 2 at time $t=1$, i.e. sets $\chi_{2,1}^{\dagger} = 1$ (we add † to denote variables under this alternative scenario). Since greenification incentives only move downstream with a lag, this does not change $\chi_{1,1}$ and $\chi_{3,1}$, which remain the same as without the policy:

$\chi^{\dagger}_{1,1} = \chi_{1,1}$ and $\chi^{\dagger}_{3,1} = \chi_{3,1}$.

Consider now time $t = 2$. Using (C.43), we get that $\pi_{3,2} = 1 - e^{-(\alpha+(1-\alpha)(\chi_{1,1}-1))Z} < 1 - e^{-\alpha Z} = \pi_{3,1}$, so that there is no further greenification in sector 3 namely: $\chi^{\dagger}_{3,2} = \widetilde{\chi}^{\dagger}_{3,2} = \chi^{\dagger}_{3,1} = \chi_{3,1}$. In sector 1, using (C.43), we get

$$\chi^{\dagger}_{1,2} = F_1\left(\left(1 - e^{-Z}\right)\chi_{3,1}\left(1-\alpha\right)\right).$$

We assume that this is still equal to $\chi_{1,1}$: That is, the distribution of greenification costs in sector 1 is such that a positive mass of varieties have fixed costs below $\left(1 - e^{-Z}\right)\chi_{3,1}e^{-Z\alpha}\left(1-\alpha\right)$, no varieties have fixed costs in the interval $\left(1 - e^{-Z}\right)\left(1-\alpha\right)\times\left(\chi_{3,1}e^{-Z\alpha}, \chi_{3,1}\right)$, and all remaining varieties have their fixed costs in the interval $\left(1 - e^{-Z}\right)\left(1-\alpha\right)\times\left(\chi_{3,1}, \chi_{3,1} + \left(1-\chi_{3,1}\right)e^{-(\alpha+(1-\alpha)\chi_{1,1})Z}\right)$. At this point, the economy has reached a steady-state and no further greenification occurs. This gives a utility flow:

$$-\ln p_3 = \left[\chi_{3,1}\alpha + \chi_{3,1}\left(1-\alpha\right)\chi_{1,1} + \left(1-\chi_{3,1}\right)\left(1-\alpha\right)\right]Z - \ln\left(1+\xi\right)$$

For $\chi_{3,1}$ sufficiently small, which is clearly feasible, this is strictly less than the utility flow z under laissez-faire. That is, industrial policy backfires.

C.10 Excessive Greenification and Path Dependence

We now build an example where, given initial conditions and the greenification cost functions, the laissez-faire allocation involves positive greenification in sector 2, but no greenification in sectors 3 and 1, whereas the optimal allocation involves less greenification in sector 2, but positive greenification in sectors 3 and 1. In other words, the laissez-faire economy exhibits excessive greenification in sector 2.

Example 3. We still assume that a Pigouvian carbon tax is in place. We choose the cost functions so that there exists a steady-state $(\chi^*_1, \chi^*_2, \chi^*_3)$ with $\chi^*_2 > 0$ and $\chi^*_1 = \chi^*_3 = 0$:

$$\chi^*_2 = F_2\left(\left(1 - e^{-Z}\right)\left(1-\alpha\right)\right) \text{ and } \chi^*_3 = 0 = F_3\left(1 - e^{-Z\left(\alpha-(1-\alpha)\chi^*_2\right)}\right).$$

The latter condition requires that the lowest fixed cost in sector 3 be above $1 - e^{-Z\left(\alpha-(1-\alpha)\chi^*_2\right)}$.

Suppose now that the economy starts with greenification shares $\chi_{2,0} = \chi^*_2 - \varepsilon$ (with $\varepsilon > 0$ but small) and $\chi_{1,0} = \chi_{3,0} = 0$. Then, we get that $\chi_{2,1}$ and $\chi_{3,1}$ solve

$$\chi_{2,1} = F_2\left(\left(1 - e^{-Z}\right)\left(1-\alpha\right)\right) = \chi^*_2 \text{ and } \chi_{3,1} = F_2\left(1 - e^{-Z(\alpha-(1-\alpha)\chi_{2,0})}\right) = 0$$

whenever ε is sufficiently small and the smallest fixed greenification cost in sector 3 lies significantly above $1 - e^{-Z\left(\alpha-(1-\alpha)\chi^*_2\right)}$ relative to ε. Then, the laissez-faire economy will reach the steady-state $(0, \chi_2^*, 0)$.

Let us compare this laissez-faire equilibrium with the social optimum. We note first that the Social Planner always wants to implement the socially optimal steady-state immediately. Then, provided that greenification costs are bounded above, a sufficiently patient Social Planner will seek to maximize steady-state utility flow given by (C.42), which is maximized with $\chi_1 = \chi_3 = 1$ regardless of χ_2. Therefore, the Social Planner immediately sets greenification to $(1, \chi_{2,0}, 1)$. The corresponding χ_2 is lower than under laissez-faire: there is excessive greenification of 2 in laissez-faire compared to the social optimum.

D Details on the Quantification

D.1 Calibrated Model

We briefly present the model that we calibrate in Section 5. We consider the set-up of Section 4.1 with heterogeneous z_i, e_i and TFP parameters A_i, but with only 2 sectors, $N = 2$, in a vertical chain where only the clean production process of sector 2 uses the upstream sector 1:

$$\begin{aligned} y_{1t}(\nu) &= A_1 \left[\ell_{d1t}(\nu) + \gamma_{1t}(\nu) e^{z_1} \ell_{c1t}(\nu)\right] \\ \text{and } y_{2t}(\nu) &= A_2 \left[\ell_{d2t}(\nu) + \gamma_{2t}(\nu) \left(\frac{e^{z_2}\ell_{c2t}(\nu)}{\alpha_2}\right)^{\alpha_2} \left(\frac{m_{2t}(\nu)}{1-\alpha_2}\right)^{1-\alpha_2}\right] \end{aligned}$$

Utility is still given by (1) but we relax the normalizations $\kappa = 1$ and $w_t = 1$ to interpret units more easily. Our equilibrium derivation still applies, of course, but with prices relative to wages. With e_i the emission rate per unit of dirty labor (e_i/A_i is emissions per unit of output) and τ^{CO_2} the carbon price per ton of CO_2, the relevant tax policy parameter is $\tilde{\tau}_{it} = e_i \tau^{CO_2}/w_t$ (scaled by wages to simplify the expressions).

We then obtain the price index in each sector as

$$\frac{p_{1t}}{w_t} = \frac{1+\tilde{\tau}_{1t}}{A_1} \min\left\{\left(\frac{e^{-z_1}}{1+\tilde{\tau}_{1t}}\right)^{\chi_{1,t-1}}, 1\right\}, \tag{D.1}$$

$$\frac{p_{2t}}{w_t} = \frac{1+\tilde{\tau}_{2t}}{A_2} \min\left\{\left(\frac{e^{-\alpha_2 z_2}}{1+\tilde{\tau}_{2t}}\left(\frac{p_{1,t}}{w_t}\right)^{1-\alpha_2}\right)^{\chi_{2,t-1}}, 1\right\}. \tag{D.2}$$

Mark-ups can be written as

$$\theta_{1t} = \max\left\{e^{z_1}(1+\tilde{\tau}_{1t}), 1\right\} \text{ and } \theta_{2t} = \max\left\{\frac{e^{\alpha_2 z_2}(1+\tilde{\tau}_{2t})}{(p_{1,t}/w_t)^{1-\alpha_2}}, 1\right\}.$$

Revenues in sector 2 are $r_{2t} = E_t$ where E_t are total expenditures in the sector, revenues in sector

1 are $r_1 = E_t \widetilde{\chi}_{2t} (1 - \alpha_2)$, as long as the green technology is used downstream (and 0 otherwise). Profits still obey $\pi_{it} = \left(1 - \theta_{it}^{-1}\right) r_{it}$ of course, so that with a set of greenification subsidies $\{q_{it}\}$, we get the law of motion for χ_{it} as $\chi_{it} = \max\left\{F_i\left(\frac{\pi_{it}}{1-q_{it}}\right), \chi_{i,t-1}\right\}$.

We obtain output in each sector from $y_{it} = r_{it}/p_{it}$, and using (C.11), we get dirty and clean labor in each sector as:

$$\ell_{dit} = \frac{1 - \hat{\chi}_{it}}{w_t (1 + \tilde{\tau}_{it})} r_{it}, \quad \ell_{cit} = \frac{\widetilde{\chi}_{i,t}}{w_t} r_{i,t} \alpha_i,$$

with $\hat{\chi}_{it} = \chi_{it}$ when clean technologies are in use (which is the case in our computation) and 0 otherwise. Finally, disutility from emissions in sector i is given by

$$a_{it} = \xi^{CO_2} e_i \frac{1 - \hat{\chi}_{it}}{w_t (1 + \tilde{\tau}_{it})} r_{it},$$

where the emissions disutility parameter ξ^{CO_2} ensures that the household's willingness to pay to avoid another ton of CO_2 emissions equals the social cost of carbon (SCC) via $\xi^{CO_2} = \frac{\kappa}{w} SCC$. Total emissions disutility is then given by $a = a_1 + a_2$.

D.2 Calibration Details

Initial Wages, Prices, Productivities, and Emissions: The initial wage w_0 equals the value-added per worker-year in LRHD transportation services in 2022 (= \$362,220 based on initial output of \$833.1 billion (BTS 2024) and employment of 2.3 million worker-years (BLS 2024 counting air, rail, water, and truck transportation (NAICS 481-484)).[D.1] The initial price of transportation services $p_{20} = \$0.23$ is calculated as a Cobb-Douglas price index over the revenue-share weighted average freight price per ton-mile of \$0.26 and the revenue-share weighted average passenger price per passenger-mile of \$0.13 in 2022 (both calculated for air, truck, railroad, and water transport from BTS Tables 1-50, 3-21, 1-40, and 3-20), where the expenditure share of freight (vs. passenger) miles is set to 0.81 in line with its value for 2022 in the data. Transportation TFP A_2=1.5914×10^6 is calculated from (D.2), where we set $\tau_{2,0}$ based on the average effective carbon price in the United States in 2021 (\$13/tCO_2, OECD 2023) and the sector's emissions intensity (0.0008 tCO_2e/\$ calculated from total emissions data from EPA 2024 and output data from BTS 2024) and assume that $\chi_{20} = 0$. The initial price of delivered GH_2 is set to $p_{10} = \$4.65$/kg based on the current (2023) estimated US fossil hydrogen production cost of \$1.45/kg from BloombergNEF (2023) plus current fossil (truck) distribution costs of \$3.20/kg estimated using the Argonne National Labo-

[D.1] We include in "heavy duty" some trucks that would technically be categorized as "medium duty" under the US Department of Transportation classification based on gross vehicle weight ratings.

ratory's HDSAM (v4.5, Elgowainy et al. 2024, as described further below) and converted to \$2022 (using US GDP deflator data from FRED). Hydrogen TFP $A_1 = 8.8779 \times 10^4$ is calculated from (D.1) analogously to the calculation of A_2 but with $\tau_{1,0}$ based on an estimated emissions intensity of 10.75 $kgCO_2/kgH_2$ with 9 $kgCO_2/kgH_2$ from natural gas-based hydrogen production (median value from IEA 2023) plus 1.75 $kgCO_2/kgH_2$ from distribution (from the HDSAM). Here we also assume that $\chi_{1,0} \approx 0$ based on estimates from the IEA (2024).

Transportation Parameters α_2 and z_2: First, the studies used to calibrate α_2 (described in Section 5.1) include Ahluwalia et al. (2021, 2020), Hoelzen et al. (2023), Ledna et al. (2024), Gilleon et al. (2022), Burnham et al. (2021), Hunter et al. (2021), and Steer (2023). Second, for each study i, we infer the implied value of $z_{2,i}$ from each paired estimate of the levelized costs of producing a transportation service with fossil vs. hydrogen fuels via the associated ratio of clean vs. dirty marginal costs:

$$\frac{mc_{c,i}}{mc_{d,i}} = \frac{(w_0)^{\alpha_2} e^{-\alpha_2 z_{2,i}} (p_{1,i})^{1-\alpha_2} / A_2}{w_0(1+\widetilde{\tau}_{2,i})/A_2},$$

with $p_{1,i}$ based on the hydrogen input price assumed in study i (with adjustments where necessary to reflect the cost of *distributed* and not dispensed hydrogen as explained below), $\widetilde{\tau}_{2,i}$ based on the tax rates assumed, and the other parameters are set as described above. We then average across $z_{2,i}$'s for each variety and compute the value added-share weighted average across varieties.

Aviation Fixed Costs: For aircraft development costs, we first take an industry estimate of \$16.4 billion (Steer 2023) for one H_2 narrow-body short- to medium-range (NB/SMR) aircraft similar to the Airbus A320 and scale it up based on typical NB/SMR aircraft family expenditure shares to yield an overall cost of \$55 billion (\$2022).[D.2] For the wide-body long-range (WB/LR) market, development costs are generally expected to be higher as the lower volumetric energy density of hydrogen necessitates additional changes in aircraft designs to accommodate increased fuel storage requirements (ICAO 2022a). We thus adjust our per-aircraft family development cost estimate upwards based on estimated hydrogen-gas turbines and blended-wing-body aircraft development costs (from Ballesteros et al. 2022) to yield a per-aircraft family cost of \$21.8 billion and scale this figure up based on typical aircraft family market shares for WB/LR aircraft, which

[D.2] Using US BTS Air Carrier Financial (Schedule P-5.2) operating expenditure data—which include both passenger and cargo operations—for 2023 we calculate an NB/SMR market expenditure share of 25.2% for the A320 family (including both A318 and A319 aircraft but not the A321/LR due to its distinctly larger size and range). Given Airbus' smaller market share in the United States, we also consider the Boeing 737 (again excluding longer range variants such as the 737-900ER) for which the corresponding expenditure share is 45.6%. We thus assume that the \$16.4 billion development costs cover 30% of the NB/SMR market.

we take to be 15%,[D.3] yielding \$145 billion in overall WB/LR aircraft development costs. For airport infrastructure adjustments, we assume a cost increase of 9.1% per revenue passenger kilometer during the adjustment period, in line with other estimates.[D.4]

Other Transportation Fixed Costs: For trucking, rail, and water–passenger, we quantify the excess costs of initial clean technology adoption based on pairs of LCO estimates for current vs. future hydrogen-fueled transportation within each variety. For refueling infrastructure, we infer (capital) cost shares from Bracci et al. (2024) and Reddi et al. (2017) and apply these to the relevant estimates of *dispensed* H_2 costs underlying each LCO estimate from the literature. For water-freight, we lack sufficient data to quantify fixed costs analogously to the other varieties. Given the challenges associated with the use of H_2 as a fuel in long-distance and cargo shipping (EMSA 2023; Ahluwalia et al. 2020), we thus set the fixed costs for this variety as double the fixed costs of passenger water travel (ferries).

Hydrogen Productivity Parameter z_1: We again construct pairs of levelized cost estimates i from fossil vs. clean production to infer $z_{1,i}$ from the corresponding clean vs. dirty marginal cost ratios (via $\frac{mc_{c,i}}{mc_{d,i}} = e^{-z_{1,i}}/(1+\widetilde{\tau}_{1,i})$). For H_2 production, we consult BloombergNEF (2023) estimates for US hydrogen production using SMR today vs. hydrogen in the future (2050). For H_2 distribution, we run the Argonne HDSAM model (v 4.5) for the Medium/Heavy-Duty market assuming either tube-Trailer or pipeline distribution for each region.[D.5] We then average the resulting $z_{1,i}$ estimates for distribution across regions and compute a weighted average value across production and distribution based on the current cost share of each in the levelized cost of (fossil) H_2, yielding $z_1 = 1.608$.

Hydrogen Sector Fixed Costs: First, we calculate the fixed costs $\phi_1(.)$ for *production* based

[D.3] BTS data suggest WB/LR expenditure shares range from, e.g., 9.2% for older models such as the B757 to 18.1% for the B777 family.

[D.4] Hoelzen et al. (2023) estimate that airport refueling system adjustment and liquefaction plant costs will add around \$0.48 to the levelized cost per kg of dispensed liquid hydrogen (LH_2) at a typical airport. We combine this estimate with projected future hydrogen aircraft fuel economy (ICCT 2022) and reference aircraft operating costs (Hoelzen et al. 2022) to calculate the corresponding cost increase per passenger kilometer. For comparison, Steer (2023) projects an 11% future increase in airport charges in a typical flight due to the investment costs required to accommodate hydrogen aircraft.

[D.5] We assume a typical transmission distance of 640 km, in line with, e.g., Tayarani and Ramji (2022) and in the central range of the broader H_2 transmission literature. For example, Frank et al. (2021) consider 50, 550, and 1500 km; diLullo et al. (2022) consider 100-3000 km, and Demir et al. (2018) consider 100 km. We also assume a typical throughput of 150 tonnes/day and leave the remaining parameters at their HDSAM benchmarks. To isolate the levelized cost of H_2 transmission, we subtract from the HDSAM's resulting cost estimates (in \$/kgH_2) the costs associated with refueling stations (accounted for in downstream fixed costs) and the capital cost component of the pipelines (included in the upstream fixed costs).

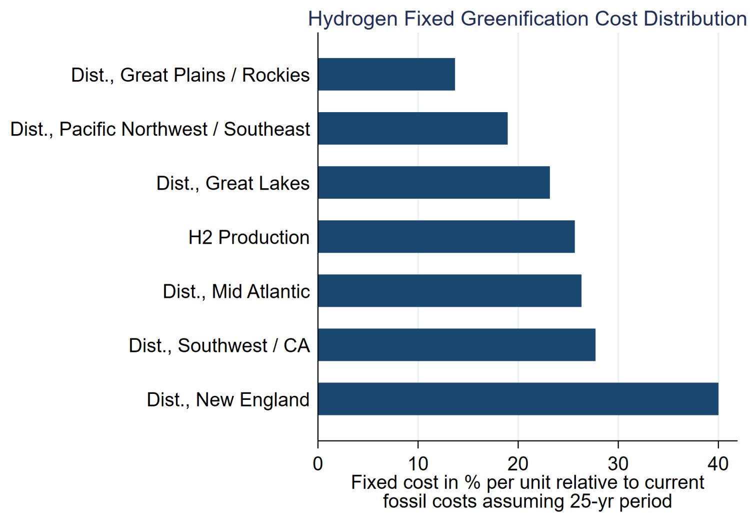


Figure D.1. Distribution of fixed greenification costs in the upstream sector

on *excess initial* current costs relative to future (2050) costs. We adjust the raw projected cost difference in three ways: First, we remove projected cost declines due to improvements in renewable electricity generation (which we take to occur outside the model), and second we account for the exponential nature of the cost declines projected by BloombergNEF over the next 25 years. Second, for distribution, we take the capital costs (per kg H_2) for pipeline construction in each region from the HDSAM model.[D.6] Third, we adjust the magnitude of the fixed costs downward based on the projected share of transportation in future hydrogen demand (which we take to be around 40% based on IEA 2023) to account in reduced form for other downstream sectors which demand H_2. Figure D.1 showcases the estimated distribution of fixed greenification costs (in percent relative to current per unit fossil costs) in the hydrogen production and distribution sector. Fixed pipeline distribution costs are lowest in the Great Plains and highest in New England due to, e.g., the relatively flat vs. rocky nature of the terrain.

Robustness: Our preferred quantification re-scales fixed costs to account for additional potential revenue sources outside our model—such as non-US demand for H_2 aircraft—that could, in reality, help offset some of the fixed technology development costs in our calibration. We now discuss the robustness of our results to this assumption. First, for transportation services, removing the scaling factor—that is, assuming that 100% of the fixed technology development

[D.6] We assume greenfield hydrogen pipeline development. Our quantification could, however, also accommodate the possibility of retrofitting existing natural gas pipelines.

costs must be compensated by US domestic demand for H_2 transportation—does not change the results meaningfully. Figure D.2 presents a version of Figure 4.b with the re-scaling factor removed, which reveals a very similar pattern of multiplicity in steady-states. It also remains the case that a temporary downstream technology development subsidy can lead to sustained and high levels of greenification, whereas the same level of upstream subsidy cannot. While the *level* of the subsidy is higher with the re-scaling factor removed (increasing from 50% to 65% to achieve greenification levels of 89% and 95% in Sectors 1 and 2, respectively), the relative effectiveness of downstream vs. upstream targeting is unchanged.

Second, we consider alternative scaling for fixed costs in hydrogen production and distribution. As there are already other sources of demand for H_2 in the US economy, such as from ammonia fertilizer production, we first consider a change from 40% to 60% as the implied share of clean H_2 technology development costs that must be borne by the LRHD transportation sector. Here we again find (i) multiplicity of steady-states as shown in Figure D.2 and (ii) that a temporary downstream subsidy is more effective at inducing greenification (with, e.g., a 65% two-period subsidy inducing greenification shares of 78% upstream and 99% downstream) than an equivalent upstream subsidy (which achieves no greenification). Finally, for a hypothetical 100% cost share from transportation (no H2 fixed cost rescaling), we find that, at a carbon price of \$300/tCO_2, (0,0) is now the only steady-state. However, in the dynamic analysis, we again find that a downstream fixed cost subsidy is more effective (with a 75% two-period subsidy leading to greenification shares of 11% upstream and 99% downstream) than an equivalently sized upstream subsidy (which achieves no greenification), showcasing the robustness of this result.

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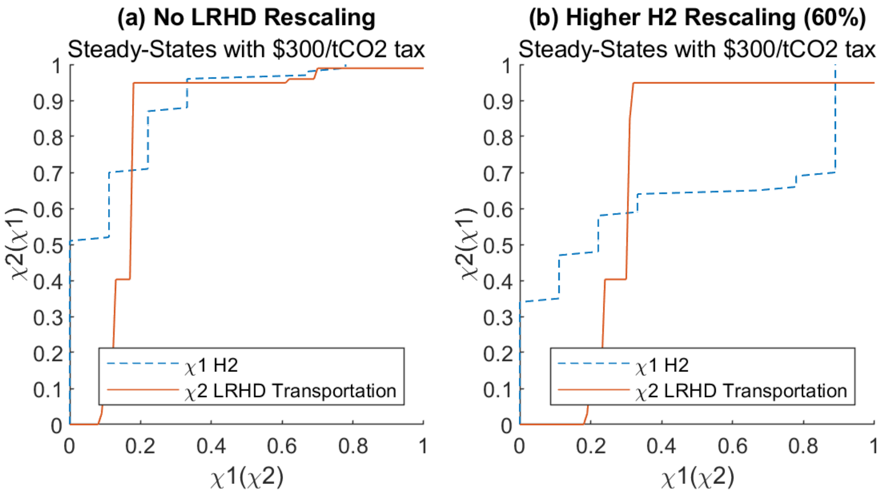

Figure D.2. Steady-state greenification shares (a) without re-scaling of LRHD Transportation fixed costs to account for global demand and (b) with less re-scaling of H_2 fixed costs (60% instead of 40%) to account for other H_2 demand

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